CURRENT STATE AND PROSPECTS FOR GRAVITATIONAL WAVE DETECTION

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Observatoire de NICE

SITZUNGSBERICHTE 1916.
XXXIII.
DER
KÖNIGLICH PREUSSISCHEN
AKADEMIE DER WISSENSCHAFTEN.

Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

$$\gamma'_{\mu\nu} = \alpha_{\mu\nu} f(x_1 + i x_4) = \alpha_{\mu\nu} f(x - t)\,. \tag{15}$$

Dabei sind die $\alpha_{\mu\nu}$ Konstante; f ist eine Funktion des Arguments $x - t$. Ist der betrachtete Raum frei von Materie, d. h. verschwinden die $T_{\mu\nu}$, so sind die Gleichungen (6) durch diesen Ansatz erfüllt. Die Gleichungen (4) liefern zwischen den $\alpha_{\mu\nu}$ die Beziehungen

$$\left.\begin{aligned} \alpha_{11} + i\alpha_{14} &= 0 \\ \alpha_{12} + i\alpha_{24} &= 0 \\ \alpha_{13} + i\alpha_{34} &= 0 \\ \alpha_{14} + i\alpha_{44} &= 0 \end{aligned}\right\}. \tag{16}$$

Von den 10 Konstanten $\alpha_{\mu\nu}$ sind daher nur 6 frei wählbar. Wir können die allgemeinste Welle der betrachteten Art daher aus Wellen von folgenden 6 Typen superponieren

$$\left.\begin{array}{lll} \text{a)}\ \begin{array}{l} \alpha_{11} + i\alpha_{14} = 0 \\ \alpha_{14} + i\alpha_{44} = 0 \end{array} & \begin{array}{l} \text{b)}\ \alpha_{12} + i\alpha_{24} = 0 \\ \text{c)}\ \alpha_{13} + i\alpha_{34} = 0 \end{array} & \begin{array}{l} \text{d)}\ \alpha_{22} \neq 0 \\ \text{e)}\ \alpha_{23} \neq 0 \\ \text{f)}\ \alpha_{33} \neq 0 \end{array} \end{array}\right\}. \tag{17}$$

$$\begin{aligned} &\text{d)}\quad \frac{1}{i} t_{22} = \frac{f'^2}{4\varkappa} \alpha_{22}^2 = \frac{1}{4\varkappa} \left(\frac{\partial \gamma'_{22}}{\partial t} \right)^2 \\ &\text{e)}\quad \frac{1}{i} t_{23} = \frac{f'^2}{4\varkappa} \alpha_{23}^2 = \frac{1}{4\varkappa} \left(\frac{\partial \gamma'_{23}}{\partial t} \right)^2 \\ &\text{f)}\quad \frac{1}{i} t_{33} = \frac{f'^2}{4\varkappa} \alpha_{33}^2 = \frac{1}{4\varkappa} \left(\frac{\partial \gamma'_{33}}{\partial t} \right)^2 \end{aligned}$$

Es ergibt sich also, daß nur die Wellen des letzten Typs Energie transportieren, und zwar ist der Energietransport einer beliebigen ebenen Welle gegeben durch

$$I_x = \frac{1}{i} t_{41} = \frac{1}{4\varkappa} \left[\left(\frac{\partial \gamma'_{22}}{\partial t} \right)^2 + 2 \left(\frac{\partial \gamma'_{23}}{\partial t} \right)^2 + \left(\frac{\partial \gamma'_{33}}{\partial t} \right)^2 \right]. \tag{18}$$

THE RADIATION FIELD

Transverse Plane Wave Solutions with “Electric” and “Magnetic” Terms

Geometric Interpretation

$$d\,s^2 \;=\; g_{ij} dx^i\, dx^j$$

$$g_{ij} \;=\; \eta_{ij} \;+\; h_{ij} \qquad \text{weak field}$$

$$\eta_{ij} \;=\; \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ 0 & & -1 & \\ & & & -1 \end{pmatrix} \qquad \text{Minkowski Metric of Special Relativity}$$

Gravity Wave Propagating in the x_1 Direction

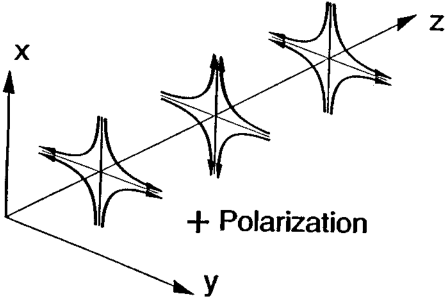
$$h_{ij} \;=\; \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & h_{22} & h_{23} \\ 0 & 0 & h_{32} & h_{33} \end{pmatrix} \qquad \text{all } h_{ij} \ll 1$$

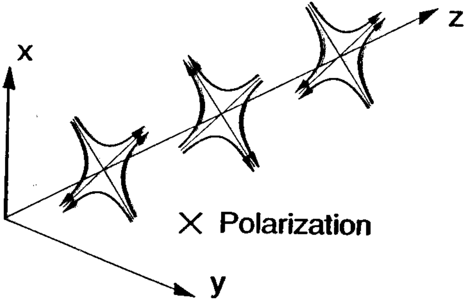
Plane Wave

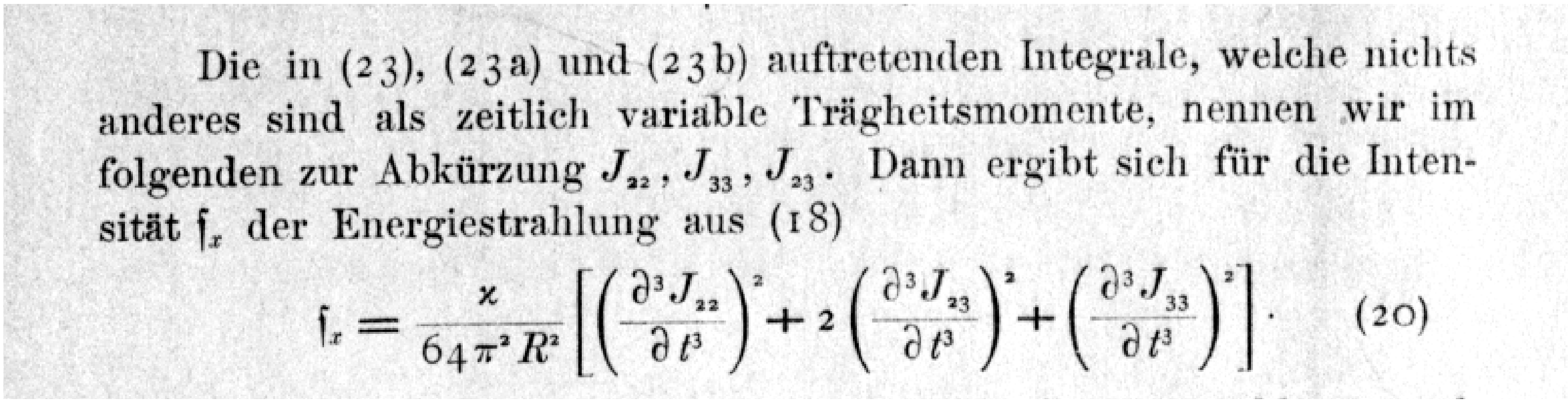
$$\mathbf{h_{22}} \;=\; -\mathbf{h_{33}} \qquad\qquad \mathbf{h_{23}} \;=\; \mathbf{h_{32}}$$

+ polarization **×polarization**

And All Only Function of $x_1 \;-\; ct$


x
z
y
+ Polarization


x
z
y
× Polarization



Die in (23), (23a) und (23b) auftretenden Integrale, welche nichts anderes sind als zeitlich variable Trägheitsmomente, nennen wir im folgenden zur Abkürzung J_{22}, J_{33}, J_{23}. Dann ergibt sich für die Intensität $\mathfrak{f}_x$ der Energiestrahlung aus (18)

$$\mathfrak{f}_x = \frac{\varkappa}{64\pi^2 R^2}\left[\left(\frac{\partial^3 J_{22}}{\partial t^3}\right)^2 + 2\left(\frac{\partial^3 J_{23}}{\partial t^3}\right)^2 + \left(\frac{\partial^3 J_{33}}{\partial t^3}\right)^2\right]. \qquad (20)$$

SPHERICALLY SYMMETRIC MOTION RADIATES GRAVITATIONAL WAVES

1918 **VI. VII. VIII**

SITZUNGSBERICHTE

DER

KÖNIGLICH PREUSSISCHEN

AKADEMIE DER WISSENSCHAFTEN

Sitzung der physikalisch-mathematischen Klasse am 7. Februar. (S. 139)
Sitzung der philosophisch-historischen Klasse am 7. Februar. (S. 141)
J. Kirchner: Archon Euthios. (S. 142)
Gesamtsitzung am 14. Februar. (S. 153)
Einstein: Über Gravitationswellen. (Mitteilung vom 31. Januar.) (S. 154)
E. Freundlich: Über die singulären Stellen der Lösungen des n-Körper-Problems. 1. Mitteilung. (Mitteilung vom 31. Januar.) (S. 168)

BERLIN 1918

VERLAG DER KÖNIGLICHEN AKADEMIE DER WISSENSCHAFTEN

IN KOMMISSION BEI GEORG REIMER

Über Gravitationswellen.

Von A. Einstein.

(Vorgelegt am 31. Januar 1918 [s. oben S. 79].)

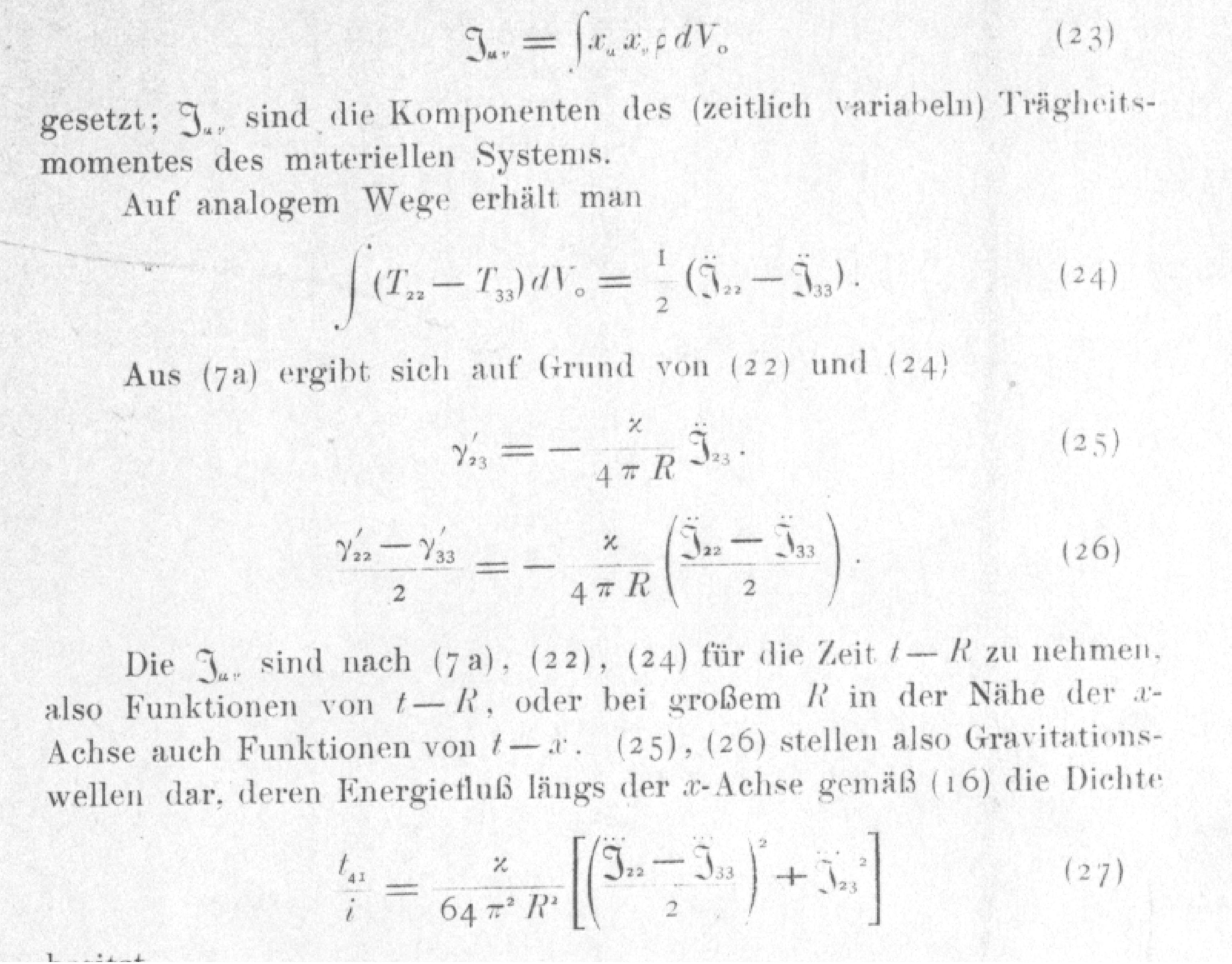
Die wichtige Frage, wie die Ausbreitung der Gravitationsfelder erfolgt, ist schon vor anderthalb Jahren in einer Akademiearbeit von mir behandelt worden[1]. Da aber meine damalige Darstellung des Gegenstandes nicht genügend durchsichtig und außerdem durch einen bedauerlichen Rechenfehler verunstaltet ist, muß ich hier nochmals auf die Angelegenheit zurückkommen.

