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Thermal modeling of continuous-wave end-pumped solid-state lasers

M. E. Innocenzi, H. T. Yura, C. L. Fincher, and R. A. Fields
Electronics Research Laboratory, The Aerospace Corporation, P. O. Box 92957, Los Angeles, California 90009

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In order to estimate deleterious effects caused by heating in continuous-wave end-pumped solid-state lasers, the heat equation has been solved for an axially heated cylinder with a thermally conductive boundary at the periphery. Steady-state thermal profiles are developed using both a full numerical solution and an analytic approximation which assumes only radial heat flow. The analytic solution, which is in good agreement with the numerical solution, is utilized to obtain an expression for the thermal focusing due to temperature-induced refractive index changes. For Nd:YAG, 1 W of pump power deposited as heat is predicted to cause a thermal focusing length comparable to the cavity length of a typical diode end-pumped laser.

In endeavoring to obtain higher output powers from continuous-wave (cw) end-pumped solid-state lasers, one must consider thermal effects that will impact optical performance. One important effect, thermal lensing, results from temperature-induced changes in the refractive index of the gain medium. In order to model the thermally induced index changes, the heat equation must be solved for the laser medium to obtain an appropriate temperature distribution. To this end, Farrukh *et al.*[1] presented a generalized analytical solution to the heat equation and obtained the temperature distributions in cylindrical end-pumped laser rods for both cw and pulsed pumping. Farrukh's solution was obtained for a convective boundary condition at the periphery. However, in many end-pumped applications, the periphery of the laser crystal is not convectively cooled; instead, the crystal boundary is in contact with a thermal sink of fixed temperature (conductive boundary condition). In this work, an approximate analytic solution (assuming radial heat flow) and a full numerical solution to the heat equation are developed for cw, axially symmetric, longitudinal pumping of a cylindrically symmetric solid-state laser rod with conductive boundary conditions. It is shown that the analytic solution is in good agreement with the full numerical solution.

The arrangement to which the present thermal modeling is applicable consists of a solid-state laser rod longitudinally pumped by a second cw laser. The periphery of the laser crystal is held at constant temperature by an actively cooled heat sink. Figure 1 shows a side and end view of a laser rod and heat sink. In the steady state

$$\nabla\cdot h(r,z) = Q(r,z), \tag{1}$$

where h is the heat flux, and $Q(r,z) = dP(r,z)/dV$ is the power per unit volume deposited as heat in the laser crystal. The heat flux is related to the corresponding temperature distribution within the crystal by

$$\mathrm{h}(r,z) = -K_c \nabla T(r,z), \tag{2}$$

where K_c is the thermal conductivity of the laser material.

Consider the case where the thermal conductivity of the heat sink is much greater than the thermal conductivity of the laser crystal. For example, the thermal conductivity of Cu, a common heat sink material, is a factor of 30 larger than that of yttrium aluminum garnet (YAG), a common solid-state laser material. If it is assumed that the heat flow is essentially radial, Eq. (1) can be integrated over a crystal volume bounded by a Gaussian surface of radius r and infinitesimal thickness Δz. This yields

$$2\pi r \Delta z h = \int_z^{z+\Delta z} \int_0^r \frac{dP(r',z')}{dV} 2\pi r' dr' dz'. \tag{3}$$

Now

$$\frac{dP(r,z)}{dV} = \alpha I_h(r,z), \tag{4}$$

where α is the absorption coefficient of the material and $I_h(r,z)$ is the intensity of the incident pump light that results in heating of the crystal. Here, it is assumed that

$$I_h(r,z) = I_{oh} \exp(-2r^2/w_p^2)\exp(-\alpha z). \tag{5}$$

In Eq. (5), I_{oh} is the incident heat irradiance on axis and w_p is the $1/e^2$ Gaussian radius of the pump beam. Substituting Eqs. (4) and (5) into Eq. (3) and performing the integration yields

