Parameter estimation for binary black holes with networks of third generation gravitational-wave detectors

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The two binary black-hole (BBH) coalescences detected by LIGO, GW150914 and GW151226, were relatively nearby sources, with a redshift of ∼0.1. As the sensitivity of advanced LIGO and Virgo increases in the next few years, they will eventually detect BBH farther away, up to redshifts of ∼2 for systems of 200 solar masses. However, these are still relatively small distances if compared with the size of the universe, or what done with traditional astrophysics. If one wants to study BBH in the epoch or reionization, or black holes born from pop III stars, more sensitive instruments are needed. Third generation gravitational-wave detectors, such as the Einstein Telescope or the Cosmic Explorer are already in an advanced R&D stage. With some variations, depending on the actual implementation, they would increase by a ten fold the sensitivity of design LIGO and Virgo, and be sensitive to BBH up to redshifts of 20. In this paper we quantify the precision with which these new facilities will be able to estimate the parameters of stellar-mass and intermediate-mass BBH as a function of their redshifts and the number of de'tectors. We show how having only 2 detectors would yield estimation of black hole intrinsic masses a factor of ∼2 larger than what can be done with 3 or 4 instruments. Larger improvements are visible for the sky localization, although it is not yet clear whether BBH are luminous in the electromagnetic or neutrino band. The measurement of the spins parameters, instead, do not get significantly better as more detected as, compatibly with the fact that one does not need polarization or redshift information to measure spins.

I. INTRODUCTION

With the detection of the binary black holes (BBH) GW150914 [1] and GW151226 [2, 3] the era of gravitational wave (GW) astrophysics has started. The first two systems detected by the LIGO and Virgo collaborations had very different masses. GW150914 was made of two black holes of roughly $30\mathrm{M}_\odot$ [4], i.e. much more massive than known stellar-mass black holes [5]. These large masses have been used to set constrains on the metallically of the progenitor stars [6]. At 14 $\mathrm{M}_\odot$ and 7 $\mathrm{M}_\odot$, the masses of GW151226 were instead in the middle of the range of masses for known BHs [3]. Very little could be said about the spins of either source [3], mostly due to the lack of visible precession.

Although very different in their physical parameters, the two events had something in common: their luminosity distance, slightly more than 400Mpc. Using the cosmology measured by the latest Planck results [7], this corresponds to a redshift of ~ 0.09 [3]. It has not been possible to verify if the binaries formed dynamically, and merged relatively quickly after forming a bound system, or if they formed in galactic fields, and inspiralled for a long time before merging [3, 6, 8–10].

Over the next few years, existing ground-based GW detectors such as LIGO [11] and Virgo [12] will steadily increase their sensitivities [13]. Once at design, toward the end of this decade, they will be a factor of 10 more sensitive than 1st generation GW detectors (initial LIGO and Virgo). Other detectors will join the network: KAGRA [14] is being built in Japan, while the construction of LIGO India [15] has been recently approved. This network of second generation (advanced) detectors will be able to probe a significant volume, and detected heavy BBH up to redshifts of unity (with heavier and well oriented systems detectable up to $z \sim 2$ [30] anyone else to cite here?). A combination of better coating, quantum squeezing and heavier masses can add another factor of ∼2 in range [16], after which current facilities will saturate their potential.

New facilities (henceforth third-generation, or 3G, detectors) will be required to substantially increase the sensitivity of advanced detectors, and allow for exploring the most remote corners of universe and detecting rare events.

The Einstein Telescope [17]More? (ET) is a European proposal for an underground 3G detector. Although its design is not yet precisely finalized, it should consist of 3 Michelson interferometers [1] with 10 Km long arms, and inter-arm angles of 60°, arranged to form a triangle. The fact that 3 interferometers are used gives to the ET more power in discriminating GW polarizations than an equivalent L-shaped detector [18]. On the other hand, the fact that they are co-located strongly reduces the capabilities to localize GW sources in the sky. Finally, if built underground, it would have good sensitivity down to a few Hz, as opposed to the 10 Hz realistically achievable with ground-based detectors.

Another possible way forward to 3G detectors is to keep orthogonal arms, but significantly increase their length. The Cosmic Explorer (CE) [19, 20]More? is a proposed ground-based 40 Km L-shaped detector. Intense R&D will be necessary, and is already ongoing, to ensure that all the major known sources of noise can

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[1] In this paper we will talk of detector to refer to the whole ET apparatus, made of 3 interferometers.

be dealt with. These include quantum noise, Newtonian noise and coating thermal noise [19].

3G detectors will have three main, related, advantages over existing instruments. First, they will allow for extremely frequent detections of common systems (such as BBH) and dramatically increase the probability of detecting more rare events, such as core-collapse supernovae (SNe). Second, they will make accessible a much larger fraction of the universe. As we will see below, 3G detectors will be sensitive to heavy BBH up to redshifts of more than 10, well within the epoch of reionization. Detection of extremely high redshift BBH might shed light on pop III stars and on primordial black holes. Finally, events at small redshifts (below unity) would be detectable with extremely large signal-to-noise ratios (SNR). For example, a CE class detector would detect systems similar to GW150914 with SNR of the order of a thousand.

