

# Positronium Decay and Correlated Photons

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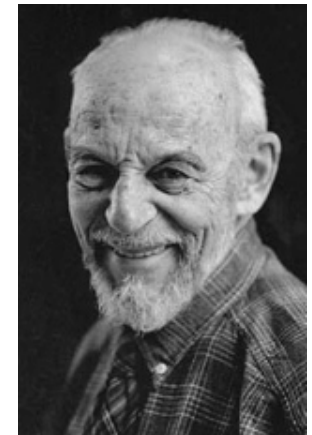
Thursday May 15, 2014



*The Short, Rich Life of Positronium*  
Bronze, 66" x 30" x 18"  
University of Michigan  
Jens Zorn 1999

# Outline

- A little about radioactivity
- A little about the atomic physics of positronium
  - M. Deutsch Phys Rev **82** 455 (1951), **83** 866 (1952), **84** 601 (1952), **85** 1047 (1952)
- Positronium two and three photon decay
  - The positronium correlated two photon state and polarization
  - Change of photon basis states
- The parity of Positronium in the ground state
- Demonstration of Positronium annihilation and photon correlations
  - Modified Wu & Shaknov experiment Phys Rev **77** 136 (1950)
- Einstein Podolsky Rosen (non)paradox
  - A. Einstein, B. Podolsky, N. Rosen Phys Rev **47** 777 (1935)
  - N. Bohr Phys Rev **48** 696 (1935)
- Why Junior Physics Lab



Martin Deutsch

# Radioactivity Primer

- Definition: Curie =  $3.7 \times 10^{10}$  decays/sec
  - The  $\text{Na}_{22}$  source in experiment  $1 \times 10^{-4}$  C:  $\gamma$  2, 0.5 Mev  $\rightarrow 1 \times 10^{-6}$  watts
- Definition: Roentgen Equivalent Man REM =  $10^{-2}$  Joules(absorbed)/kg =  $10^{-2}$  Sieverts Eq Man
- Background at sea level:  $2 \times 10^{-5}$  REM/hr  $\rightarrow 5 \times 10^{-4}$  REM one day of exposure
- Background in jet plane at 30kft:  $5 \times 10^{-4}$  REM/hr
- Dose from chest X-ray:  $10^{-2}$  REM

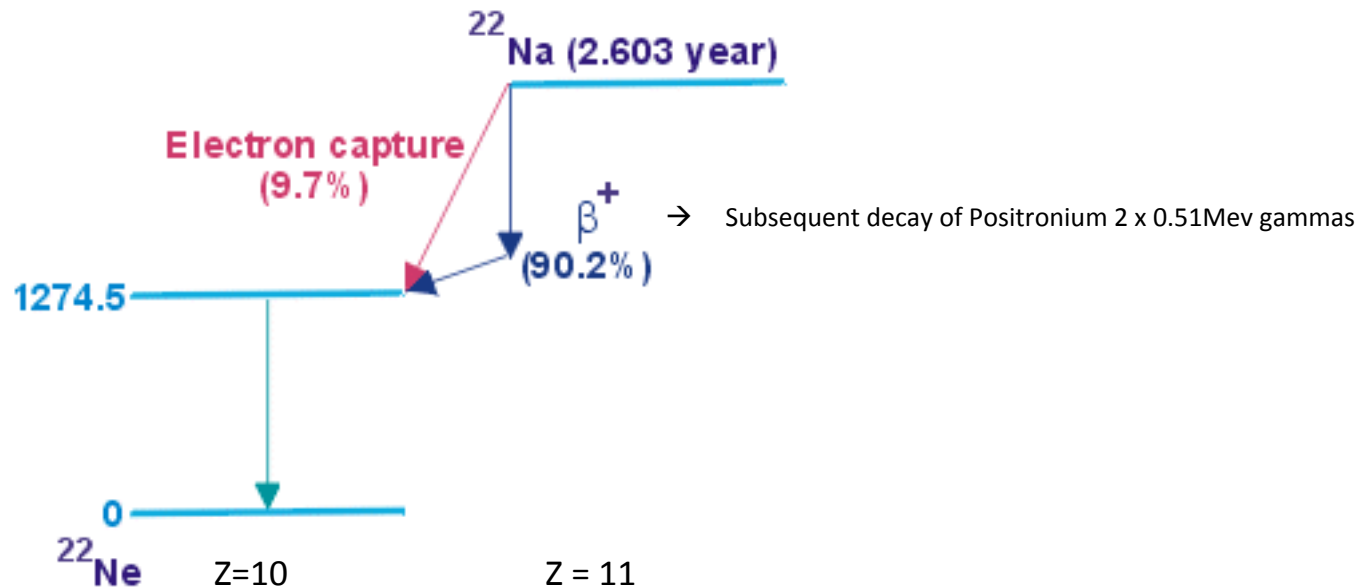
## Allowed dose in a year REM

Whole body	5
extremities	50
eye	15
Baby whole body	0.5

Full body dose from unshielded  $\text{Na}_{22}$  source at 2 meters  $\sim 1 \times 10^{-4}$  REM/hr  
Full body dose with lead shielding is indistinguishable from background

# Na<sub>22</sub> production and decay scheme

**Production:** Bombard magnesium with 66MeV Protons for days at 100 microamps. Converts proton to neutron and emission of positron. Then, chemistry to separate sodium from other elements and time to allow for decay of short lived nuclei.





