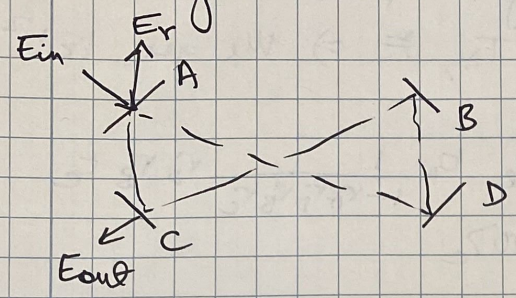
10/31/23 Cavity Calculations: Mirror Transmission



$E_{in} = E_0 |00\rangle_\ell \rightarrow$ laser basis

$= E_0 \sum_{m,n} |mn\rangle_c \langle mn|00\rangle_\ell$ → cavity basis

$E_r = -r_A E_0 \sum_{\substack{m,n \\ \neq 0,0}} |mn\rangle_c \langle mn|00\rangle_\ell$

$-r_A E_0 |00\rangle_c \langle 00|00\rangle_\ell$

$+E_0 T_A r_D r_B r_C \frac{(-1)}{1 + r_A r_D r_B r_C} \sum_{\substack{m,n \\ \neq 0,0}} |mn\rangle_c \langle mn|00\rangle_\ell$

$+E_0 T_A r_D r_B r_C \overbrace{\frac{1}{1 - r_A r_D r_B r_C}}^{\mathcal{F}/\pi} |00\rangle_c \langle 00|00\rangle_\ell$

Assuming: All other $|mn\rangle_c$ where $m,n \neq 0,0$ are anti resonant ($e^{i\phi} = -1$) and $|00\rangle_c$ is resonant ($e^{i\phi} = 1$)

Taking r of mirrors ≈ 1 to first order

$\Rightarrow E_r \approx -E_0 |00\rangle_\ell + T_A \frac{\mathcal{F}}{\pi} E_0 |00\rangle_c \overbrace{\langle 00|00\rangle_\ell}^{\varepsilon} - \frac{T_A}{2} E_0 \sum_{\substack{m,n \\ \neq 0,0}} |mn\rangle_c \langle mn|00\rangle_\ell$ ← Neglect

$\Rightarrow P_r = P_0 \left[1 + T_A^2 \frac{\mathcal{F}^2}{\pi^2} |\varepsilon|^2 - 2 T_A \frac{\mathcal{F}}{\pi} |\varepsilon|^2\right]$

$\Rightarrow |\varepsilon|^2 = \frac{\left(\frac{P_r}{P_0} - 1\right)}{T_A \frac{\mathcal{F}}{\pi} \left(T_A \frac{\mathcal{F}}{\pi} - 2\right)}$

$\Rightarrow \boxed{|\varepsilon|^2 = \frac{1 - P_r/P_0}{T_A \frac{\mathcal{F}}{\pi} \left(2 - T_A \frac{\mathcal{F}}{\pi}\right)}}$

From measuring $\frac{P_r}{P_0}$ (driving ramp into laser PZT to measure cavity coupling), T_A, $\mathcal{F}$ $\Rightarrow$ We get $|\varepsilon|^2$

Then, $E_{out} = E_0 \, |00\rangle_c \, {}_c\langle 00|00\rangle_\ell \, t_A \frac{1}{1 - r_A r_D r_B r_C} r_D r_B t_C$

$$\approx E_0 \, \varepsilon \, \frac{\mathcal{F}}{\pi} t_A t_C \, |00\rangle_c$$

$$\Rightarrow P_{out} = P_{in} |\varepsilon|^2 \frac{\mathcal{F}^2}{\pi^2} T_A T_C$$

$\therefore$ From measuring P_{out}, P_{in}, T_A, $\mathcal{F}$, $|\varepsilon|^2$

$$\Rightarrow \boxed{T_C = \frac{P_{out}}{P_{in} |\varepsilon|^2 \frac{\mathcal{F}^2}{\pi^2} T_A}}$$