$$h(r,z) = \left(\frac{\alpha P_{\mathrm{ph}}}{2\pi}\right) \exp(-\alpha z) \left(\frac{1-\exp(-2r^2/w_p^2)}{r}\right), \tag{6}$$

where $P_{\mathrm{ph}} = \pi w_p I_{oh}/2$ is the fraction of pump power that results in heating. Substituting Eq. (6) into Eq. (2) and integrating to the crystal boundary, r_b, the final expression for the steady-state temperature difference is obtained as

$$\Delta T(r,z) = \frac{\alpha P_{\mathrm{ph}} \exp(-\alpha z)}{4\pi K_c} \times \left[\ln\left(\frac{r_b^2}{r^2}\right) + E_1\left(\frac{2r_b^2}{w_p^2}\right) - E_1\left(\frac{2r^2}{w_p^2}\right)\right], \tag{7}$$

where $\Delta T(r,z) = T(r,z) - T(r_b,z)$ and E_1 is the exponential integral function.[2] In most cases, $E_1(2r_b^2/w_p^2)$ is small and can be omitted. It is noted that Eq. (7) is identical to the solution obtained by solving the cylindrically symmetric heat equation

$$\nabla_r^2 T + \nabla_z^2 T = -Q/K_c, \tag{8}$$

where one neglects the longitudinal (i.e., z) derivatives with respect to the corresponding radial derivatives.

In order to assess the assumption of radial heat flow, a finite difference numerical solution has been obtained for

09 September 2026 13:37:29

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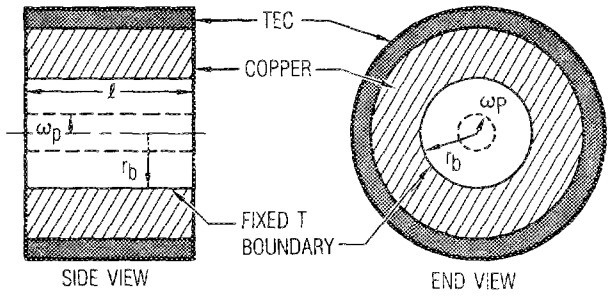


FIG. 1. Side view and end view of an applicable laser rod and thermoelectrically cooled (TEC) heat sink. The length of the rod is ℓ, the rod radius is r_b, and the $1/e^2$ radius of the Gaussian pump beam is w_p.

Eq. (8) by defining a matrix of nodal points within the crystal. These nodal points are connected by thermal conductances which are a linear function of the thermal conductivity of the material. The power input to a given node is expressed as

$$P_{\text{node}} = \frac{2\alpha P_{\text{ph}} V_{\text{node}}}{\pi w_p^2} \exp\left(\frac{-2r^2}{w_p^2}\right) \exp(-\alpha z), \qquad (9)$$

where V_{node} is the volume associated with the node. The solution is iterated until a strict energy balance criterion is met (the energy flowing into each node is balanced by the energy flowing out.) The actual numerical code employed to solve Eq. (8) is part of the SINDA thermal analysis package.[3]

At the end surfaces of the cylinder, no fixed boundary temperature is defined; cooling is facilitated by free convection to air and by radiation. Convection effects were included in the model, but radiation effects were negligible for the temperature excursions considered here. The numerical solution is in good agreement with the analytic solution for all r and z.

As an example of the agreement between the analytic and numerical solutions, Fig. 2 shows a comparison of temperature profiles calculated using both methods for the crystal surface closest to the pump laser. The upper curve is the analytic result and the lower curve is the numerical result. As would be expected, since axial heat flow and convection from the end surfaces are neglected in the analytic model, it produces slightly larger temperature differences than the numerical solution. For the cases considered, the agreement between the thermal profiles is within 10%.[4] Since there is good agreement between the analytic and numerical models, the thermal focusing within the laser rod can be approximated using the analytic solution.