# Positronium and Hydrogen energy levels

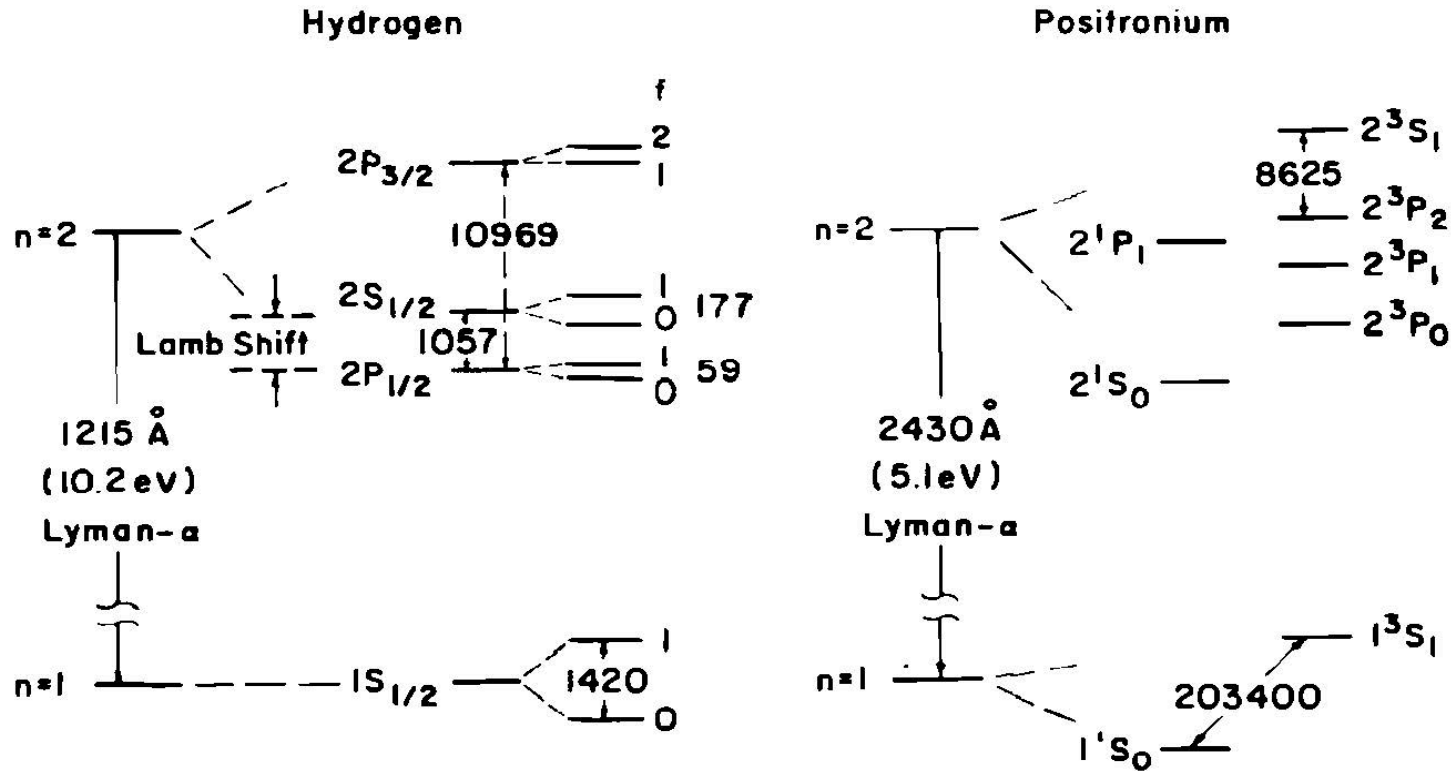
Electron  $-e$   $m_e$   $s = 1/2$   $\mu = \mu_0$   
 Proton  $+e$   $1836m_e$   $l = 1/2$   $\mu = 1.519 \times 10^{-3} \mu_0$

$$a_0 = \frac{h^2}{(2\pi e)^2 m'}$$

$$E_{n=1} = -\frac{(2\pi)^2 e^4 m'}{2 h^2}$$

$$m' = \frac{m_1 m_2}{m_1 + m_2}$$

Electron  $-e$   $m_e$   $s = 1/2$   $\mu = \mu_0$   
 Positron  $+e$   $m_e$   $s = 1/2$   $\mu = \mu_0$



*Figure 1* Schematic comparison of the  $n = 1$  and the  $n = 2$  energy levels of hydrogen and positronium. Energy level differences are given in MHz to the nearest MHz; for the exact experimental and theoretical values see Drell (1979) for hydrogen, and consult the text for positronium.

# Positronium decay

- S=0 state decays from s state: two photons

$$\Gamma_{2\gamma}^{(0)}(n^1S_0) = \sigma v |\Psi_n(0)|^2 = \frac{\pi m c^2 \alpha^5}{h n^3} = 8 \times 10^9 / \text{sec} \quad (n=1) \quad \alpha = \frac{2\pi e^2}{hc} \approx \frac{1}{137}$$

- S=1 state decays from s state: three photons

$$\Gamma_{3\gamma}^{(0)}(n^3S_1) = \frac{4}{9}(\pi^2 - 9) \frac{\pi m c^2 \alpha^6}{h n^3} = 7 \times 10^6 / \text{sec} \quad (n=1)$$

- The two photon state of positronium

- **Zero** momentum, angular momentum, rest mass
- Only possibility back to back equal energy photons with total angular momentum zero

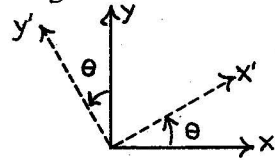
$$|\Psi_{2\gamma}^{n=1}\rangle = \frac{1}{\sqrt{2}} |RR\rangle \pm \frac{1}{\sqrt{2}} |LL\rangle$$

# Polarization algebra

Need to express two photon states  $|RR\rangle$  and  $|LL\rangle$  in linear polarization  $x$  and  $y$  basis rather than circular polarization basis to couple to demonstration.