Several authors have analyzed many of the scientific goals that would be achievable with 3G detectors. However they mostly covered the ET and focussed on binary neutron stars (BNS), since those were thought to be most commonly detectable sources of GW in the universe. Examples include perform tests of general relativity [5, 21], measure cosmological parameters [22–24], and measure the equation of state of neutron stars [22, 25].

In this paper we consider the capabilities of 3G networks to characterize BBHs. We show how having a network of 3G detectors will be of fundamental important to extract key parameters of the sources, such as their masses. We consider several hypothetical networks of 3G observatories, made of 2, 3 or 4 detectors. We generate astrophysical populations of BBH and show how precisely their parameters can be estimated by each of the networks. We then find that in order to measure the masses of BH in binaries one must use more than 2 detectors. This happens because what is measured with GW are the redshifted masses. The intrinsic masses must be derived from those, by obtaining the redshift from the luminosity distance measurement. Since the distance information is encoded in both polarizations of the GW signal, and strongly correlated with the inclination angle, more than 2 detectors are required. We will show how the estimation of the (intrinsic) mass parameters will get a factor of 2 (for nearby events) to a factor of several several better if 4 3G detectors are used instead of 2. We also show that, unsurprisingly, the same effect does not apply to the dimensionless spins, since they do not get redshifted.

This paper is structured as follows. In Sec. II we describe the networks used in the study. In Sec. III we describe the simulated BBH events and make some considerations on the role of multiple detectors. In Sec. IV we expand on some details about the mass measurement. The main results are reported in Sec. V, while some caveats are listed in Sec. VI. Conclusions and future work are summarized in Sec. VII.

II. NETWORKS

In this study, we considered 5 different network configurations, listed here below in increasing number of instruments.

- **LC** Two CE detectors, one in the location of LIGO Livingston and one in China (see table I for details).
- **LE** One CE in Livingston and one ET in Europe.
- **LAE** Two CE instruments, one in Livingston and the other in Australia. One ET in Europe.
- **LCAE** Three CE instruments, one in Livingston one in Australia and one in China. One ET in Europe.
- **LCAI** Four CE instruments, one in Livingston, one in Australia, one in China and one in India.

We stress that the coordinates and orientations we used, Tab. I are not meant to represent realistic locations. In particular we did not check for geographical constraints or seismic noise levels. These kind of detailed studies will off course have to be performed before a site is selected. However, for the purposes of this study the exact locations do not matter, and approximate positions (up to a few thousands kilometers) are enough.

For each detector we generated simulated gaussian noise using the power spectral densities for ET-D and CE given in refs. [19, 26], and shown in Fig. 1, together with the advanced LIGO one, for scale. While the Einstein Telescope could have good sensitivity down to a few Hertz, we started the analysis for all interferometers at the same frequency of 10Hz. However this will not change our conclusions, since those mostly depend on the geometry of the networks and not on the details of each instrument.

	Longitude	Latitude	Orientation	Type
L	-1.58	0.533	2.83	CE
C	1.82	0.67	1.57	CE
I	1.34	0.34	0.57	CE
E	0.182	0.76	0.34	ET-D
A	2.02	-0.55	0	CE

TABLE I. The coordinates of the interferometers used in this study. Orientation is the smallest angle made by any of the arms and the local north direction. All angles are in radians. The last column reports the type on instrument, Cosmic Explorer or Einstein Telescope.

III. INJECTIONS

In this section we describe the generation of the simulated BBH events for each network.

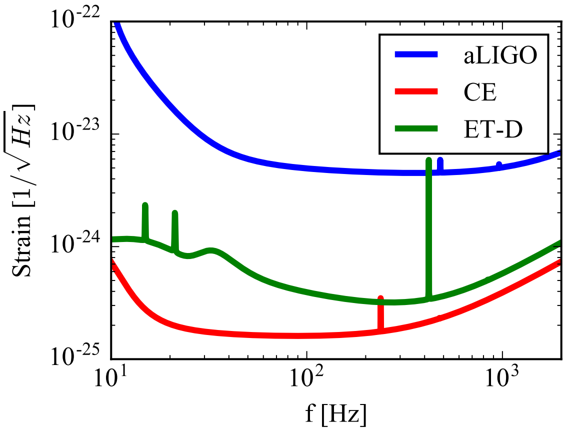

FIG. 1. The amplitude spectral density of the ET-D and CE, compared with the design advanced LIGO.

We assumed intrinsic, or source-frame, total masses are uniform in the range $[12, 200]$ $M_\odot$ with a minimum mass ratio of $1/3$ [2], consistently with the range of validity of the of the waveform family we used (see below). The lower limit of the mass range is due to the evidence that stellar mass black holes have masses above $\sim 6M_\odot$. The upper limit is somehow arbitrary, since no observational evidence exists of intermediate-mass black holes.

Spins were uniform in magnitude in the range $[0, 0.98]$ and randomly oriented in the unit sphere.

The redshifts were uniform in comoving volume, assuming a standard ΛCDM cosmology [7], in the range $z \in [0, 20]$. This implies we assumed the star formation rate (SFR) is not a strong function of the redshift, which off course is only a very rough approximation. However the main goal of this paper is not as much to report astrophysical uncertainties, as to show how those uncertainties depend on the GW network used. We thus assumed this to be a sufficient working hypothesis. If our readers have a particular SFR on mind, they will be able to use our figures in the range of redshift where they expect most sources. The redshift distribution we used in shown in Fig. 2.