Wie damals beschränke ich mich auch hier auf den Fall, daß das betrachtete zeiträumliche Kontinuum sich von einem »galileischen« nur sehr wenig unterscheidet. Um für alle Indizes

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

Sind die Bedingungen (15) erfüllt, so stellt (14) eine mögliche Gravitationswelle dar. Um deren physikalische Natur genauer zu durchschauen, berechnen wir deren Dichte des Energiestromes $\frac{t_{41}}{i}$. Durch Einsetzen der in (15) gegebenen $\gamma'_{\mu\nu}$ in Gleichung (9) erhält man

$$\frac{t_{41}}{i} = \frac{1}{4\varkappa} f'^2 \left[\left(\frac{\alpha_{22} - \alpha_{33}}{2}\right)^2 + \alpha_{23}^2\right]. \qquad (16)$$



$$\mathfrak{J}_{\mu\nu} = \int x_\mu x_\nu \rho \, dV_0 \qquad (23)$$

gesetzt; $\mathfrak{J}_{\mu\nu}$ sind die Komponenten des (zeitlich variabeln) Trägheitsmomentes des materiellen Systems.

Auf analogem Wege erhält man

$$\int (T_{22} - T_{33})\, dV_0 = \frac{1}{2}\left(\ddot{\mathfrak{J}}_{22} - \ddot{\mathfrak{J}}_{33}\right). \qquad (24)$$

Aus (7a) ergibt sich auf Grund von (22) und (24)

$$\gamma'_{23} = -\frac{\varkappa}{4\pi R}\ddot{\mathfrak{J}}_{23}. \qquad (25)$$

$$\frac{\gamma'_{22} - \gamma'_{33}}{2} = -\frac{\varkappa}{4\pi R}\left(\frac{\ddot{\mathfrak{J}}_{22} - \ddot{\mathfrak{J}}_{33}}{2}\right). \qquad (26)$$

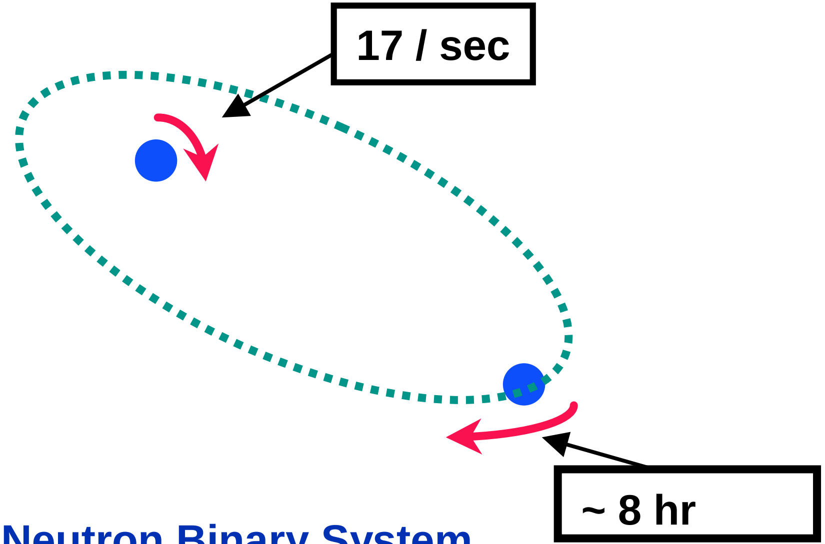
Die $\mathfrak{J}_{\mu\nu}$ sind nach (7a), (22), (24) für die Zeit $t - R$ zu nehmen, also Funktionen von $t - R$, oder bei großem R in der Nähe der x-Achse auch Funktionen von $t - x$. (25), (26) stellen also Gravitationswellen dar, deren Energiefluß längs der x-Achse gemäß (16) die Dichte

$$\frac{t_{41}}{i} = \frac{\varkappa}{64\pi^2 R^2}\left[\left(\frac{\dddot{\mathfrak{J}}_{22} - \dddot{\mathfrak{J}}_{33}}{2}\right)^2 + \dddot{\mathfrak{J}}_{23}^{\,2}\right] \qquad (27)$$

THE QUADRUPOLE FORMULA A FACTOR OF 2 TOO SMALL

Neutron Binary System – Hulse & Taylor

PSR 1913 + 16 -- Timing of pulsars



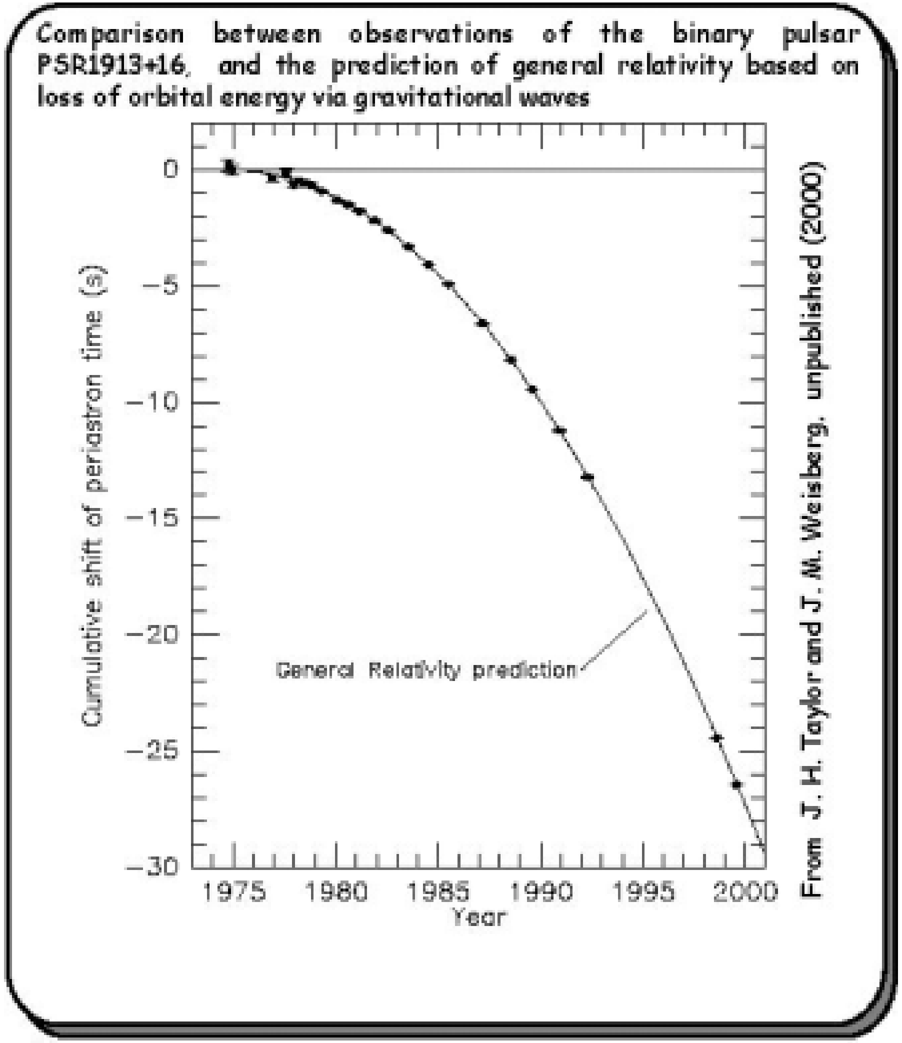
Neutron Binary System

- separated by 10^6 miles
- $m_1 = 1.4 m_\odot$; $m_2 = 1.36 m_\odot$; $\varepsilon = 0.617$

Prediction from general relativity

- spiral in by 3 mm/orbit
- rate of change orbital period

Emission of gravitational waves


Comparison between observations of the binary pulsar PSR1913+16, and the prediction of general relativity based on loss of orbital energy via gravitational waves

From J. H. Taylor and J. M. Weisberg, unpublished (2000)

Direct detection of gravitational waves from astrophysical sources

- Physics
  - Observations of gravitation in the strong field, high velocity limit
  - Determination of wave kinematics – polarization and propagation
  - Tests for alternative relativistic gravitational theories
- Astrophysics
  - Measurement of coherent inner dynamics – stellar collapse, pulsar formation….
  - Compact binary coalescence – neutron star/neutron star, black hole/black hole
  - Neutron star equation of state
  - Primeval cosmic spectrum of gravitational waves
- Gravitational wave survey of the universe

Measurement challenge

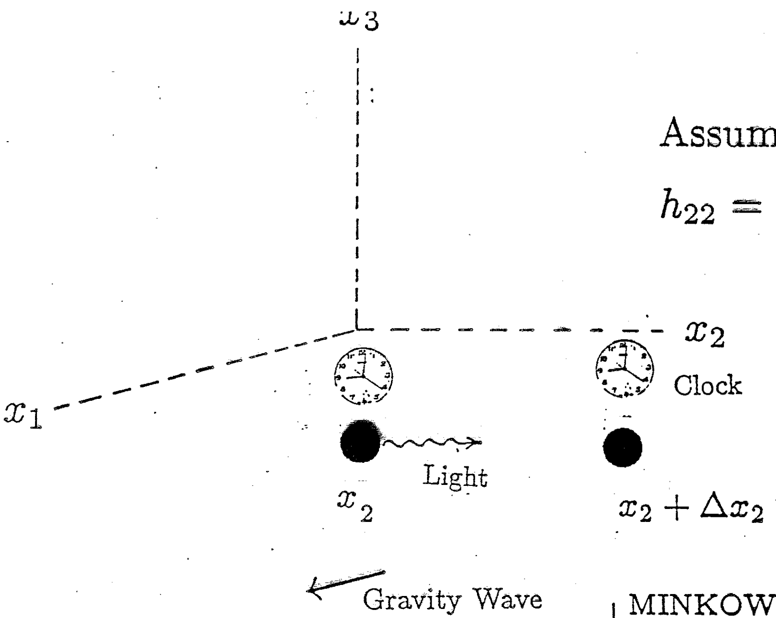
- Needed technology development to measure:

$$h = \Delta L/L < 10^{-21}$$

$$\Delta L < 4 \text{ x } 10^{-18} \text{ meters}$$

Assume + Polarization

$h_{22} = h \sin(kx_1 - \omega t)$



$$\Delta s^2 = 0 = c^2 \Delta t^2 - \Big(\overset{\text{MINKOWSKI}}{\overset{\downarrow}{1}} + h \sin(kx_1 - \omega t)\Big) \Delta x_2^2$$