Table 1: Probability amplitudes for photon polarization states

Angle convention: (Beam emerges toward reader.)



FROM STATE

	$ x\rangle$	$ y\rangle$	$ x'\rangle$	$ y'\rangle$	$ R\rangle$	$ L\rangle$
$\langle x $	1	0	$\cos \theta$	$-\sin \theta$	$-i/\sqrt{2}$	$i/\sqrt{2}$
$\langle y $	0	1	$\sin \theta$	$\cos \theta$	$1/\sqrt{2}$	$1/\sqrt{2}$
$\langle x' $	$\cos \theta$	$\sin \theta$	1	0	$\frac{1}{\sqrt{2}}e^{i(\theta - \frac{\pi}{2})}$	$\frac{1}{\sqrt{2}}e^{-i(\theta - \frac{\pi}{2})}$
$\langle y' $	$-\sin \theta$	$\cos \theta$	0	1	$\frac{1}{\sqrt{2}}e^{i\theta}$	$\frac{1}{\sqrt{2}}e^{-i\theta}$
$\langle R $	$i/\sqrt{2}$	$1/\sqrt{2}$	$\frac{1}{\sqrt{2}}e^{-i(\theta - \frac{\pi}{2})}$	$\frac{1}{\sqrt{2}}e^{-i\theta}$	1	0
$\langle L $	$-i/\sqrt{2}$	$1/\sqrt{2}$	$\frac{1}{\sqrt{2}}e^{i(\theta - \frac{\pi}{2})}$	$\frac{1}{\sqrt{2}}e^{i\theta}$	0	1

# Change of Basis

U

$$\begin{pmatrix} \langle xx|\Psi \rangle \\ \langle xy|\Psi \rangle \\ \langle yx|\Psi \rangle \\ \langle yy|\Psi \rangle \end{pmatrix} = \begin{pmatrix} \langle xx|RR \rangle & \langle xx|RL \rangle & \langle xx|LR \rangle & \langle xx|LL \rangle \\ \langle xy|RR \rangle & \langle xy|RL \rangle & \langle xy|LR \rangle & \langle xy|LL \rangle \\ \langle yx|RR \rangle & \langle yx|RL \rangle & \langle yx|LR \rangle & \langle yx|LL \rangle \\ \langle yy|RR \rangle & \langle yy|RL \rangle & \langle yy|LR \rangle & \langle yy|LL \rangle \end{pmatrix} \begin{pmatrix} \langle RR|\Psi \rangle \\ \langle RL|\Psi \rangle \\ \langle LR|\Psi \rangle \\ \langle LL|\Psi \rangle \end{pmatrix}$$

Example matrix element :  
assumes independent operations  
on the two photons

$$\langle xy|RL \rangle = \langle x|R \rangle \langle y|L \rangle$$

$$U = \frac{1}{2} \begin{pmatrix} -1 & 1 & 1 & -1 \\ -i & -i & i & i \\ -i & i & -i & i \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