Every time a set of proposed parameters is randomly generated from the distributions described above, we calculate the network SNR it would produce in the network under consideration, and only keep the source if the SNR in the range $[10, 600]$. However we notice that selection was only seldom used. The reason is that for all the networks, given our mass range and redshift distribution, most of the proposed sources had SNR inside this range. This is why most sources will be the same for all networks.

In this paper we do not deal with confusion noise and detectability of sources. Work exists in the contest of ET for binary neutron stars [27–29] where it is shown that even overlapping events can be detected very efficiently (since the overlap in time needs not to correspond to an overlap in frequency). We will assume that the same is true for BBH and only use the SNR as a probe for detectability. A full mock data challenge should be put in place for a network of 3G detectors to fully support this assumption, which is beyond the scope of this study.

Some of the key properties of the population of detectable BBH, and a few differences with second-generation detectors (i.e. advanced LIGO type instruments) are highlighted in Ref. [30].

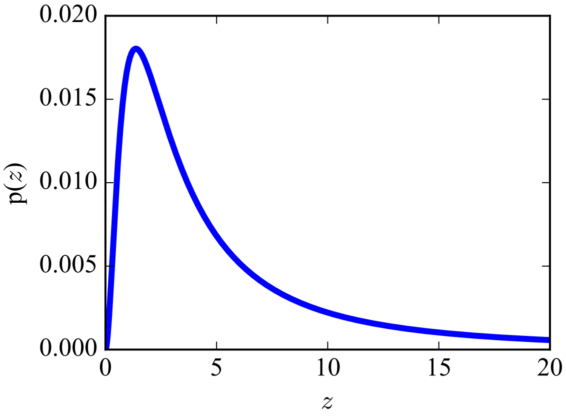

FIG. 2. The redshift distribution used to generate the simulated events. This curve is also used as prior for the parameter estimation algorithm.

For each of the networks above, we selected roughly 200 events, which were analyzed with `lalinference` [31], the parameter estimation algorithm used to characterize the BBH detected in the first science run of advanced LIGO [3, 4].

We used a simplified precessing approximant, `IMRPhenomPv2` [32–34] both for simulating the signals to add in the data, and as templates for the parameter estimation algorithm.

It is worth stressing that IMRphenomPv2 does not include higher modes, while one might expect those to be relevant for high mass systems. While our choice was mostly driven by the lack of better waveform approximants, which contain all the relevant physics and still are fast enough to compute, we can defend it by noticing that the importance of higher order modes is enhanced by large mass asymmetries [35], while we keep the mass ratio of the simulated signals above $1/3$. While this does not imply a study similar to this should be repeated as better waveform models become available, it reassures us that the results we obtain are a good first investigation to assess the capabilities and network requirements for 3G networks.

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[2] We define the mass ratio in the range $[0, 1]$.

IV. INTRINSIC MASS MEASUREMENT

What is measured by GW detectors are the redshifted BH masses, from which one needs to calculate the intrinsic, astrophysical interesting, masses. The two are related by the simple relationship:

$$m^s = \frac{m^{det}}{1+z} \tag{1}$$

where "s" stands for source frame and "det" for detector frame. With m here we indicate any dimensionfull mass quantity (component masses, total mass, chirp mass). Thus, in order to get the posterior distribution for the source-frame masses one needs to estimate the redshift of the GW source.

Unfortunately, the redshift is extremely hard to measure from GW observations alone. While it could be doable for systems with a neutron star, if the equation of state of nuclear matter were known [22, 25, 36], no methods have been suggested to extract the redshift directly from BBH systems. On the other hand, GW provide a direct measurement of the luminosity distance to the source. From this, the redshift can be calculated if one assumes the cosmology is known. This is the approach followed to calculate the redshifts quoted for GW150914 and GW151226 [3], where the latest cosmology measured by Planck was used [7].

We assume the same approach will be followed for 3G detectors, and the luminosity distance will still play a pivotal role to measure the redshift, and hence the intrinsic masses. This could change if some intrinsic properties of black-holes in BBH were discovered in the next few years that can be used to directly extract the redshift from GW measurements, however up to date no such properties seem to exist.

This one of the first examples in which there become apparent a coupling between one extrinsic parameter (distance) and some intrinsic parameters (masses), while these two groups have traditionally been considered quite independent, in the sense that the measurement of one would not affect the other.

Later in the life-span of advanced detectors, and even more so with 3G instruments, i.e. when cosmological distances are reached, a good estimation of the luminosity distance is paramount to measure masses.

A. The role of polarizations

It is well know that, within general relativity, gravitational waves have two polarizations [37]. The luminosity distance to the source and the inclination of the orbital plane w.r.t. the line of sight enter in different ways in the two polarizations:

$$h_+ \propto \frac{(\cos\iota + 1)^2}{2D_L}$$

$$h_\times \propto \frac{\cos\iota}{D_L}$$

Being able to measure both polarizations will thus help breaking correlations and improve the estimation of the luminosity distance, which in turn is necessary to estimate the source-frame masses. This is an important reason why more than one 3G detector should be built: measuring distances, and hence intrinsic masses, would be extremely hard otherwise.