LIGHT RAY

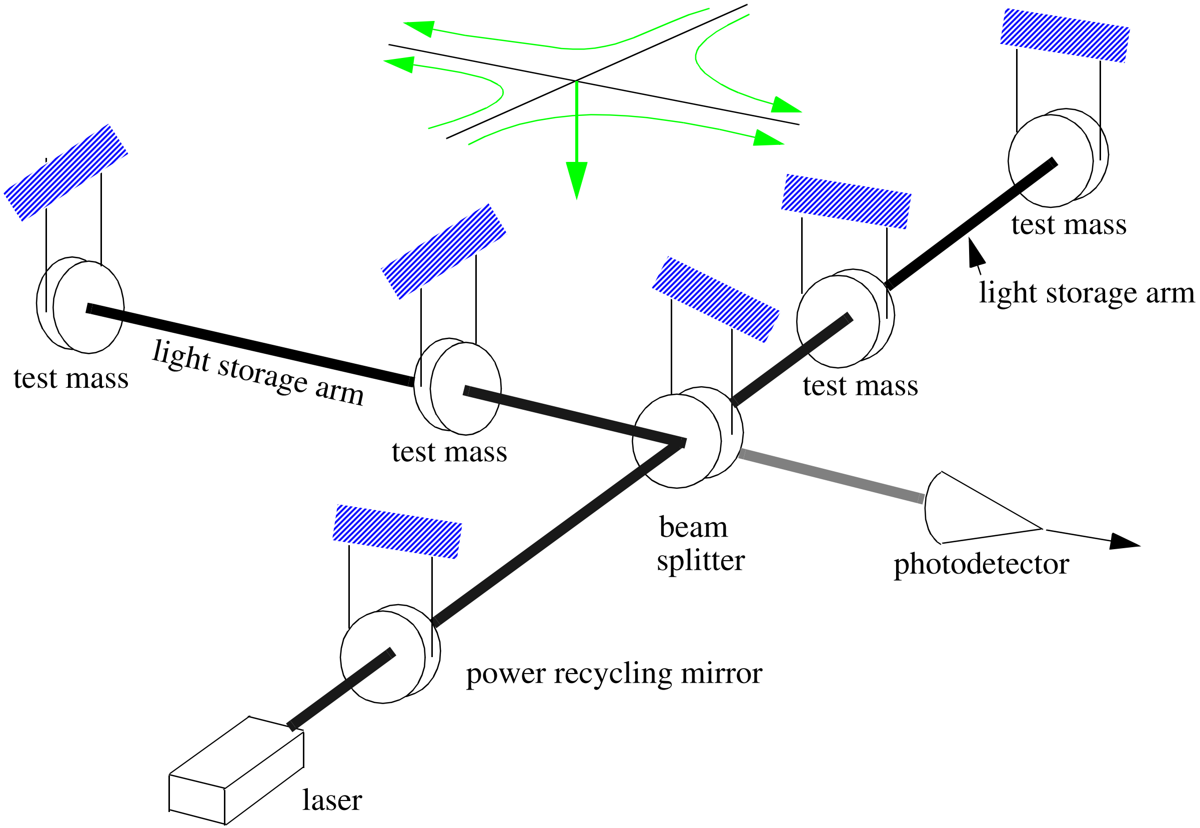
Let $\Delta t \ll \dfrac{1}{\omega} \qquad h \ll 1$

$$c\Delta t \cong \Big(1 + \frac{h}{2} \sin(kx_1 - \omega t)\Big) \Delta x_2$$

↖ *INFERRED DISTANCE BETWEEN POINTS*

$$\frac{\delta(c\Delta t)}{\Delta x_2} = \frac{h}{2} \sin(kx_1 - \omega t)$$ Time Dependent **Strain**

$$\frac{\Delta l}{l} = \frac{h}{2}$$ The Measurable Quantity


test mass
light storage arm
test mass
light storage arm
test mass
test mass
beam
splitter
photodetector
power recycling mirror
laser

FRINGE SENSING

wavelength 1 x 10 $^{-6}$ m

$$h = \frac{x}{L} \sim \frac{\lambda}{L b \sqrt{\dot{N}\tau}}$$

arm length = 4000 m

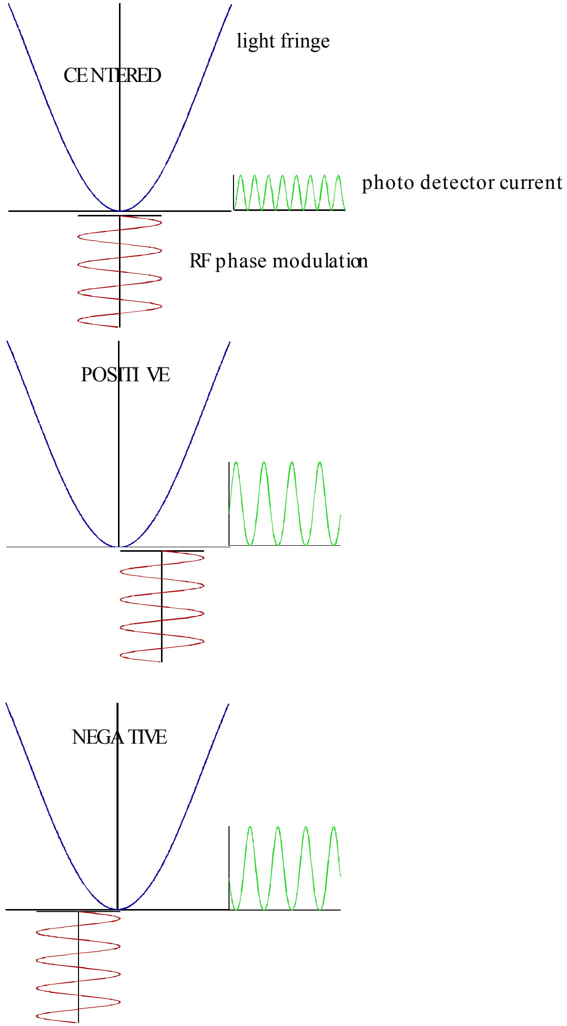
integration time

equivalent # of passes = 100

number of quanta/second at the beam splitter

300 watts at beam splitter = 10^{21} identical photons/sec

$h = 6 \times 10^{-22}$ integration time 10^{-2} sec



PENDULUM THERMAL NOISE

Pendulum Brownian motion

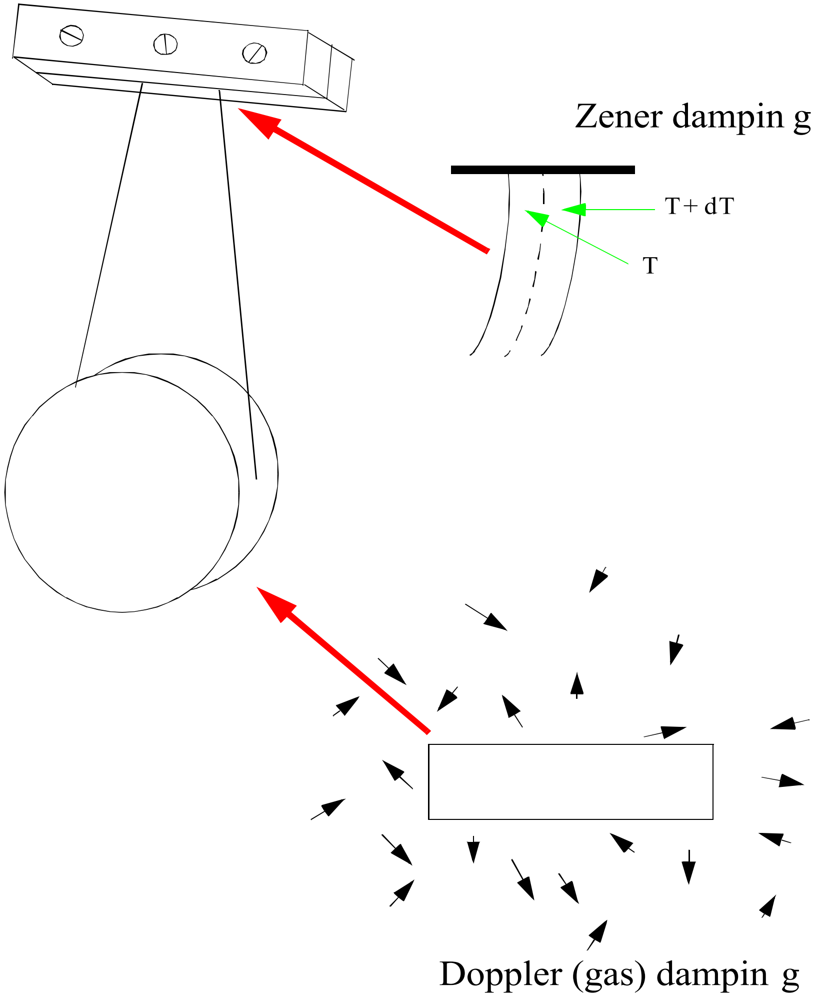
Dissipation leads to fluctuations

Tc = coherence or damping time
= Q x period of oscillator

Exchange with surroundings:

$$E(\text{thermal}) = \frac{kT\ t}{Tc}$$

Large Tc => smaller fluctuations



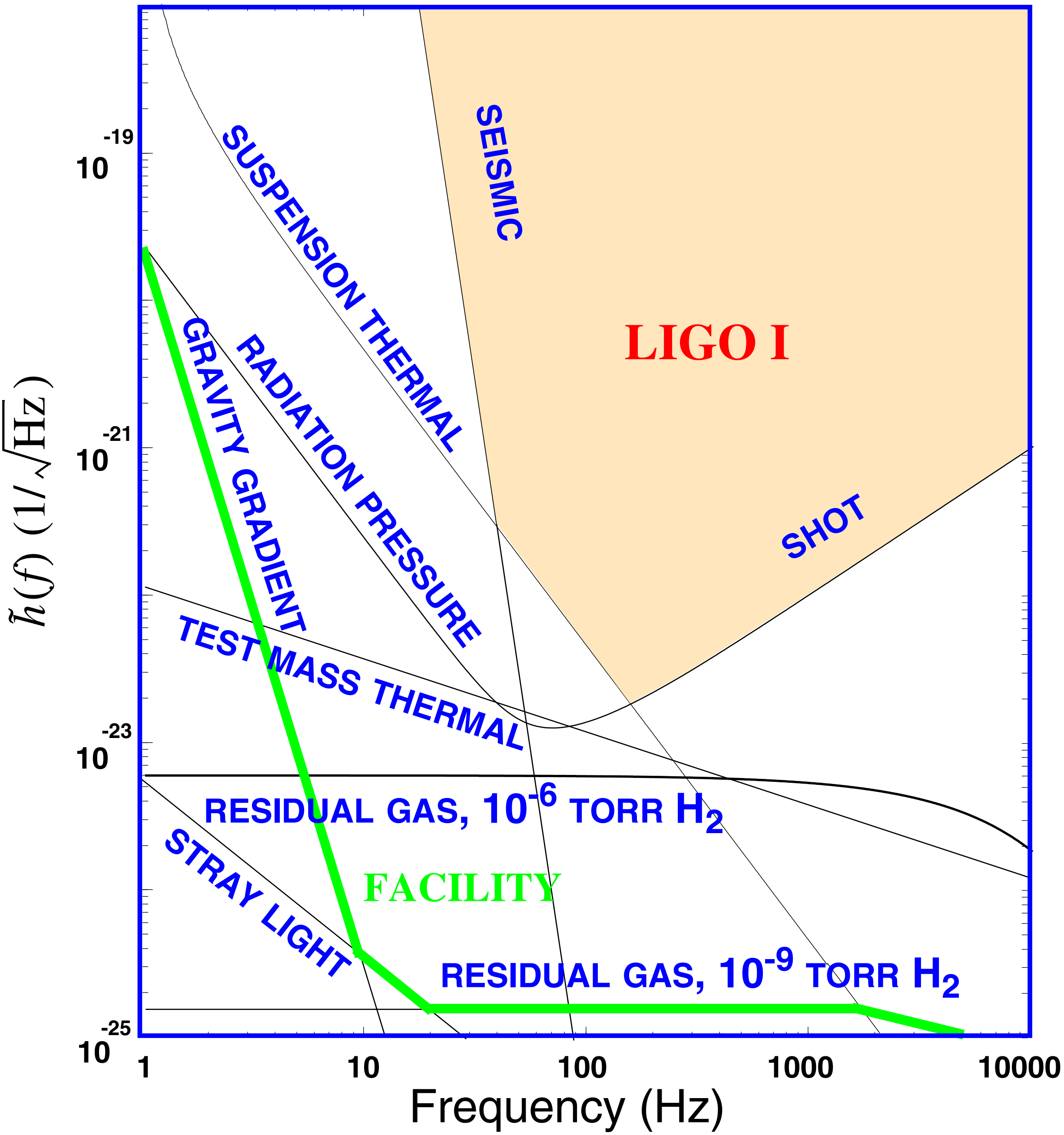
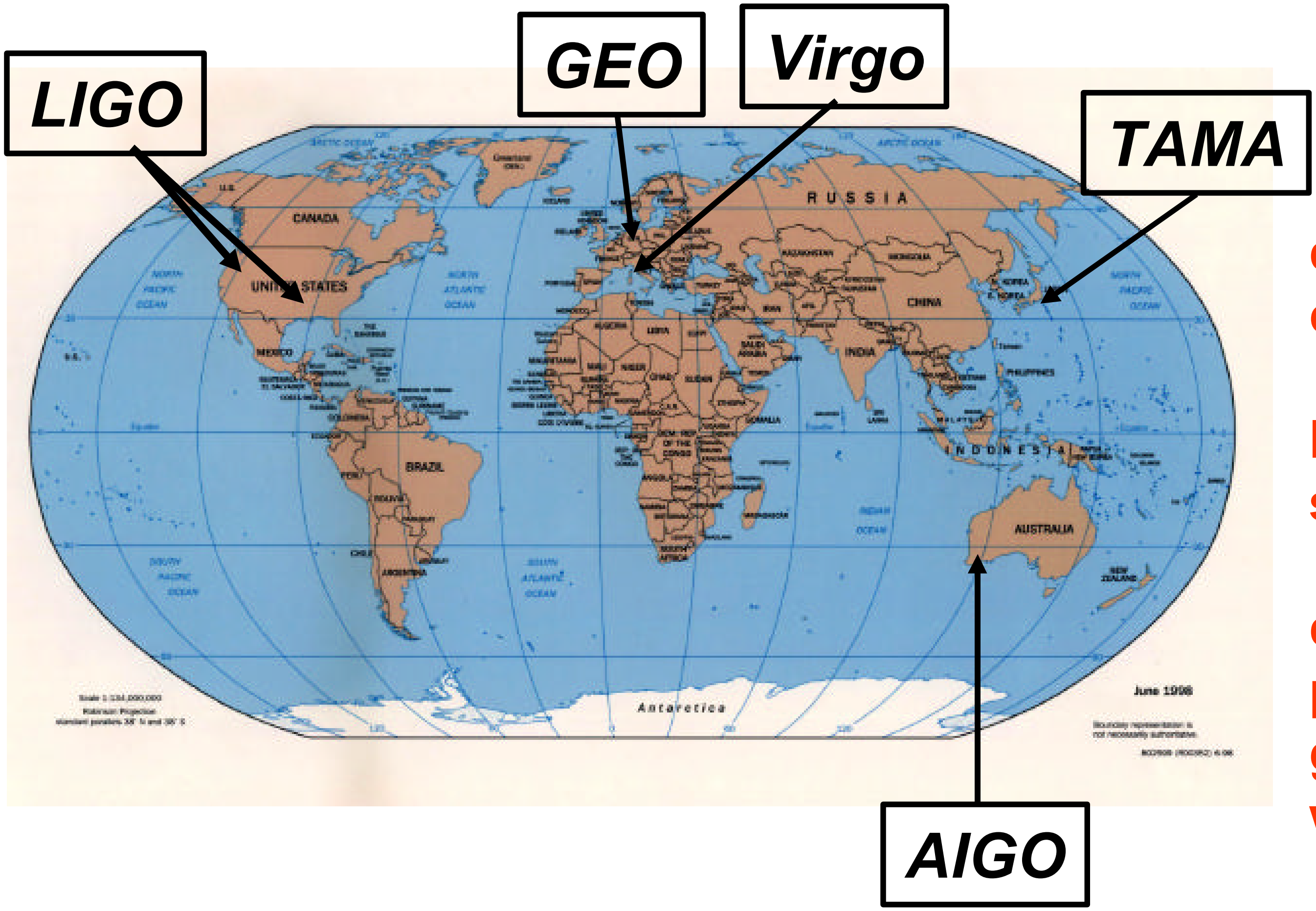