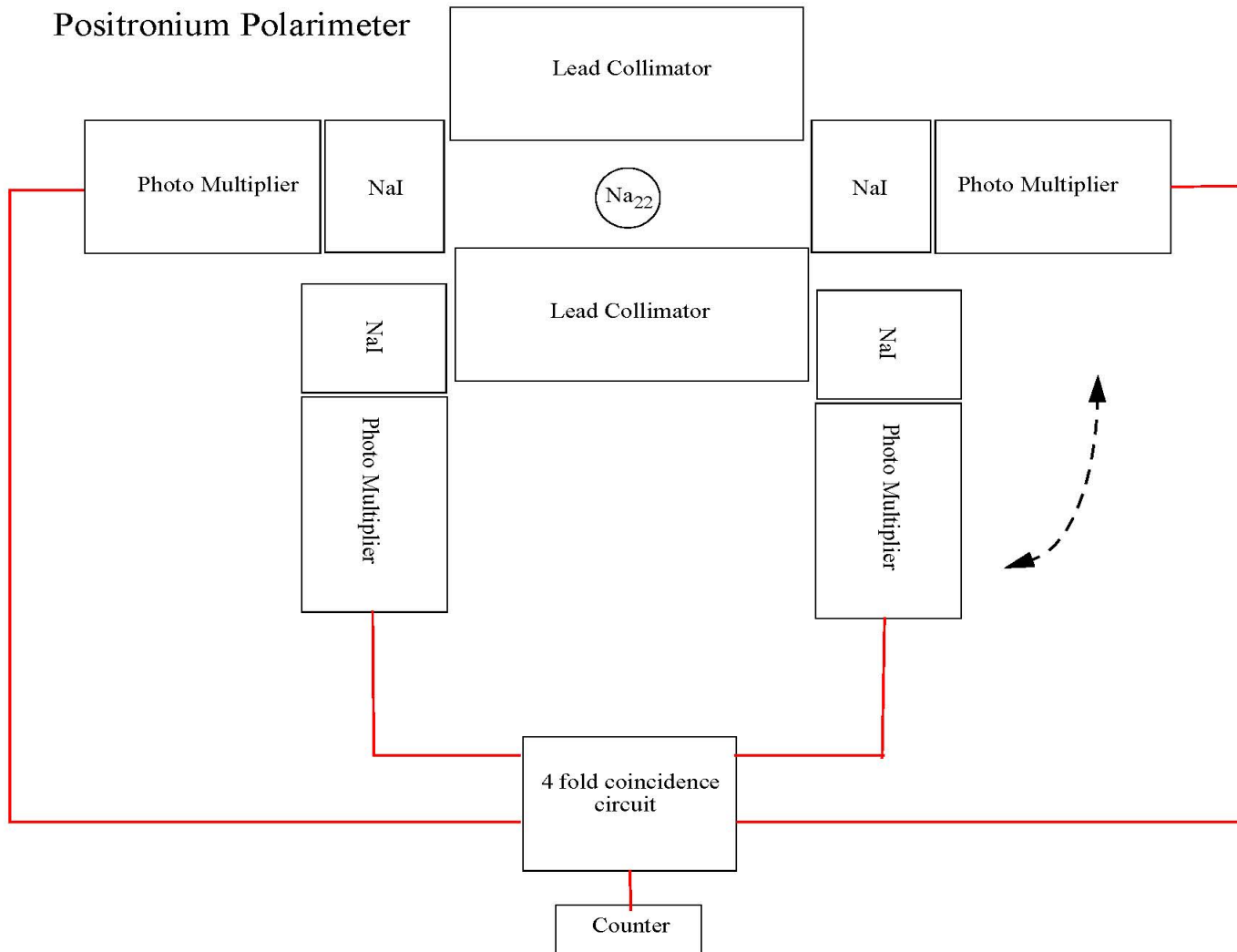
$$\begin{pmatrix} -1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = U \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$$

even parity

$$\begin{pmatrix} 0 \\ -i/\sqrt{2} \\ -i/\sqrt{2} \\ 0 \end{pmatrix} = U \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ -1/\sqrt{2} \end{pmatrix}$$