Having a network as large as possible is a goal already being pursued for 2G detectors. However the *reason* is very different. The main driver for having more than 2 advanced detectors is to reduce the uncertainty in the sky localization of the GW sources [13, 38–44], thus increasing the chances of identifying electromagnetic or neutrino counterparts to CBC and other sources. For 3G detectors, the very measurement of BH masses requires a network that can disentangle GW polarizations.

V. RESULTS

In this section we report the uncertainties in the estimation of some key parameters and show how those depend on the configuration and size of the networks of Sec. II. Unless otherwise said, we will quote the 90% confidence interval divided by the true value of the parameter, and quote uncertainties in %. Occasionally we will report the actual 90% confidence interval.

A. Distance and sky position

Let us first report the uncertainties in the estimation of two important extrinsic parameters: the luminosity distance and the sky position. As we said, both quantities should be affected by the number of detectors in the network. The effect of the numbers of detectors on sky localization uncertainties for 2G detectors has been already addressed in several papers, mostly for binary neutron stars, e.g. [39, 41]. Work is ongoing to also include 3G detectors [45].

While for CBC sources with one or two neutron stars the interest on their sky positions is fully justified by the fact that those systems are expected to be progenitors of short GRBs [46], there is still not a clear connection between BBH and EM radiation. However, we think it is still interesting to report those numbers for three reasons: a) While unlikely, it is not impossible that BBH will in fact emit some energy in the EM, or neutrinos, as some mechanisms have been proposed [47] after the discovery of GW150914 and the alleged EM sub-threshold Fermi trigger [48]; b) the trends we will see should be indicative of what one can expect for BNS and c) the positions of detected BBH could be used to study the large-scale structure of the universe [49, 50].

With this this in mind, in Fig. 3 we show violin plots for the 90% confidence interval for the sky position, in deg^2. In each violin, the red horizontal line marks the median. Each panel only uses events in a given redshift range, specified at the top, and the label in the x axis specifies the network. The cuts across redshift bins are somehow arbitrary, and mostly chosen to ensure each bin had enough sources.

As expected, the uncertainties are smaller for nearby sources (simply because they will on average be louder) and decrease with the number of detectors

We see how events up to redshift of 3 can be localized within 100 deg^2 even with only 2 3G instruments. However for systems farther away one really needs at least 3 detectors to keep the uncertainty below that threshold for most events. With 4 instruments, even events at $z > 6$ can be localized within 10 deg^2 most times. For nearby sources, localizations within 1 deg^2 would be typical for 4-instrument networks and frequent with 2-instrument networks. The best networks would even be able to localize a large fraction of events to within a tenths of deg^2, dramatically increasing the chances of identifying eventual EM or neutrino counterparts. This, together with the small distance uncertainty for relatively nearby events (see below), will significantly reduce the number of galaxies in the volume box.

We also see that the improvement going from a 3-detector to a 4-detector network is smaller than from 2 to 3. This has already been seen for 2G detectors [38] and can been understood as follow. Given that 2 polarizations must be measured, a 2-detector network is just enough. Adding a 3rd detectors makes the problem overdetermined, which dramatically increases the polarization resolution. After that, adding a 4th detector helps percentually less, mostly by better discrimination of the signal in the noise [3].

In Fig. 4 we present a similar plot for the 90% confidence interval relative uncertainties on the luminosity distance (percent). It is clear how uncertainties below 10% will only be achieved for sources with $z \lesssim 3$. If it is indeed the case that sources are distributed uniform in comoving volume, i.e. with a distribution of redshift similar to the one shown in Fig. 2 that peaks at $z \sim 2$, then typical uncertainties can be expected to be below 10% for a 4-detector network, and a factor of 2 larger for 2-instrument networks. As the redshift of the sources increase, so do the uncertainties. For sources at $z > 6$, the 2-instrument networks have a significant fraction of events with relative uncertainties above 100%, although the median stays below that value.

We notice that the network LE does typically better than LC. This happens because the ET has more polarization discrimination power than a simple L-shaped detector such as the CE we used for the detector in China [18]. Thus, a LE network should effectively been slightly better than a network with 2 L-shaped detectors (such as LC). This is exactly what the plot confirms. We don't see the same happening for the 4-instrument networks. We explain this by noticing that since the polarization problem is already over-determined in 4-instrument networks, the small extra SNR that a CE can yield matters more than the extra polarization content of ET.

We stress than in our results we have assumed weak lensing errors can be corrected [24, 51]. Should this not be true, weak lensing could add a systematical error to the measured luminosity distance. Since in this paper we are dealing with statistical errors and their dependence on the network size, and since all networks would be affected in the same way, we neglected the possibility of weak lensing errors. They should, however, be taken into account in future work as they will dominate the total (statistical plus systematic) error budget for the distance measurement at redshifts larger than a few.

B. Spins

Before we move to the masses, we report the uncertainties on the measurement of spin parameters, another key quantity for the characterization of compact objects, and black holes in particular. In a companion paper [52] we have reported the measurability of spins in heavy BBH with networks of 2G detectors and found that they will be hard to measure for individual events.