$\tilde{h}(f)$ $(1/\sqrt{\text{Hz}})$
10^{-19}
10^{-21}
10^{-23}
10^{-25}
1
10
100
1000
10000
Frequency (Hz)
SUSPENSION THERMAL
SEISMIC
LIGO I
GRAVITY GRADIENT
RADIATION PRESSURE
SHOT
TEST MASS THERMAL
RESIDUAL GAS, 10^{-6} TORR H_2
STRAY LIGHT
FACILITY
RESIDUAL GAS, 10^{-9} TORR H_2

Table 1: Initial detector parameters

Parameter	*Nominal Initial Interferometer*
Arm length	4000 m
Laser type @wavelength	Nd:YAG λ = 1064 nm
Input power at recycling cavity	6 W
Contrast defect 1-c	$< 3 \times 10^{-3}$
Mirror loss	$< 1 \times 10^{-4}$
Power recycling gain	30
Arm cavity storage time	880 μ sec
Cavity input mirror transmission	3×10^{-2}
Mirror mass	10.7 kg
Mirror diameter	25 cm
Mirror internal Q	1×10^{6}
Pendulum Q (structure damping)	1×10^{5}
Pendulum period (single)	1 sec
Seismic isolation system	T(100Hz) = -110dB

Interferometers
international network

Simultaneously detect signal (within msec)

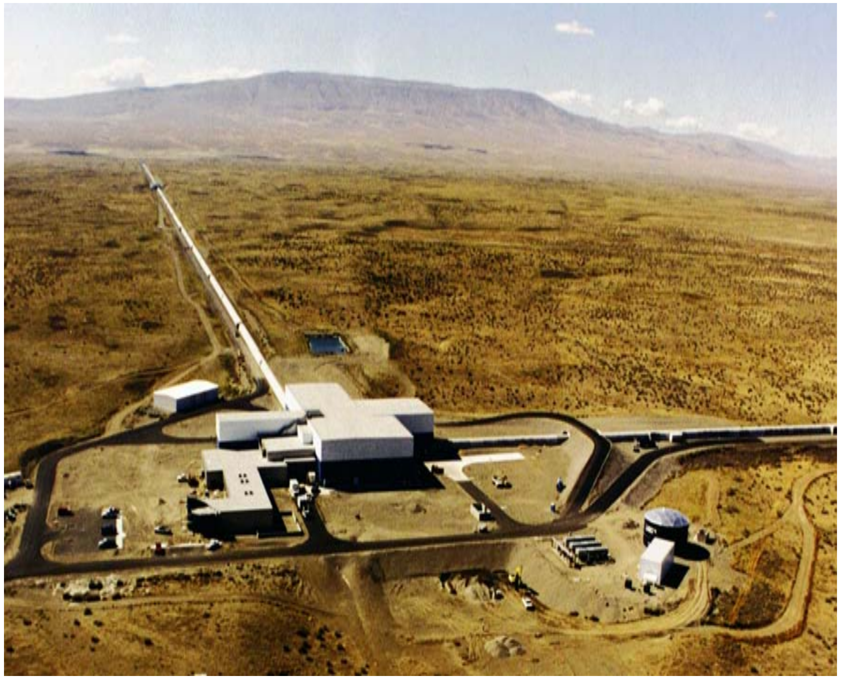


detection confidence

locate the sources

decompose the polarization of gravitational waves

LIGO Observatory Facilities



LIGO Hanford Observatory [LHO]

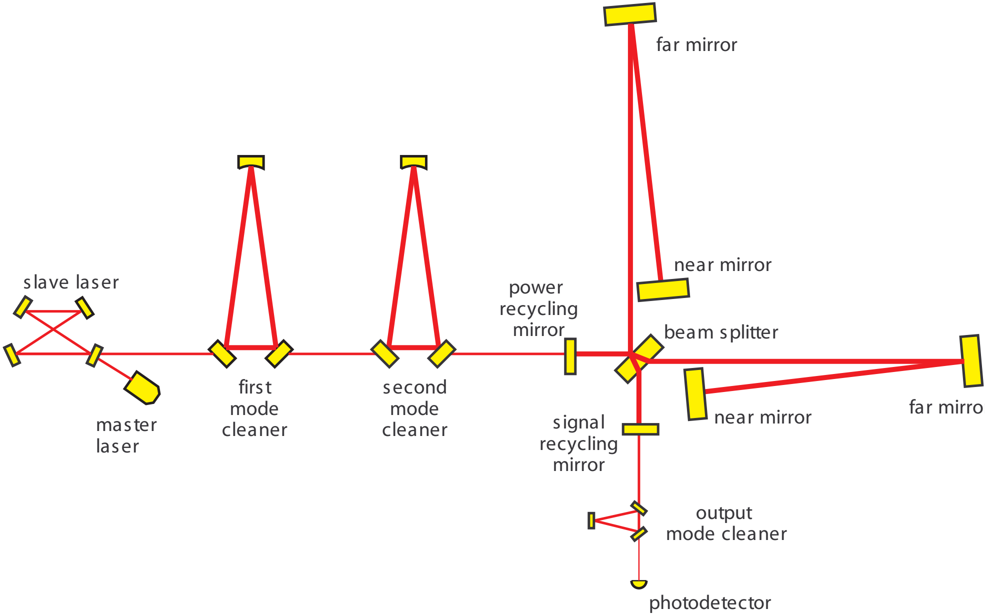
26 km north of Richland, WA

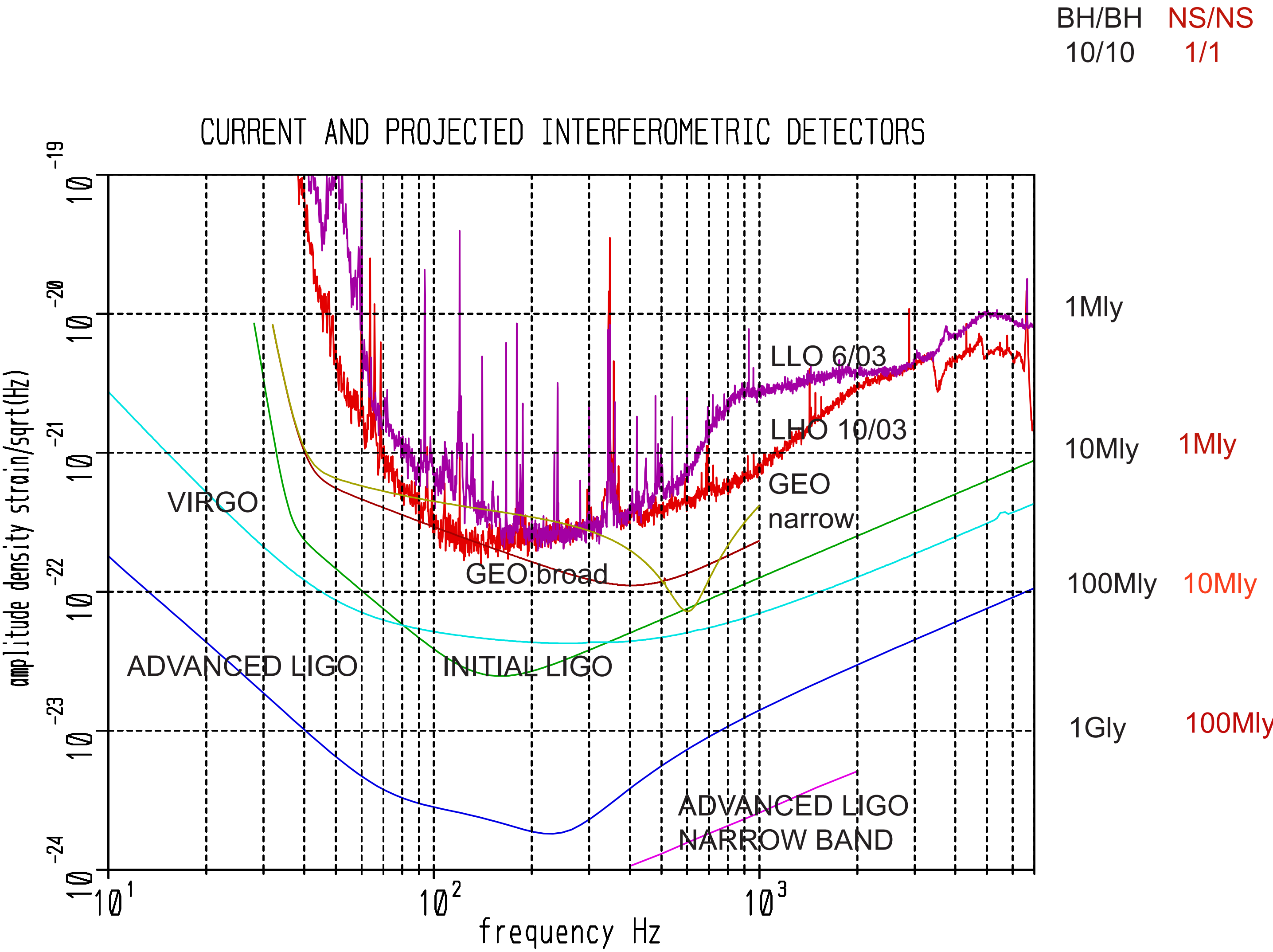
2 km + 4 km interferometers in same vacuum envelope

LIGO Livingston Observatory [LLO]