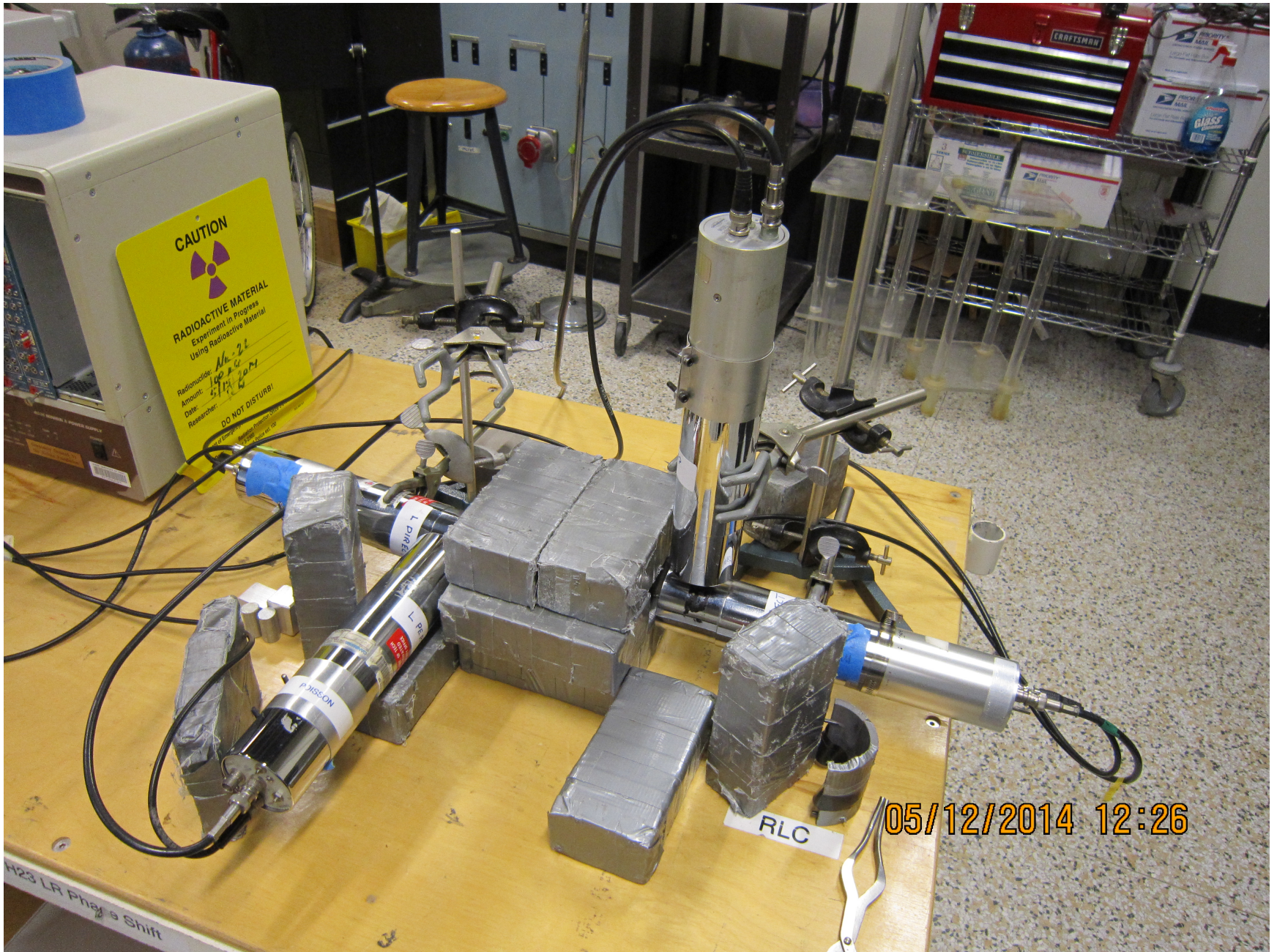
odd parity

# Positronium Polarimeter

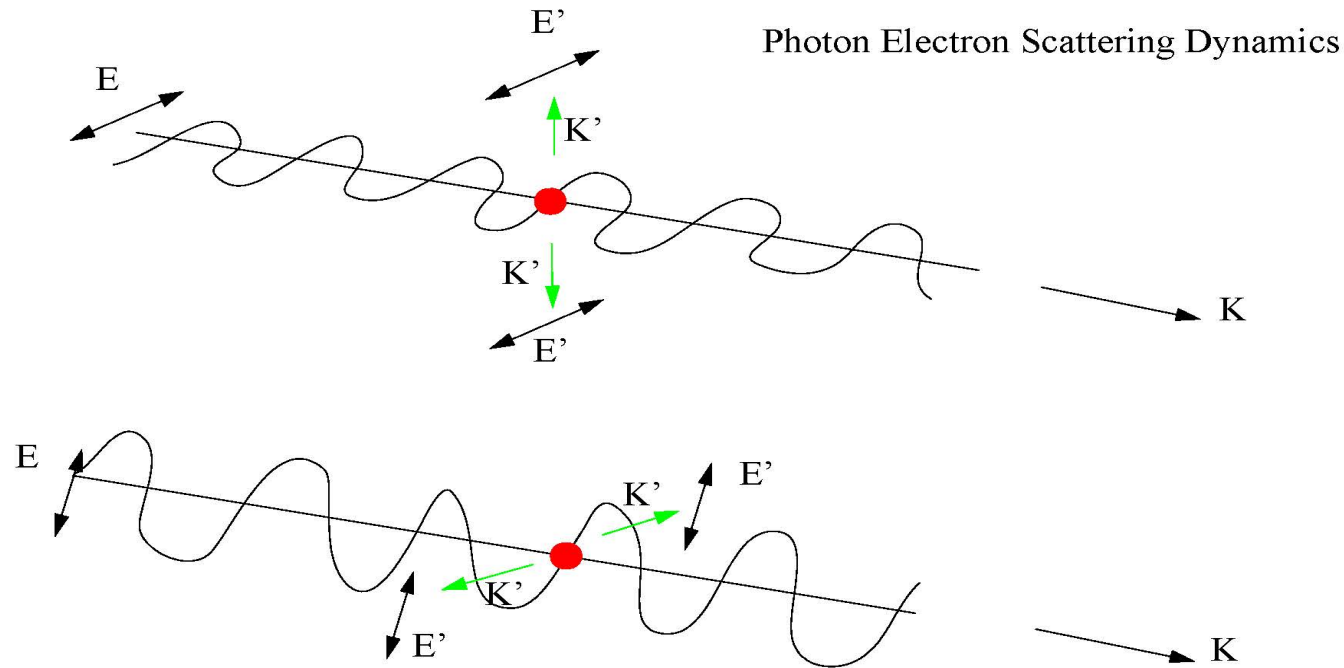




# Polarimeter



# Photon electron scattering dynamics

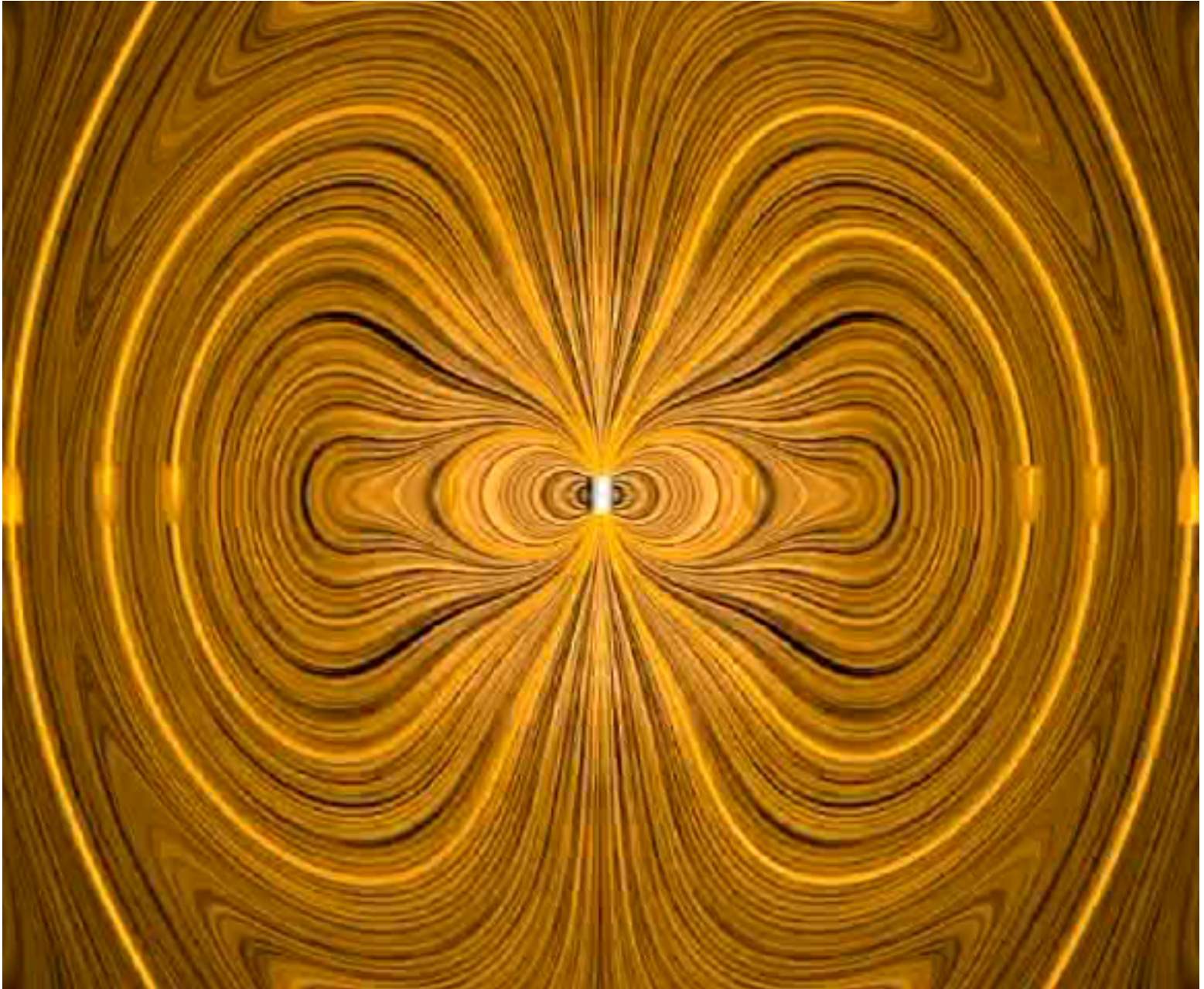


$$\sigma_{\text{Thomson}} = \frac{8\pi}{3} \left( \frac{e^2}{mc^2} \right)^2 = 6.65 \times 10^{-25} \text{ cm}^2$$

$$\frac{d\sigma}{d\Omega} = \left( \frac{e^2}{mc^2} \right)^2 |\boldsymbol{\epsilon}_{\text{scat}} \cdot \boldsymbol{\epsilon}_{\text{inc}}|^2$$