In Fig. 5 we show the 90% confidence interval for the measurement of the spin of the primary (top) and secondary (bottom) BH for all the networks we used, with different symbols. We see that that spin measurement is *not* a strong function of the number of detectors (modulo the small extra SNR that having more detectors gives) and no clear trends are visible. The vertical histograms on the right side show the actual distribution of the uncertainties for one of the networks (LCAI, but they all look very similar). The dashed red and green line report the 10^{th} and 50^{th} percentiles, respectively. Comparing with similar plots for 2G detectors [52], we find that 3G detectors can estimate spins better. For example, for 10% (50%) of systems the magnitude of the primary will be estimated with uncertainties below 0.17 (0.5). For 2G detectors and BBH in the total mass range $[60 - 100]\mathrm{M}_{\odot}$, the 10% percentile is at 0.7 [52]. The spin of the secondary is typically hard to measure, with the posterior distribution filling most of the prior for a large fraction of systems: 10% (50%) of the events have errors below 0.5 (0.8).

3G detectors can thus measure spins better for a population of BBH than 2G instruments. This is due to a combination of 2 main factors: a) most of BBH detected by 3G networks will have SNRs of several tens [30]; and b) for 3G detectors, the distribution on inclinations angles will favor edge-on systems [30], for which spin precession is clearly visible, if present, which reduces spin-mass correlations [52].

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[3] Ref. [38] also consider a 5 2G detector network, which effectively show how the improvement plateaus after the 4-th detector.

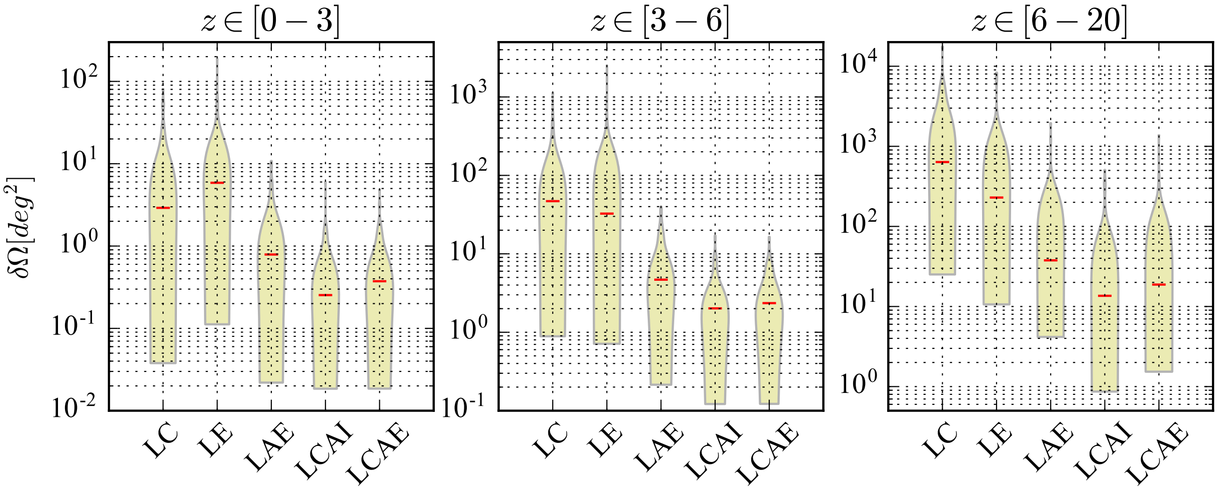

FIG. 3. Violin plot for the 90% sky localization uncertainties (y axis). Each panel focuses on a different redshift bin, indicated at the top. The labels in the x axis identify the 3G networks.

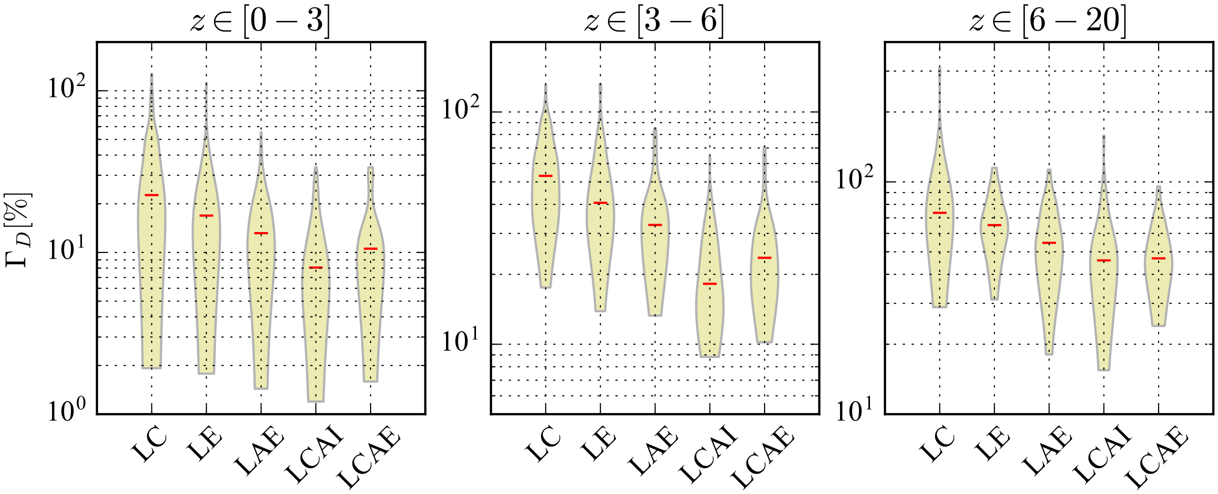

FIG. 4. Violin plot for the 90% relative uncertainties for the luminosity distance (y axis). Each panel focuses on a different redshift bin, indicated at the top. The labels in the x axis identify the 3G networks.