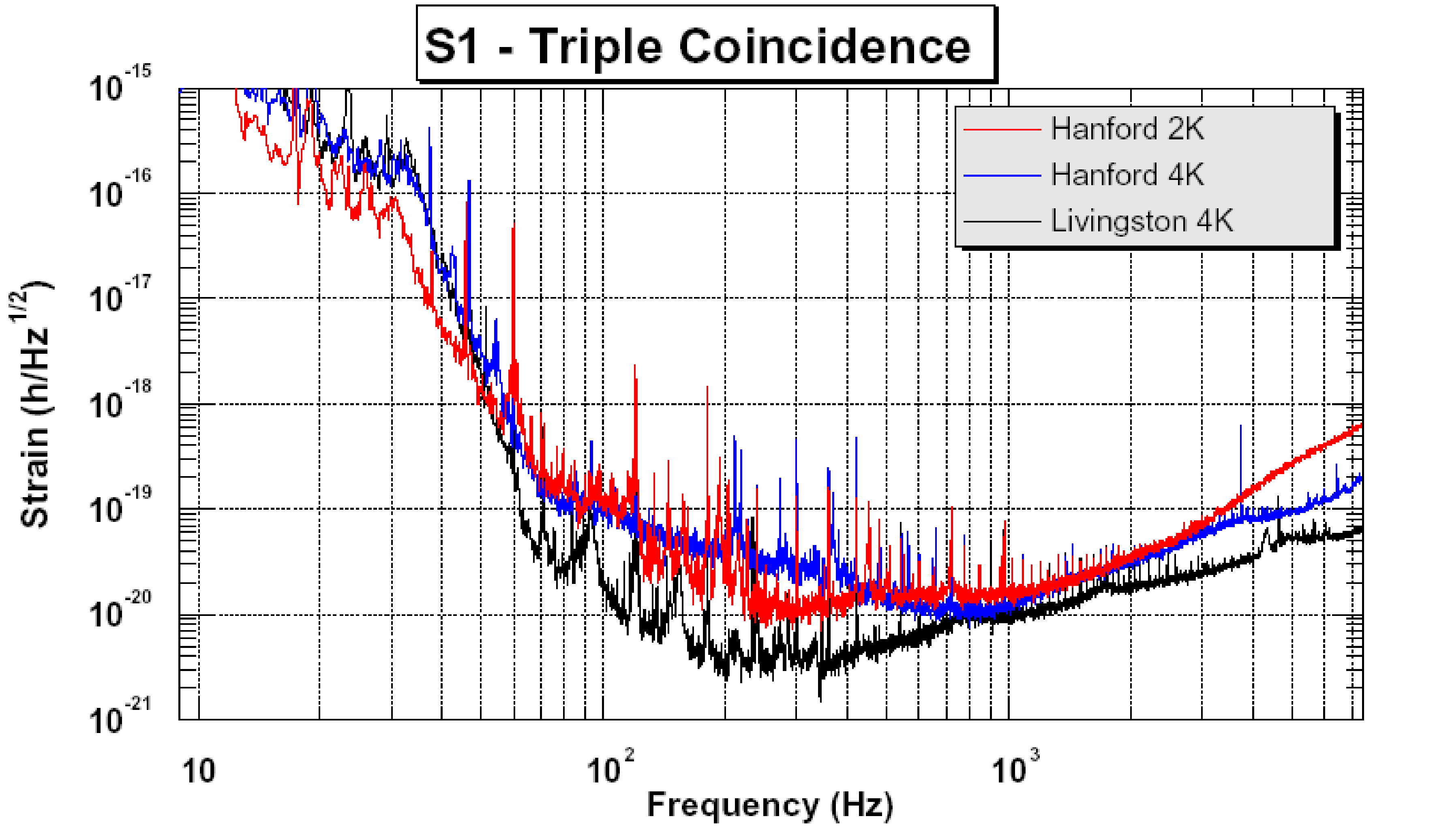
42 km east of Baton Rouge, LA

Single 4 km interferometer


far mirror
near mirror
slave laser
power
recycling
mirror
beam splitter
far mirror
first
mode
cleaner
second
mode
cleaner
master
laser
signal
recycling
mirror
near mirror
output
mode cleaner
photodetector

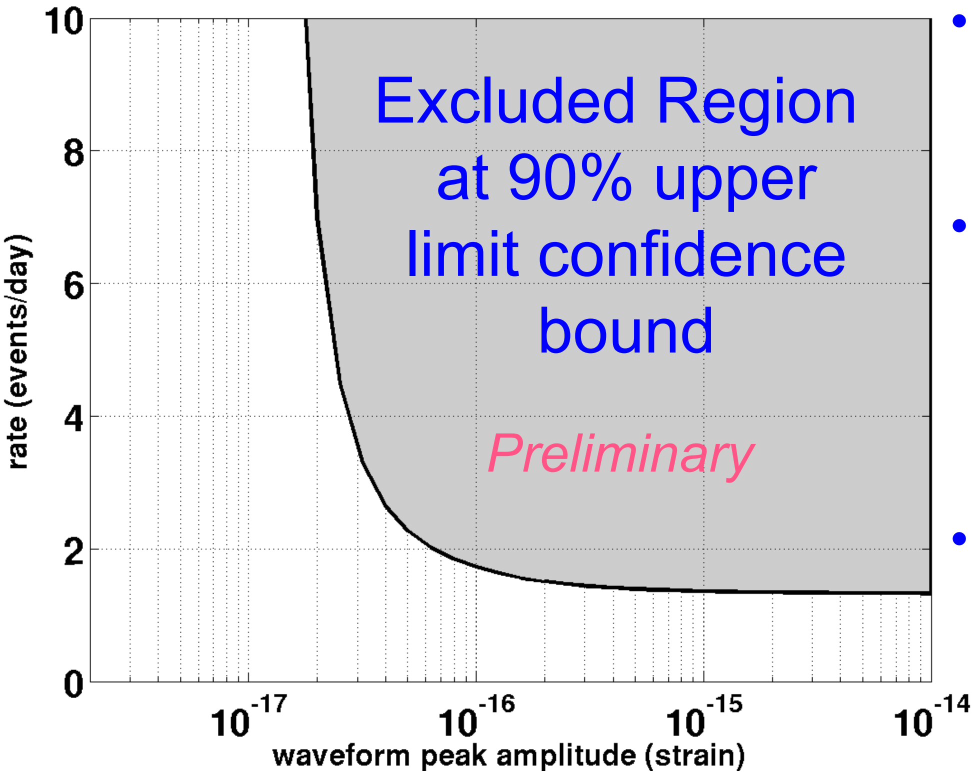

BH/BH NS/NS
10/10 1/1
CURRENT AND PROJECTED INTERFEROMETRIC DETECTORS
amplitude density strain/sqrt(Hz)
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{1}
10^{2}
10^{3}
frequency Hz
LLO 6/03
LHO 10/03
GEO
narrow
VIRGO
GEO broad
ADVANCED LIGO
INITIAL LIGO
ADVANCED LIGO
NARROW BAND
1Mly
10Mly 1Mly
100Mly 10Mly
1Gly 100Mly

S1 run: Aug 23-Sept 9



Burst Searches -- Preliminary Results

Burst Working Group



Work in progress:

- Correlations with gamma ray bursts
- Observed Type II SNe

- *Able to exclude gravitational wave bursts* of peak strength *h* above rate *r*
- Burst model --
  - » 1 ms width Gaussian pulse
  - » Linear polarization with random orientation
  - » Arriving from random directions
- *Upper limit in <u>strain sensitivity</u> with regard to prior (cryogenic bar) observations:*
  - » *Within 5X of IGEC 2000[1] results*
    - – *ICEG observation time was much longer than S1 - 90 days (triple bars)*
  - » *Within 25X of Astone[2] et al. 2001 sensitivity*

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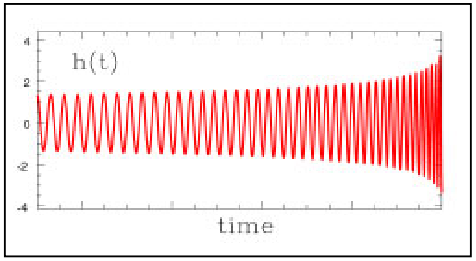
[1] *Int.J.Mod.Phys. D9 (2000) 237*

[2] *Class.Quant.Grav. 19 (2002) 5449*

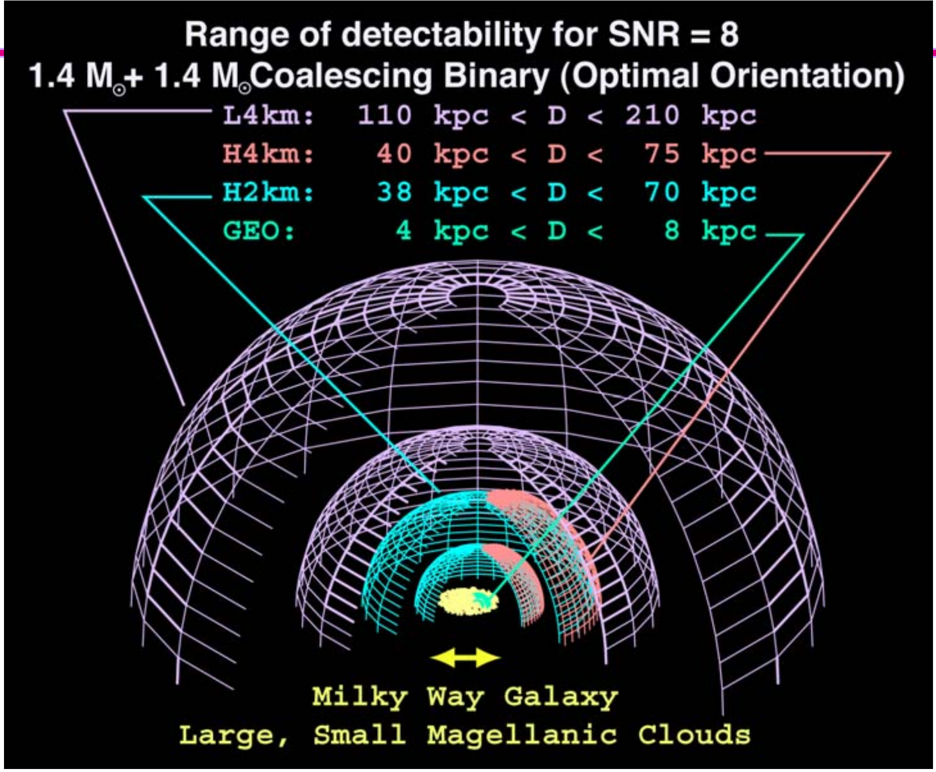
Coalescing Binaries

Inspiral Sources Working Group

- Three targets:
  - » Neutron star binaries (1-3 M_{sun})
  - » Black hole binaries (> 3 M_{sun})
  - » MACHO binaries (0.5-1 M_{sun})
- Search method
  - » template based matched filtering



- Status
  - ✓ Neutron star search complete
  - » MACHO search under way
  - » Black hole search will be done in next science run, S2



Range of detectability for SNR = 8

$1.4\ M_\odot + 1.4\ M_\odot$ Coalescing Binary (Optimal Orientation)

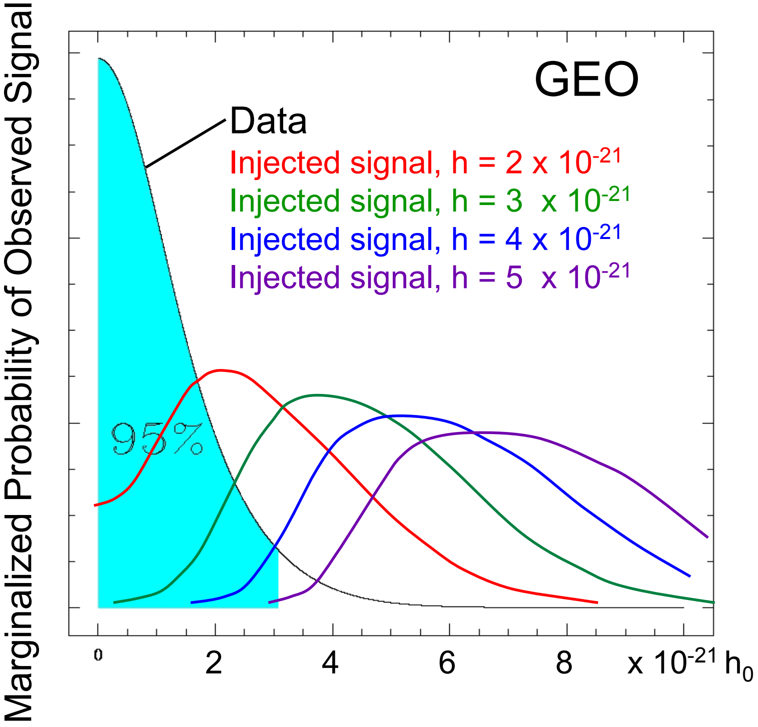
L4km: 110 kpc < D < 210 kpc
H4km: 40 kpc < D < 75 kpc
H2km: 38 kpc < D < 70 kpc
GEO: 4 kpc < D < 8 kpc