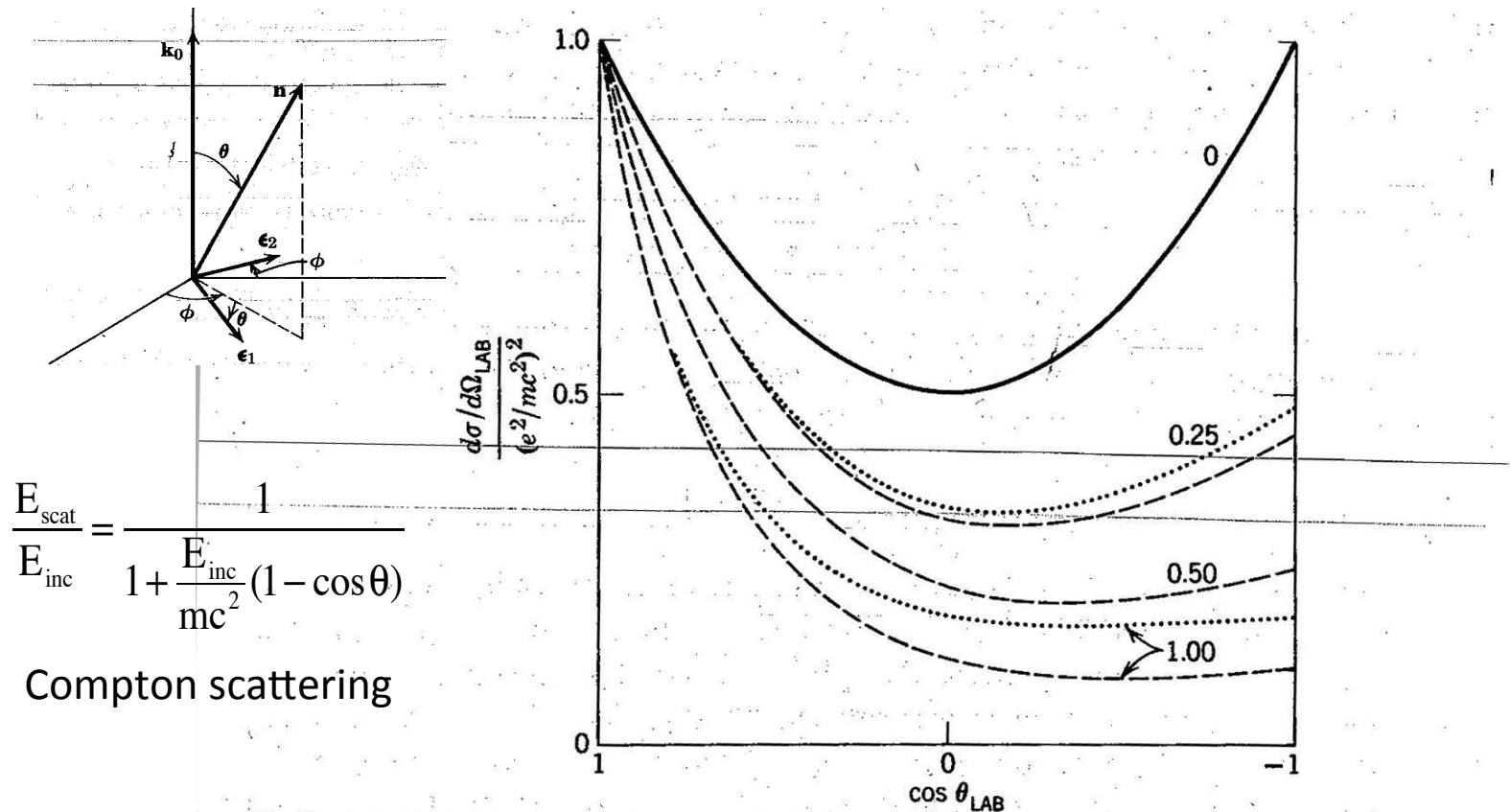


# Electric dipole radiation





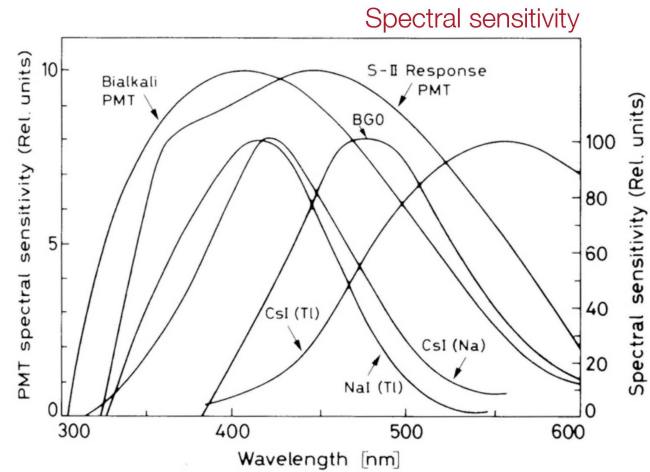
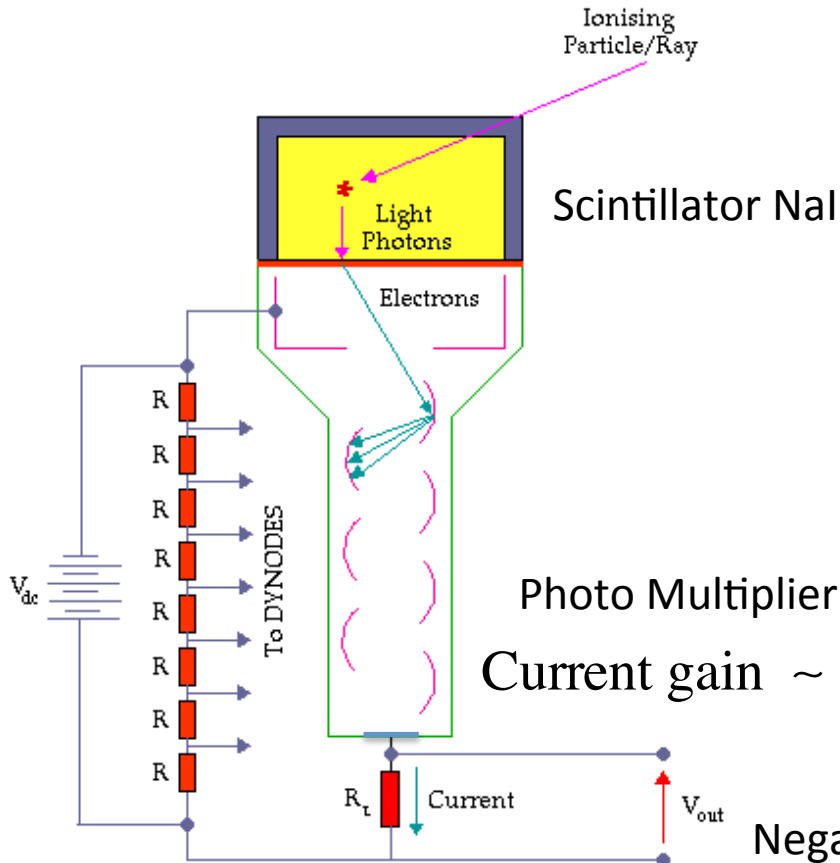
# Photon – electron scattering : Compton scattering cross section and angular distribution

