On the CLF loop drifts

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In this text I describe the theory of the CLF-pump phase loop and try to explain where the drift might come from.

I. SYMPTOMS

See https://emvogil-3.mit.edu/lasti/index.php?callRep=11703 for a description of the issue. Or the following summary:

1. We modulate the AOMs of the CLF using the Moglabs at 95MHz sharp and 105MHz sharp (displayed frequency on the interface of the Moglabs). Let $f_1 = 10$MHz and $f_{\rm mog}$ be the effective detuning output by the Moglabs (if it were perfect, we have $f_{\rm mog} = f_1$, but we know that the Moglabs does not always output the displayed frequencies for an unknown reason);
2. We attempt to lock the CLF-pump phase loop on the OPO CLF REFL PD. The demodulation frequency is $f_{\rm demod} = 2f_1 = 20$ MHz sharp.
3. We also close the LO loop by demodulating at f_1. We can verify that the loops are closed by using the internal scopes of the RP, checking that all the corresponding error signals are centered around 0 V (we have preemptively verified that when the loops are open, these error signals are free-running and not at 0).
4. We see that the squeezing level on the SR785 is slowly periodically drifting from SQZ to aSQZ with a periodicity of $f_{\rm drift} = 25 \sim 29$ mHz. Plugging the same signal on the oscilloscope, we can identify that same slow drift on the envelope of the 10MHz beatnote between the CLF and the LO (residually coming out of the same port where SQZ is measured).
5. We then change the demod frequency of the RP from $2f_1$ to $f_{\rm demod} = 2f_1 + f_{\rm drift} = 20$ MHz +25 mHz.
6. Closing again all the loops and verifying that they are close, we look at the SQZ spectrum again and check that it is not drifting anymore.

II. THEORETICAL MODEL

To explain this from the equations, let us better understand how the CLF loop works. We consider the detected CLF beatnote on the photodiode as the photocurrent $i_{\rm beat}(t)$ such that

$$i_{\rm beat}(t) = i_0 \cos\big(2\Omega_{\rm mog} t + \phi_{\rm beat}(t)\big) \tag{2.1}$$

where we use the angular frequency Ω_j in lieu of the corresponding frequency f_j for any index j. The CLF-pump phase information is contained in $\phi_{\rm beat}(t)$, a slow-varying function of time because it contains some fluctuations which need to be stabilized, as we describe below. $i_{\rm beat}$ is sent into the input of the RP, where it is internally beat with the RF signal $s_{\rm demod}(t)$ for demodulation, such that

$$s_{\rm demod}(t) = s_0 \cos(\Omega_{\rm demod} t + \phi_{\rm demod}) \tag{2.2}$$

where $\Omega_{\rm demod}$ and $\phi_{\rm demod}$ can be specified by the user. We consider the geneneral case where the demodulation frequency is not exactly the Moglab's beatnote frequency $2\Omega_{\rm mog}$ and write

$$\Omega_{\rm demod} = 2\Omega_{\rm mog} + \delta\Omega_{\rm demod} \qquad \text{with} \qquad \delta\Omega_{\rm demod} \ll 2\Omega_{\rm mog} \tag{2.3}$$

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Typically, this $\delta\Omega_{\text{demod}}$ can be the mismatch between the Moglabs displayed frequency and its output frequency (up to a multiplicative factor of 2).

As with any phase-control loop, the goal of this CLF-pump loop is to control the relative phase $\phi_{\text{beat}} - \phi_{\text{demod}}$. The demodulated signal $\epsilon(t)$ in the RP is the error signal: it is given by

$$\begin{aligned}\epsilon(t) &= i_{\text{beat}}(t)s_{\text{demod}}(t) = i_0 s_0 \cos\big(2\Omega_{\text{mog}}t + \phi_{\text{beat}}(t)\big)\cos(\Omega_{\text{demod}}t + \phi_{\text{demod}}) \\ &= \frac{i_0 s_0}{2}\cos(\delta\Omega_{\text{demod}}t - \phi_{\text{beat}}(t) + \phi_{\text{demod}}) + \text{term in} \sim 4\Omega_{\text{mog}} \text{ (lowpassed)}\end{aligned} \tag{2.4}$$

By definition of a control loop, the loop is closed if and only if the error signal is put to 0. Then, we have that the argument of the cosine above must be always equal to, say, $\pi/2$. In other words, the condition on the phases is

$$\phi_{\text{beat}}(t) = \frac{\pi}{2} + \phi_{\text{demod}} + \delta\Omega_{\text{demod}}t \tag{2.5}$$

Absorbing the $\pi/2$ term in the definition of the demod phase, and plugging this back into the CLF beatnote signal, we get

$$i_{\text{beat}}(t) = i_0\cos\big((2\Omega_{\text{mog}} + \delta\Omega_{\text{demod}})t + \phi_{\text{demod}}\big) = i_0\cos(\Omega_{\text{demod}}t + \phi_{\text{demod}}) \tag{2.6}$$

In other words, when the CLF-pump loop is closed, the frequency of each CLF beam (incident on the OPO and created by parametric amplification) is detuned from the carrier by $\pm\Omega_{\text{demod}}/2$. The parametric amplification now happens with a sidebands which are separated from the carrier by Ω_{demod}, so no longer $2\Omega_{\text{mog}}$, and ϕ_{demod} contains the information on the squeezing ellipse.