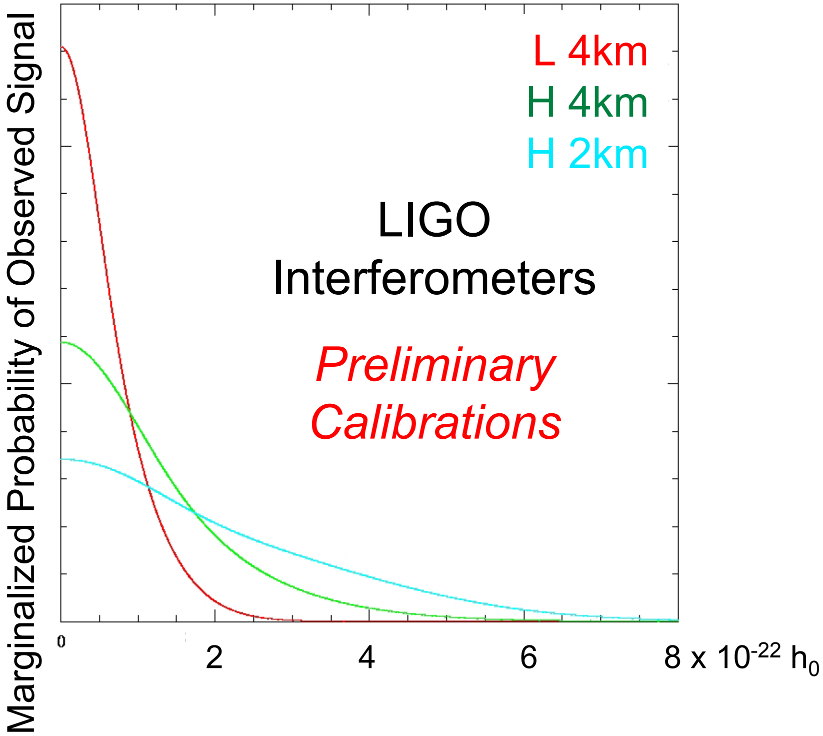
- Limit on binary neutron star coalescence rate:
  - » *$R_{90\%}$ (Milky Way) < 2.3 / (0.35 x 295.3 hr) = 170 /yr*
- Use triggers from H 4km and L 4km interferometers: *T = 295.3 hours*
  - » Monte Carlo simulation efficiency: *ε = 35%*
  - » 90% confidence limit = *2.3 / (ε T)*
- *26X lower than best published observational limit -- 40m prototype at Caltech[1]:*
  - » *$R_{90\%}$ (Milky Way) < 4400 /yr*

[1] *1994 data, Allen et al., Phys.Rev.Lett. 83 (1999) 1498*

Preliminary results -- (EM pulsar) PSR J1939

Periodic Sources Working Group

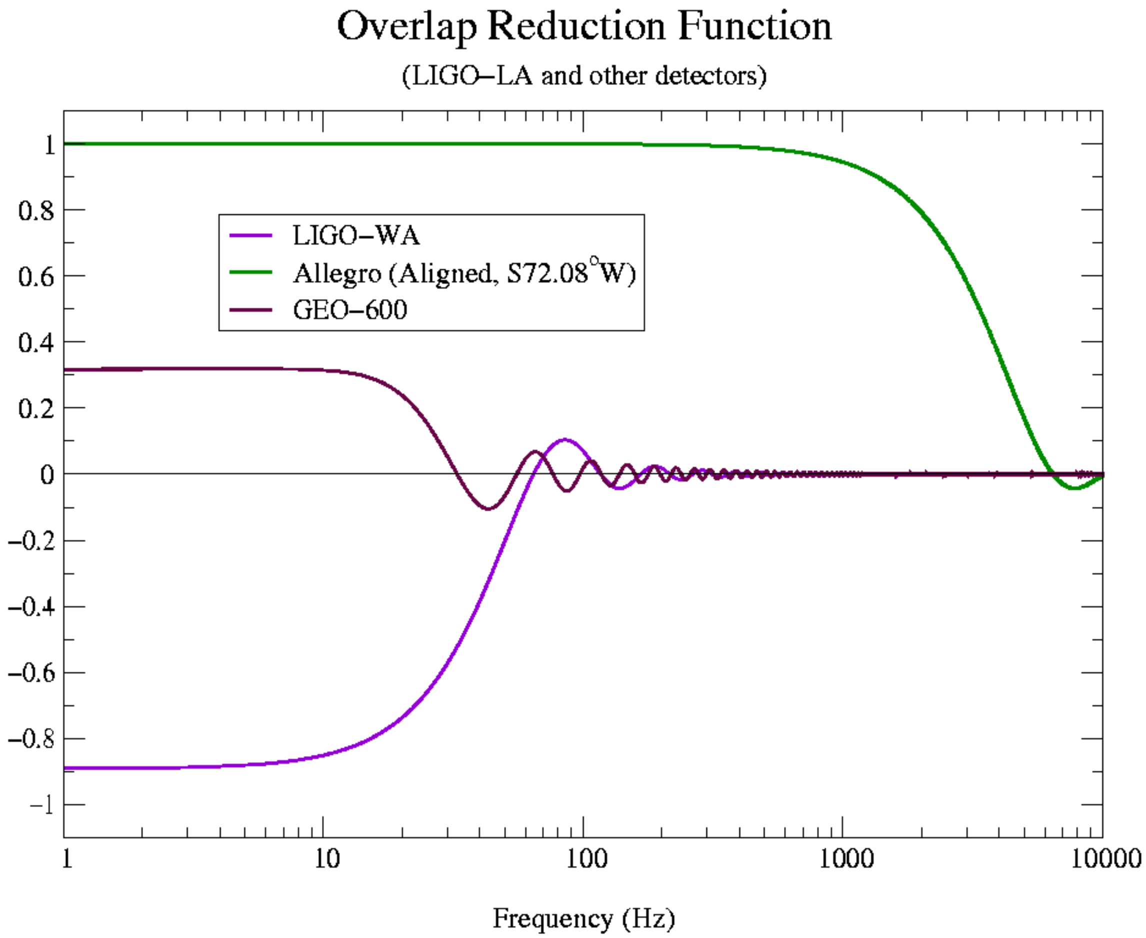




- No evidence of signal from PSR J1939 at f = 1283.86 Hz
- 95% of the probability lies below:
  - GEO: $h_{max} < 3 \times 10^{-21}$
  - H 2km: $h_{max} < 5 \times 10^{-22}$
  - H 4km: $h_{max} < 3 \times 10^{-22}$
  - ***L 4km:*** $h_{max} < 2 \times 10^{-22}$ ***(***$\varepsilon < 7 \times 10^{-5}$ ***@ 3.6 kpc)***

Overlap reduction function

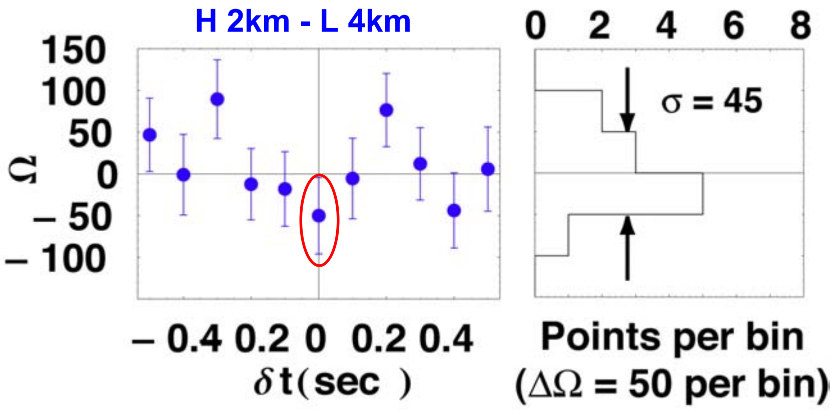
Specifies the reduction in sensitivity due to the separation and orientation of the two detectors:

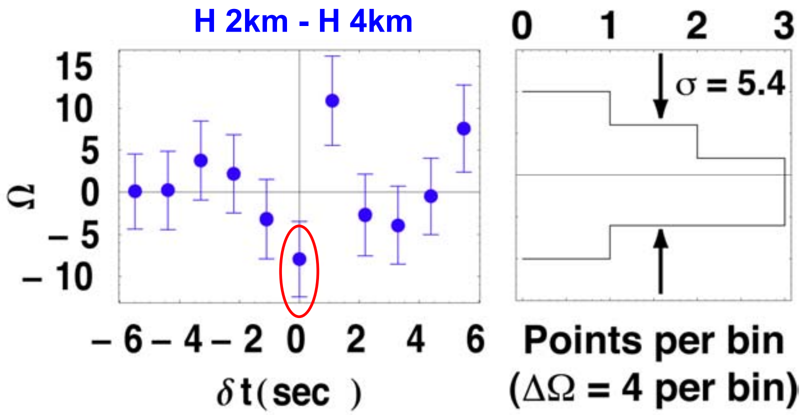


Stochastic Gravitational Wave Background

Stochastic Background Sources Working Group

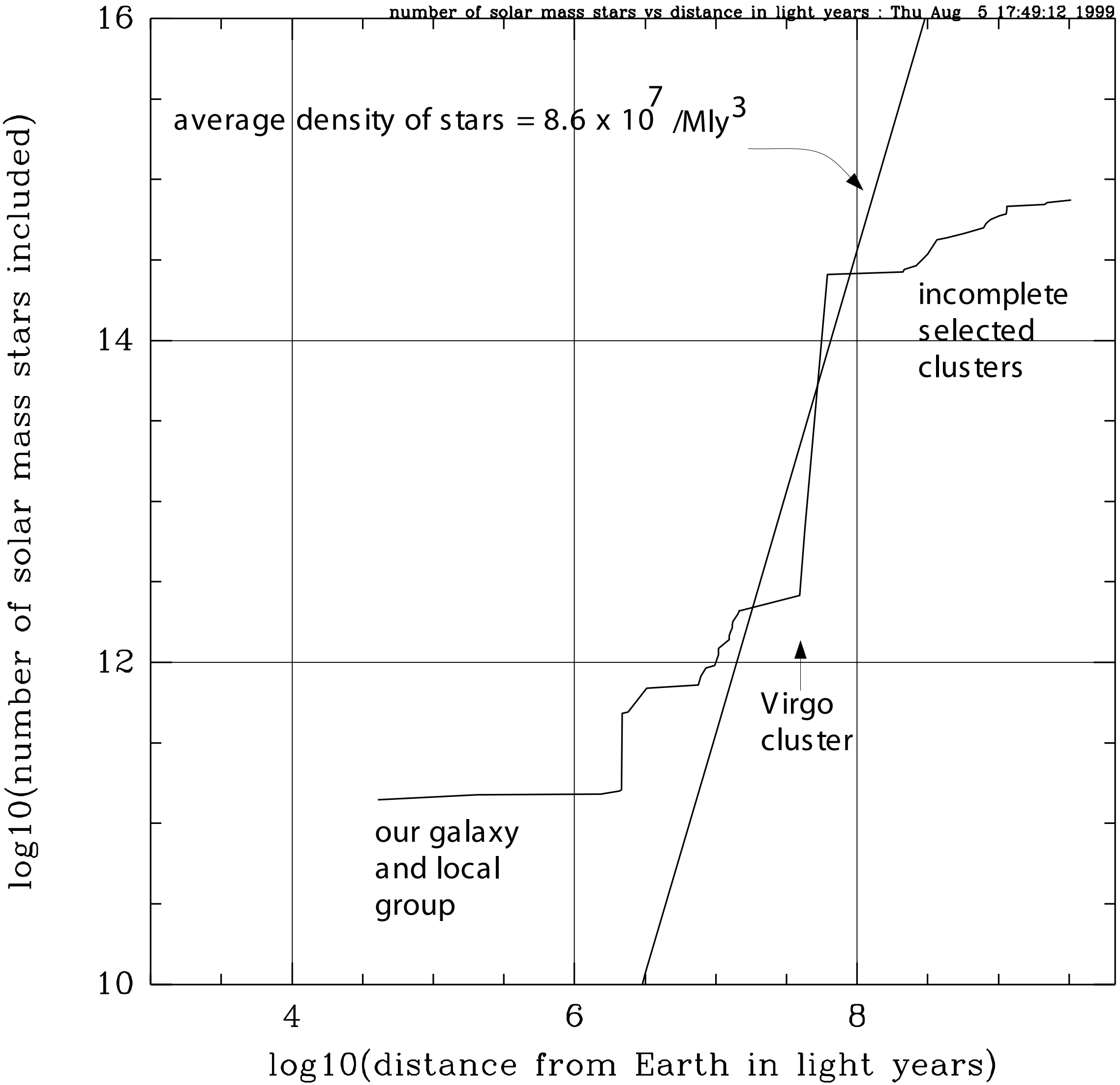
- Preliminary results from 7.5 hr of data -