C. Masses

We now move to the main result of this study, namely the measurement of BBH intrinsic masses. Let us start with the chirp mass, Fig. 6. As expected, uncertainties are lower for larger networks, whereas for the 2-detector LC network only half of the signals in the closest redshift bin have uncertainties below the 10% level.

The best measurements are obtained with the LCAI network, which can yield uncertainties below 10% all the way to redshift of several.

Similar conclusions hold when considering the component masses, Figs. 7 and 8. It is worth stressing that even for nearby loud events 3G networks will not typically get sub-percent uncertainties in the estimation of the component masses. This is *not* due the uncertainty in the distance, but only to the fact that component masses are correlated in GW signals. Even if the redshifted masses were considered, uncertainties would still be above 1% for all the signals we analyzed.

In Ref.[53] it was noticed how BBH of a few tens of solar masses could be observed in both the eLISA band and in the band of ground-based detectors. Ref. [54] showed how eLISA measurements of masses (and sky position) can be used as priors in the parameter estimation analysis with a network of 5 2G ground based detectors, improving e.g. the measurement of spins. That might be less true while considering eLISA+3G detectors, since the BBH events detectable by eLISA would be at $z \lesssim 0.4$ [53], for which the SNR in 3G detectors would be extremely

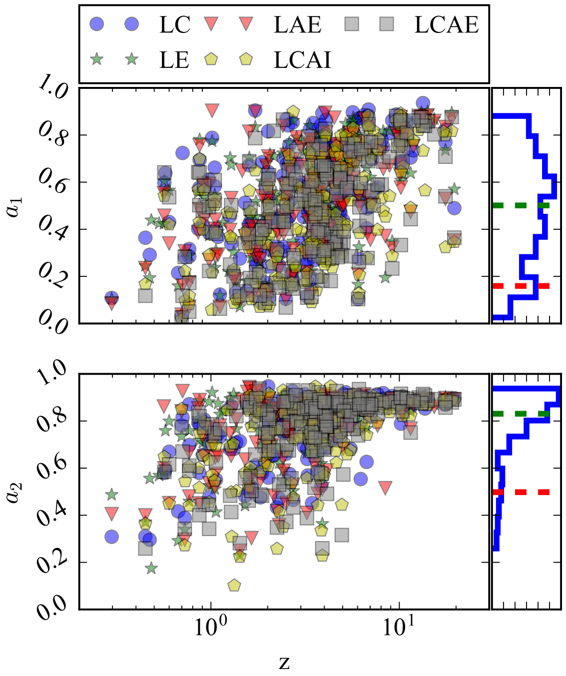

FIG. 5. The distribution of uncertainties for the spin magnitude of the primary (top) and secondary (bottom) black hole versus the injected redshift.

large. In fact, component masses with 3G would be estimated at the few % level for the closest sources, Figs. 7 and 8, comparable with what eLISA would do.

For systems in the redshift range $[0, 3]$, where most events could live, given the redshift prior, the uncertainties with LC are roughly 1.5-2 times larger than what yielded by 4-instrument networks. This ratio stays roughly the same in the other redshift bins for m_1 and m_2, while it gets larger for $\mathcal{M}$.

One of the most intriguing possibilities with 3G detectors, is to detect BBH from the epoch of reionization ($z \in [6, 20]$). For events at those distances, more than two instruments will be mandatory to estimate the component masses with uncertainties below 100%. In that range, 4-detector networks would give uncertainties of only a few tens of percent.

It is worth making a final remark: given that we simulated signals uniform in comoving volume, the events at redshift of a few dominate our population, and we did not get anything closer than $z \sim 0.3$. This is much higher than either GW151226 or GW150914. Off-course this does not imply that nearby events will not be detected very often by 3G networks, but just that they will be detected *less* often that events farther away. In a different paper, we will consider some of the research enabled by SNRs of thousands.

VI. CAVEATS

Through this work we have made a few assumptions and choices, driven by limitations in computing power or simply by lack of better alternatives. Here we list them with the hope that this will guide future work, by the author of other groups.

- We have used a waveform family without higher harmonics. While this is probably acceptable given the limited mass-ratio range we considered, it should be improved as fast waveforms with higher harmonics become available.
- We have started the analysis at 10Hz for all detectors, while the ET-D would be sensitive to lower frequencies, down to a few Hz. However this is not bound to have changed the conclusions of the paper.
- We have not considered potential systematic errors arising from weak lensing, de facto assuming lensing can be corrected. While this is acceptable when comparing the statistical performances of proposed networks, it should be taken into account if the full astrophysical capabilities of 3G networks are studied.
- We have used an arbitrary upper limit for the mass distribution of the sources we simulated. This is due to the lack of observational evidence, and should be revised as the advanced detectors expand our knowledge of CBCs and BHs.
- We have assumed sources are uniformly distributed in comoving volume. Different star formation rates should be folded in, which would change the redshift distribution of the detected sources.