Let us now study the CLF-LO loop. On the homodyne readout, we have an effective CLF-LO photocurrent beatnote $i_{\text{CLF-LO}}$ which is at half of the frequency of the previous beatnote and with a phase $\phi_{\text{CLF-LO}}(t)$, which is a function of the previous ϕ_{demod}.

The CLF-LO beatnote is

$$i_{\text{CLF-LO}}(t) = i_1\cos\left(\frac{\Omega_{\text{demod}}}{2}t + \phi_{\text{CLF-LO}}(t)\right) \tag{2.7}$$

We always demodulate this photocurrent at the frequency Ω_1 using the internal signal $s_1(t)$ such that

$$s_1(t) = s_1\cos(\Omega_1 t + \phi_{\text{demod CLF-LO}}) \tag{2.8}$$

Therefore, the error signal of the CLF-LO loop $\epsilon_{\text{CLF-LO}}(t)$ after lowpass is

$$\epsilon_{\text{CLF-LO}}(t) = \frac{i_1 s_1}{2}\cos\left[\left(\Omega_1 - \frac{\Omega_{\text{demod}}}{2}\right)t - \phi_{\text{CLF-LO}}(t) + \phi_{\text{demod CLF-LO}}\right] \tag{2.9}$$

Thus, when the loop is closed and absorbing the $\pi/2$ term as before in the definition of $\phi_{\text{demod CLF-LO}}$, we get

$$\phi_{\text{CLF-LO}}(t) = \left(\Omega_1 - \frac{\Omega_{\text{demod}}}{2}\right)t + \phi_{\text{demod CLF-LO}} \tag{2.10}$$

What does this mean in terms of measured squeezing angle drift? Theoretically, when the CLF-LO loop is properly locked, $\phi_{\text{CLF-LO}}(t)$ becomes independent of time and is a function of the actual squeezing angle ϕ_{SQZ}: $\phi_{\text{CLF-LO}}(t) = f(\phi_{\text{SQZ}})$. However, comparing with the above, this means that

$$f(\phi_{\text{SQZ}}) = \left(\Omega_1 - \frac{\Omega_{\text{demod}}}{2}\right)t + \phi_{\text{demod CLF-LO}} \tag{2.11}$$

In other words, we see that unless the two frequencies in brakets are equal, the observed squeezing angle will drift in time with a frequency given by the difference between half the demodulation frequency of the CLF-pump loop ($\Omega_{\text{demod}}/2$) and the demodulation frequency of the CLF-LO loop (Ω_1). Note that the phase of the Moglabs does not appear here. This means that, should any drift occur, it should come from a difference in demodulation frequencies internal to the RP, specified by the user.

In this analysis, we have always assumed that the error signals were centered in 0 when the loops are closed. It is possible that this is not the case, if quantization is an issue for putting the CLF-pump phase loop's error signal to 0.

III. EXPERIMENTAL TEST AND VALIDATION OF THE MODEL

Armed with this analysis and the important formula eq. (2.11), we can simply compute the effective difference in frequencies $|f_{\text{demod}} - 2f_1|$ where f_{demod} is the demodulation frequency output by the internal IQ module of the RP for the CLF-pump loop (called `iq_clf0.frequency`), and f_1 is the demodulation frequency output by the internal IQ module of the RP for the CLF-LO loop (called `iq_clf1.frequency`). Even though we specified `demod_freq = 20e6` and `demod_freq = 10e6` for these two loops respectively, the RP stores the frequency as a 32 float unsigned integer with maximum value 125 MHz. This means that the output frequencies will not exactly be 20 MHz and 10 MHz sharp. Computing this in a Notebook yields the result in fig. 1. We see that the difference is $|f_{\text{demod}} - 2f_1| = 29$ mHz, which was exactly the drift periodicity measured in step 4 of section I, and also on the figure https://emvogil-3.mit.edu/lasti/uploads/11703_20240714014537_HD_DC_slow_drifts.jpeg.

```
clf.iq_clf0.frequency-clf.iq_clf1.frequency*2
```

```
-0.029103830456733704
```

FIG. 1. Computation of the exact difference between the two demodulation frequencies, when the user-specified frequencies are $f_{\text{demod}} = 20$ MHz and $f_1 = 10$ MHz. The difference is non-zero because the values are stored as 32-bit integers in the RP's memory. One finds 29 mHz, which is exactly the measured frequency drift.

More generally, since the Red Pitaya's acquisition frequency is 125 MHz, the smallest increment of frequency that can be output is $125\ \text{MHz}/2^{32} \simeq 29.1$ mHz, which is again what was measured experimentally.

IV. CONCLUSION

The squeezing angle drift of 29 mHz which was observed in section I is independent of the Moglab's frequency offset. Rather, it is explained by the fact that the Red Pitaya stores the user-specified frequencies as 32 bit floats, which implies an elementary increment of 29 mHz. Thus the observed drift can be explained as a "rounding error" by the Red Pitaya between the specified demodulation frequencies between the CLF-pump and the CLF-LO loops.

To mitigate this issue, we should ideally choose demodulation frequencies which are integer multiples of $125\ \text{MHz}/2^{32} = 29.103830456733704$ mHz.