- Introduce non-astrophysical time lags (>20ms) to determine backgrounds *(off-source)*
  - *δt = 0 sec (on-source)* measurements consistent with *off-source* backgrounds
- Extrapolated S1 H 2km - H 4km result covers **240 Hz bandwidth, is ~10X better** than best published result for *direct measurement* of Ω_{GW} (Astone et al., 1999, cryogenic bar, 907 Hz).
- Ultimate sensitivity for LIGO I: ~ 1 x 10^{-5} for T_{obs} = 4 months

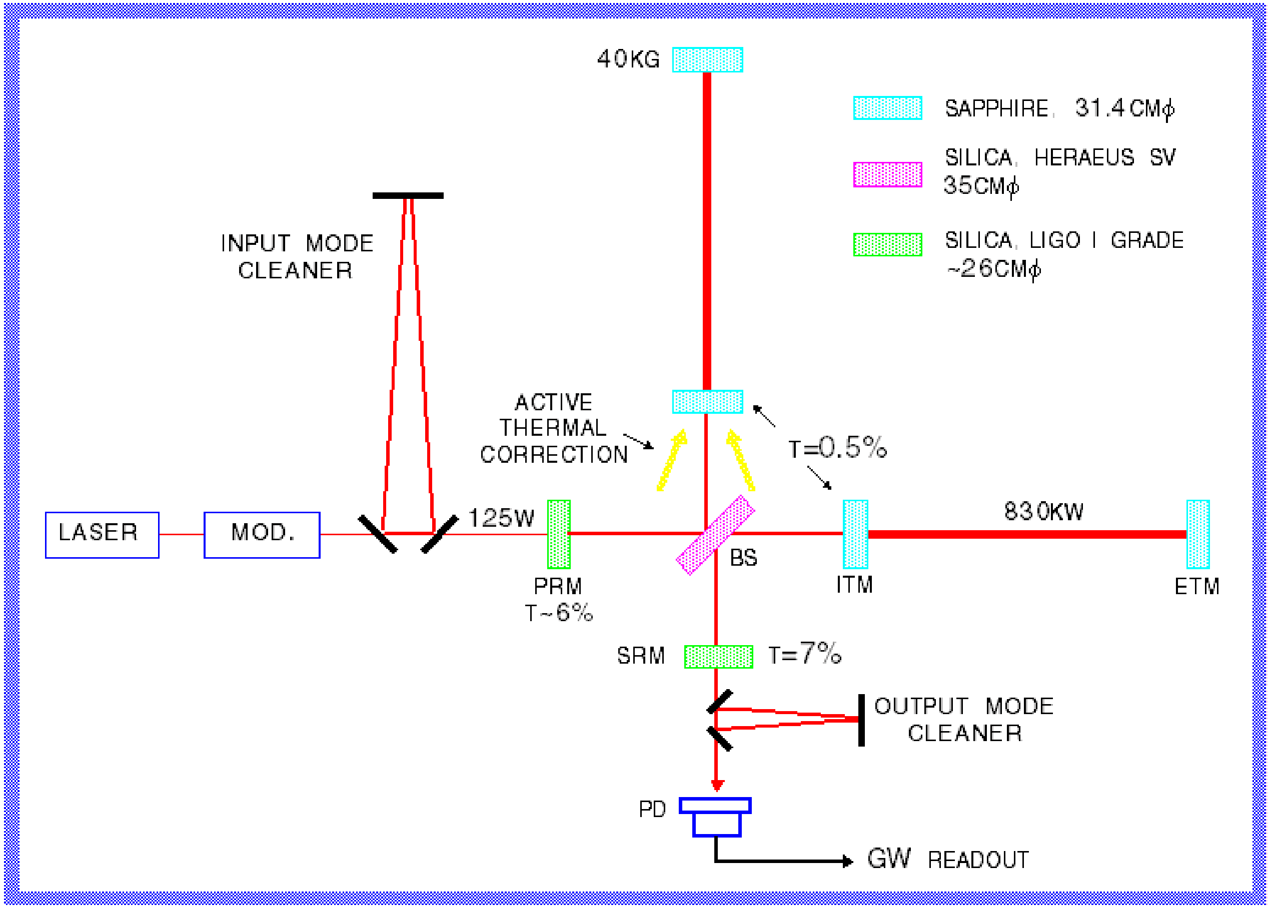
Interferometer Pair	Measurement Bandwidth	**Extrapolated Upper Limit for S1 (by scaling 7.5 hrs to 150 or 100 hrs)**	**T_{obs}**
H 2km - H 4km	40 Hz < f < 300 Hz	**Ω_{GW} < 5 (90% C.L.)**	**150 hr**
H 4km - L 4km	40 Hz < f < 314 Hz	**Ω_{GW} < 70 (90% C.L.)**	**100 hr**
H 2km - L 4km	40 Hz < f < 314 Hz	**Ω_{GW} < 50 (90% C.L.)**	**100 hr**



DATA: Cosmology of the Local Group G.Lake
Astrophysical Quantities C.W. Allen

Shoemaker Desaintfuscier Brillet

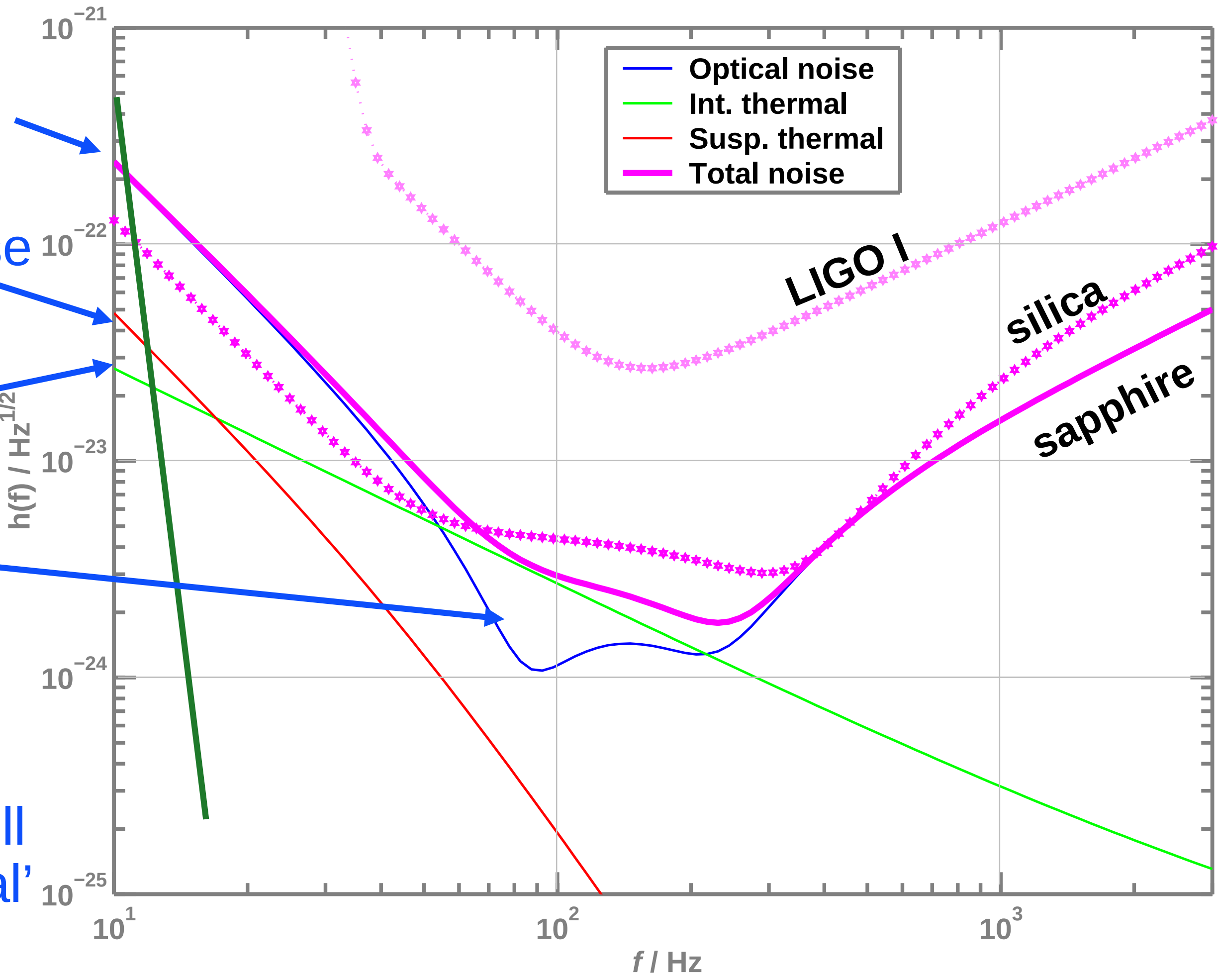
Advanced Interferometer Concept



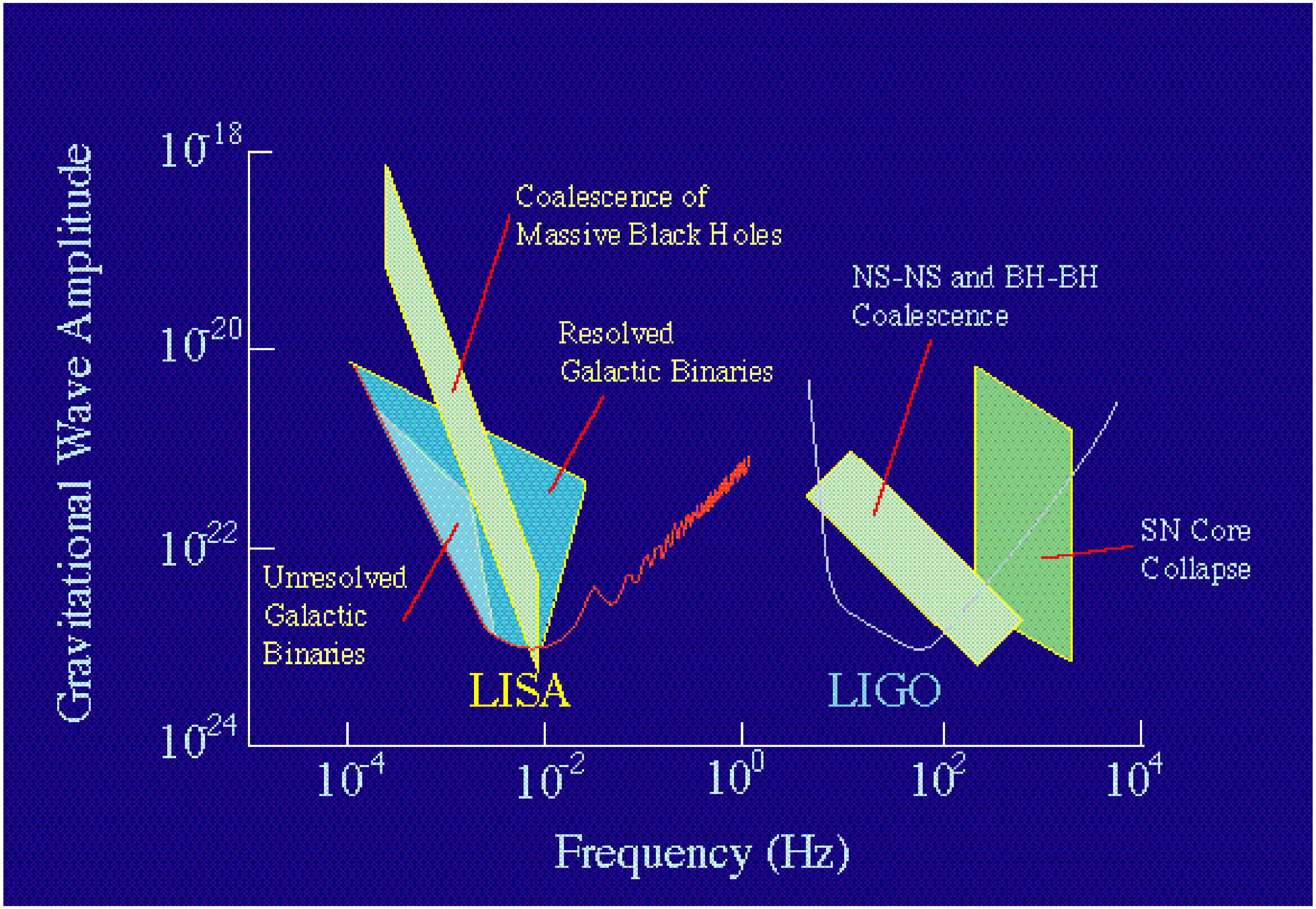
- » Signal recycling
- » 180-watt laser
- » 40 kg Sapphire test masses
- » Larger beam size
- » Quadruple suspensions
- » Active seismic isolation
- » Active thermal correction
- » Output mode cleaner