VII. CONCLUSIONS

The detection of GW from two binary black hole coalescences has clearly show how advanced ground-based detectors will detect tens of systems per year. The LIGO and Virgo detectors will reach their design sensitivity over the next few years, when stellar-mass or heavy BBH will be visible up to redshifts of roughly 1 (IMBH would be visible up to z∼2). New facilities will be necessary to extend the reach of ground-based detectors to redshifts of several.

R&D is already ongoing, and several solutions for third generation detectors have been suggested. The Einstein Telescope design consists of 3 60-degree 10-Km arms interferometers arranged to form a triangle. Built underground, it would be sensitive down to a few Hz. The Cosmic Explorer still keeps the L-shape of existing instruments, but with increased arm-length of 40Km. The sensitivity of both instruments would be over a factor of 10 better than the *design* sensitivity of advanced LIGO.

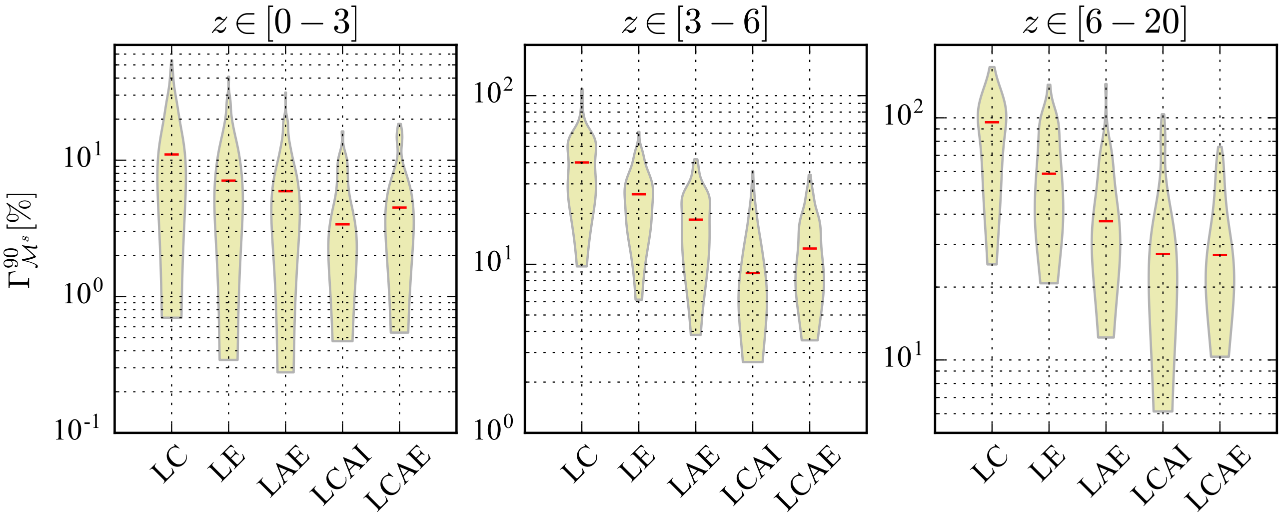

FIG. 6. Violin plot for the 90% relative uncertainties for the source frame chirp mass (y axis). Each panel focuses on a different redshift bin, indicated at the top. The labels in the x axis identify the 3G networks.

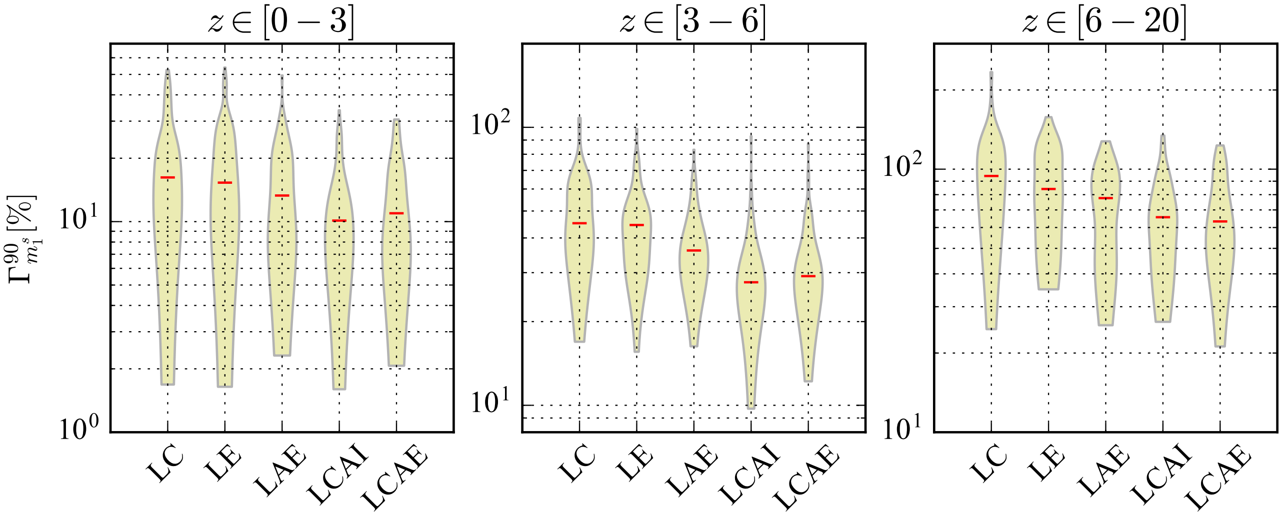

FIG. 7. Violin plot for the 90% relative uncertainties for the source frame m_1 (y axis). Each panel focuses on a different redshift bin, indicated at the top. The labels in the x axis identify the 3G networks.