To this end, it is helpful to consider the phase change associated with focusing, $\Delta\phi_f$, which can be written as

$$\Delta\phi_f = kr^2/2f, \qquad (10)$$

where f is the effective focal length[5] and k is the wave number of the emitted light. The total phase change $\Delta\phi$ that is accumulated by the laser light for a single pass through the laser rod is given by

$$\Delta\phi(r) = \int_0^{\ell} k\Delta n(r,z)dz, \qquad (11)$$

where $\Delta n(r,z)$ is the change in refractive index of the laser

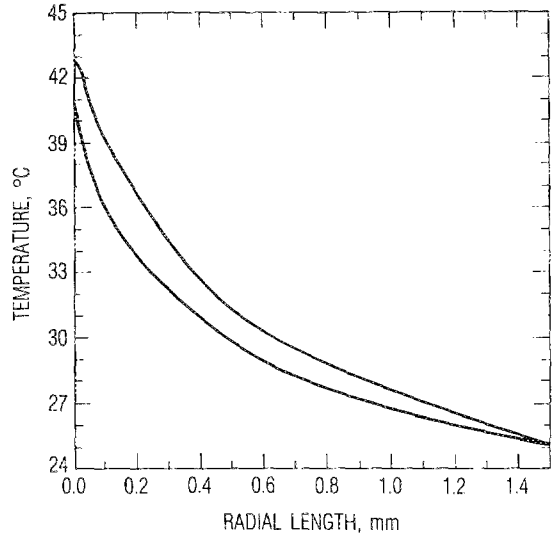


FIG. 2. Comparison of calculated temperature profiles at $z = 0$ (the crystal surface closest to the pump laser). The upper curve is the analytical result and the lower curve is the numerical result. The profiles were computed for a diode-pumped Nd:YAG lasing at 1.3 μm for 2 W of pump power and 703 mW of TEM_{00} output.

medium resulting from temperature effects, and ℓ is the length of the rod. Changes in the index of refraction (n) accrue from three temperature-dependent effects: thermal variations of the index due to dn/dT [i.e., $\Delta n(r,z)_T = \Delta T(r,z)(dn/dT)$], thermally induced stress, and thermal deformation of the rod. In flashlamp pumped YAG, the variation of n due to dn/dT accounts for 74% of the thermally induced focusing.[6]

To arrive at the phase change associated with $\Delta n(r,z)_T$, the appropriate term must be extracted from the expression for $\Delta T(r,z)$. By expanding the $E_1(2r^2/W_p^2)$ term in Eq. (7) as a power series,[2] the term that is quadratic in r of $\Delta T(r,z)$ can be obtained directly. Inserting this term into Eq. (11), and integrating with respect to z, the phase change associated with thermal focusing can be expressed as

$$\Delta\phi_f = \frac{kP_{\text{ph}}(dn/dT)}{2\pi K_c w_p^2}[1 - \exp(-\alpha\ell)]r^2. \qquad (12)$$

Equating Eqs. (10) and (12) gives the effective focal length for the entire rod:

$$f = \frac{\pi K_c w_p^2}{P_{\text{ph}}(dn/dT)}\left(\frac{1}{1-\exp(-\alpha\ell)}\right). \qquad (13)$$

From Eq. (13) it can be seen that the effective focal length depends on the square of the pump beam radius, w_p, and is independent of the crystal size, r_b. Thus, in order to minimize the thermal focal length and maximize the performance at a given pump power, one should utilize the largest possible pump radius that maintains an acceptable pump beam to laser beam mode match. As a numerical example, an effective thermal focal length is calculated in Nd:YAG for the temperature results of Fig. 2. Employing Eq. (13) with $K_c = 0.13$ W/(cm K), $w_p = 100\,\mu$m, $P_{\text{ph}} = 1.08$ W, dn/dT (YAG)[6] $= 7.3\times10^{-6}$ K^{-1}, $\alpha = 4.1$ cm^{-1}, and $\ell = 5.4$ mm, a value of $f = 5.8$ cm is obtained. It is noted that this focal length is comparable to the resonator length of a typical diode end-pumped laser. Investigations are currently in

09 September 2026 13:37:29

progress to experimentally measure thermal focal lengths and compare them to the results obtained from our model.

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09 September 2026 13:37:29