Projected Performance

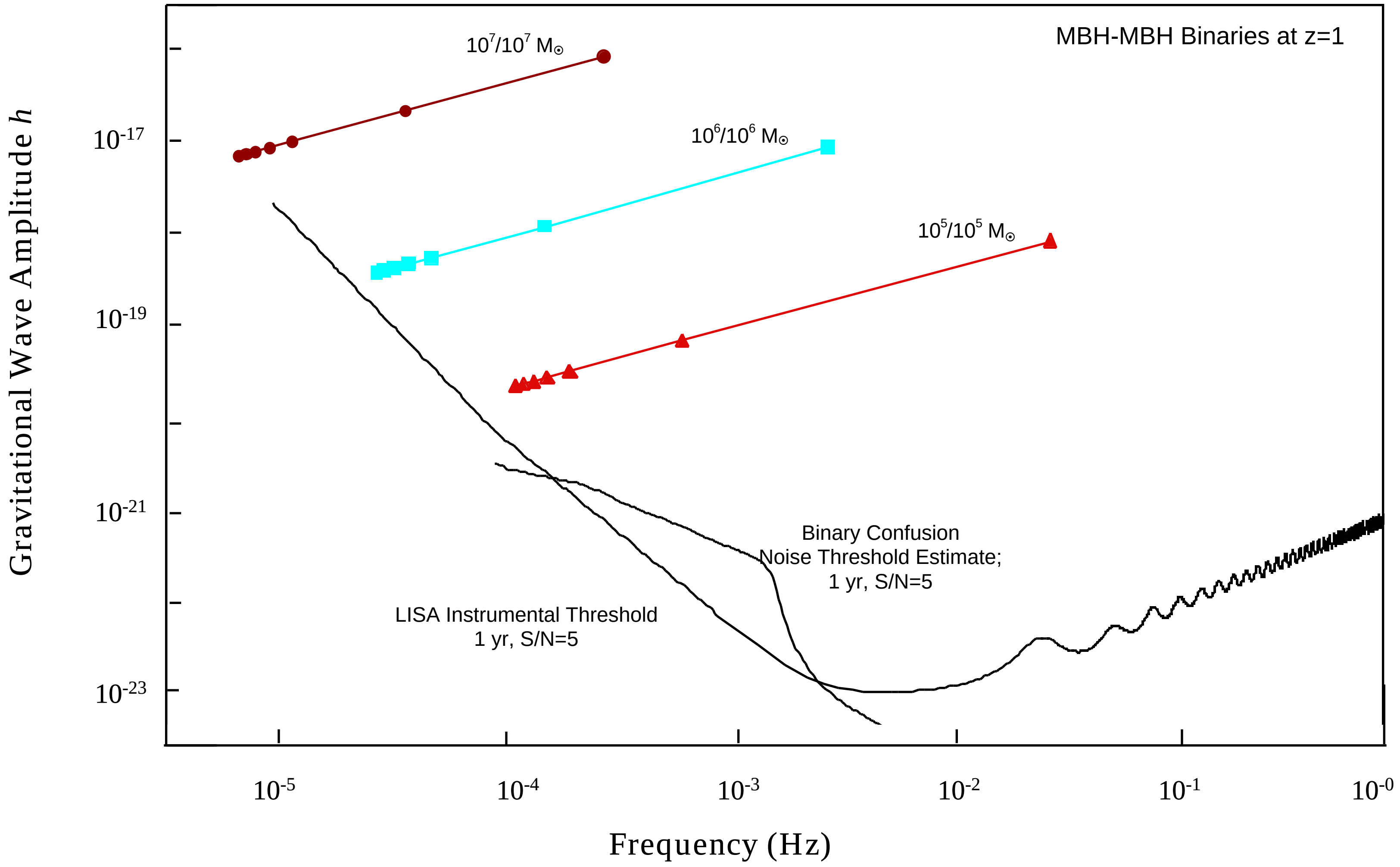
- Seismic ‘cutoff’ at 10 Hz
- Suspension thermal noise
- Internal thermal noise
- Unified quantum noise dominates at most frequencies
- ‘technical’ noise (e.g., laser frequency) levels held in general well below these ‘fundamental’ noises



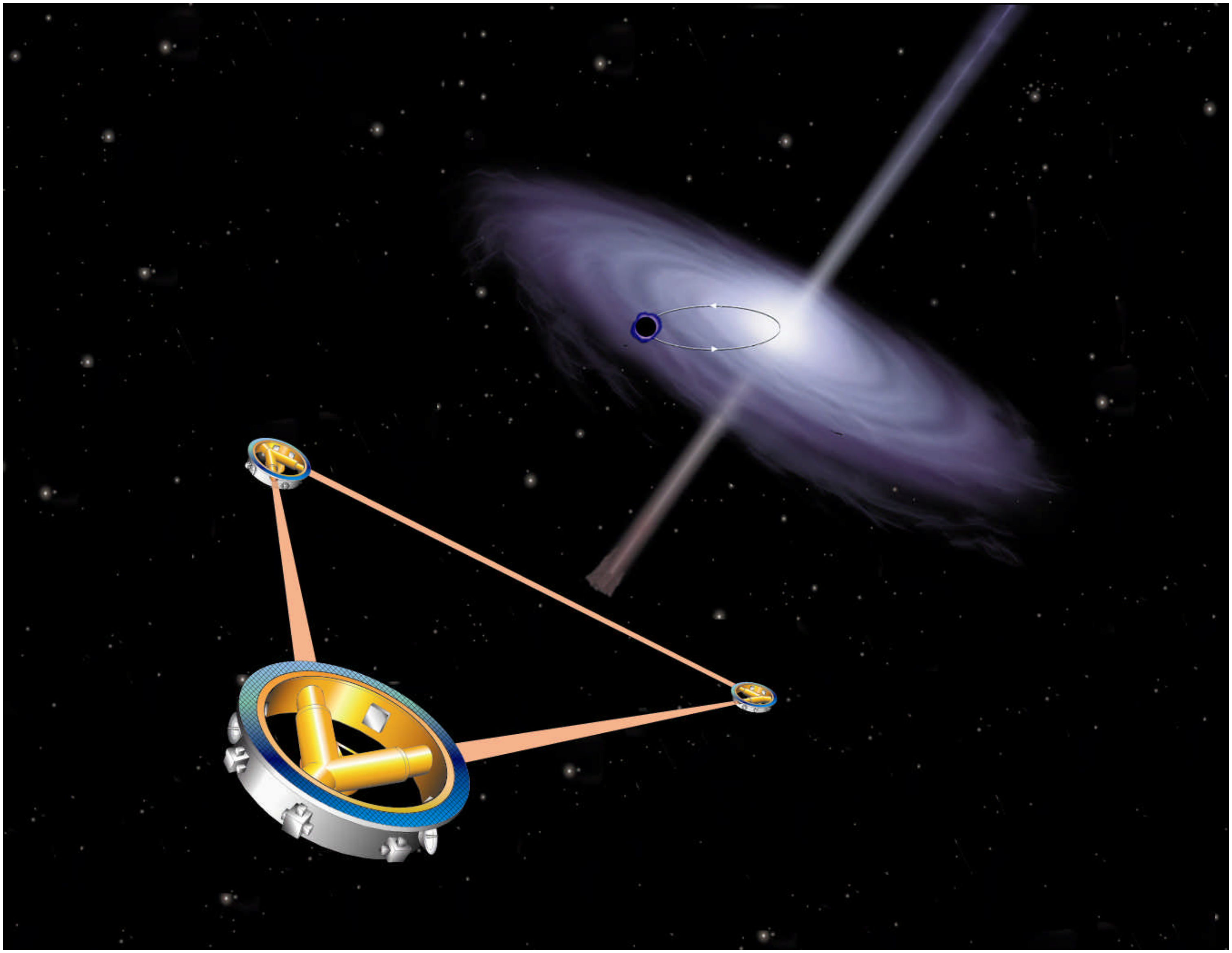
The Gravitational-Wave Spectrum



Massive Black Holes in Merging Galaxies

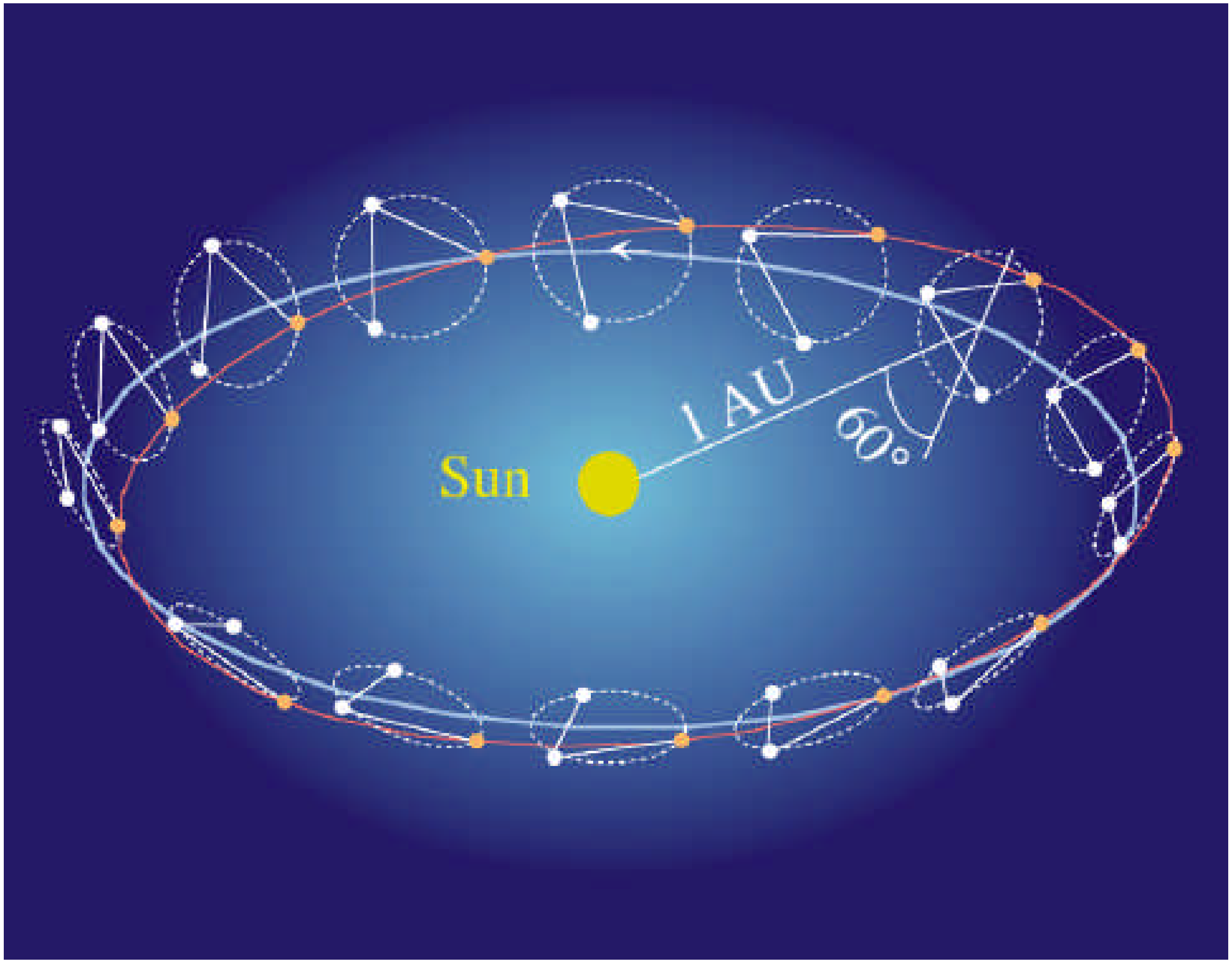


Mission Concept



Spacecraft Orbits

- Spacecraft orbits evolve under gravitational forces only
- Spacecraft fly “drag-free” to shield proof masses from non-gravitational forces



Optical System