Third generation detectors would thus be sensitive to BBH up to redshifts of 10 and above, and would be able to target stellar mass or heavy black holes born from the first generation of stars. The large horizons of these instruments would allow to perform extremely precise measurements of mass and spins, for nearby loud events, and to reconstruct the mass evolution of BH through cosmic history.

The estimation of masses is complicated by the fact that mass parameters get redshifted in gravitational-wave signals, so that what one measures is not the mass, but $(1 + z)$ times the mass; thus a measure of the redshift must be used to convert detector-frame masses into intrinsic masses. However, GWs do not directly yield a measurement of the redshift, but rather of the luminosity distance, from which the redshift can be obtained if the cosmology is known. Since information about the distance is encoded in both polarizations of the GW signal, one can expect that at least 2 detectors are necessary to properly measure it, and hence measure the intrinsic masses.

In this paper we have shown how well some key parameters of BBH can be measured for several hypothetical networks of 3G detectors, made of 2, 3 and 4 instruments. We generated distributions of BBH with intrinsic total masses in the range $[12, 200]$ $\mathrm{M}_\odot$ and random spins, uniformly distributed in comoving volume. The simulated BBH signals were then added into simulated

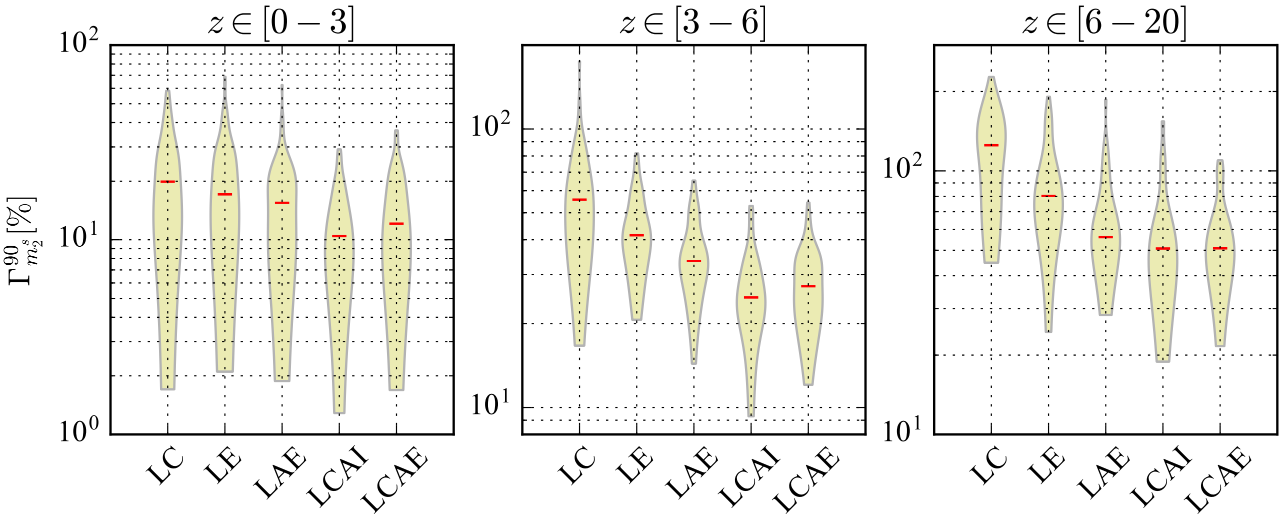

FIG. 8. Violin plot for the 90% relative uncertainties for the source frame m_2 (y axis). Each panel focuses on a different redshift bin, indicated at the top. The labels in the x axis identify the 3G networks.

noise of the 3G networks, and their parameters estimated using a nested sampling algorithm.

As expected, we found that the component masses and the chirp mass are estimated better as more detectors are added. More precisely, we found that the median uncertainty is between a factor of 1.5 and 2 larger for 2-instrument networks than for 3- or 4-instrument ones. For nearby events ($z < 3$), typical 90% confidence interval uncertainties for the component masses will be around $10-20\%$, but uncertainties of a few % will be common. For sources at large redshifts ($z > 6$) more than 2 instruments are necessary to have median uncertainties significantly below 100%. Similar conclusions hold for the chirp mass.

We have verified that the estimation of spins is not affected by the network details, which is expected since spins enter the waveforms in "redshift-free" combinations. Given that nearby events ($z < 3$) can be extremely loud in 3G detectors, and that inclination angles will be isotropically distributed, precise spin estimation will be possible. Furthermore, events up to redshift of a few can be localized in the sky to within 10 deg^2 even with 2 instruments only, and with medians of ~ 0.3 deg^2 if 4 detectors are available. Although it is not yet clear if BBH will emit EM or neutrino signals, we report these numbers, and their ratios as indicative of the improvements that would be reached for binary neutron stars.

We did not perform explicit simulations to assess the measurability of eventual deviations from general relativity. However it is clear how the possibility of accessing BBH events at SNRs of several hundreds would open new avenues for precise tests of general relativity.

This study should be updated as more realistic waveforms become available, or if significant updates are made to the design of 3G detectors. However, it already clearly shows that more than 2 3G detectors should be built to maximize their science output.

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