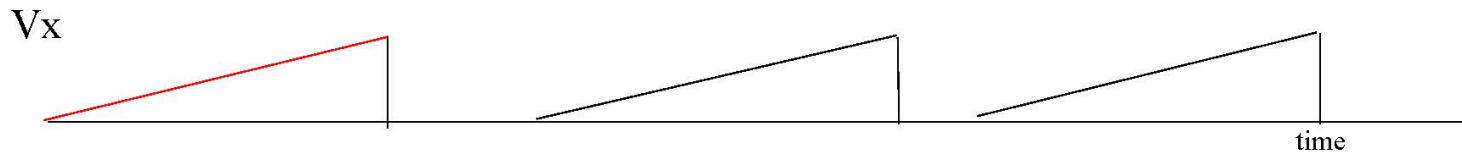
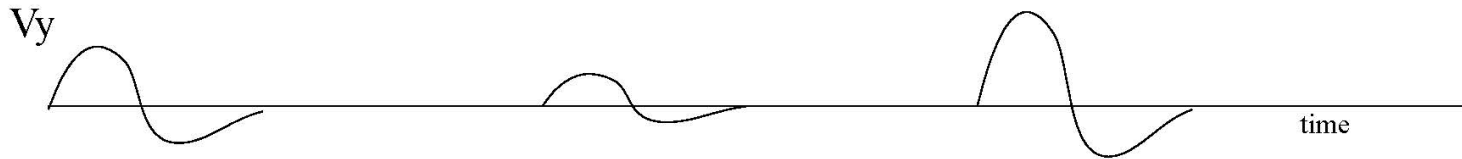
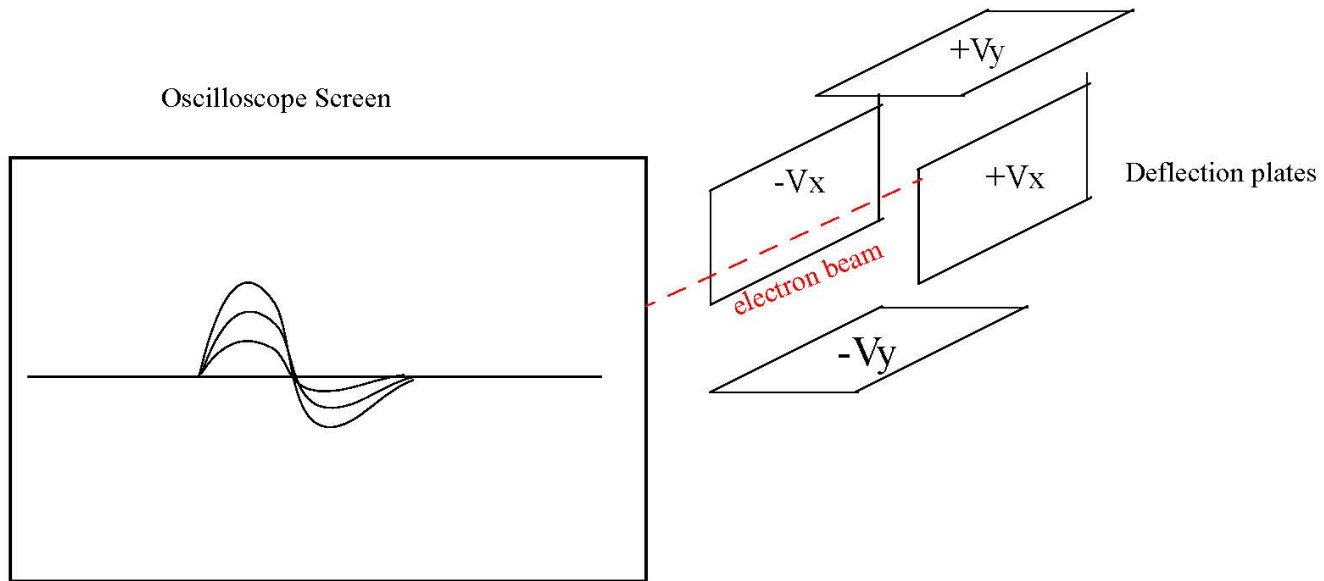


**Fig. 14.13** Differential scattering cross section of unpolarized radiation by a point charged particle initially at rest in the laboratory. The solid curve is the classical Thomson result. The dashed curves are the quantum-mechanical results for a spinless particle, with the numbers giving the values of  $\hbar\omega/mc^2$ . For  $\hbar\omega/mc^2 = 0.25, 1.0$  the dotted curves show the results for spin  $\frac{1}{2}$  point particles (electrons).

# Gamma ray detector

## Inorganic Crystals – Light Output & PMT Sensitivity



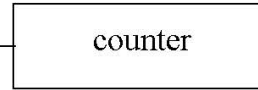
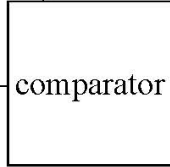


Schematic of an Oscilloscope

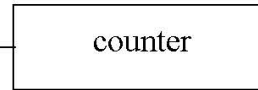
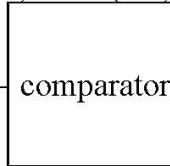


pulses in

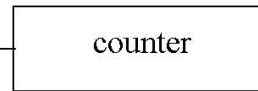
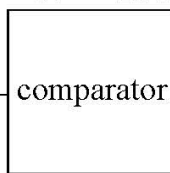
$(n-1)dV \rightarrow ndV$



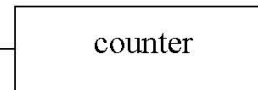
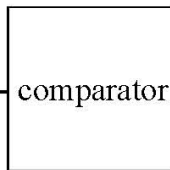
$(n-2)dV \rightarrow (n-1)dV$



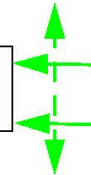
$dV \rightarrow 2dV$



$0 \rightarrow dV$



to oscilloscope  
move from counter to counter

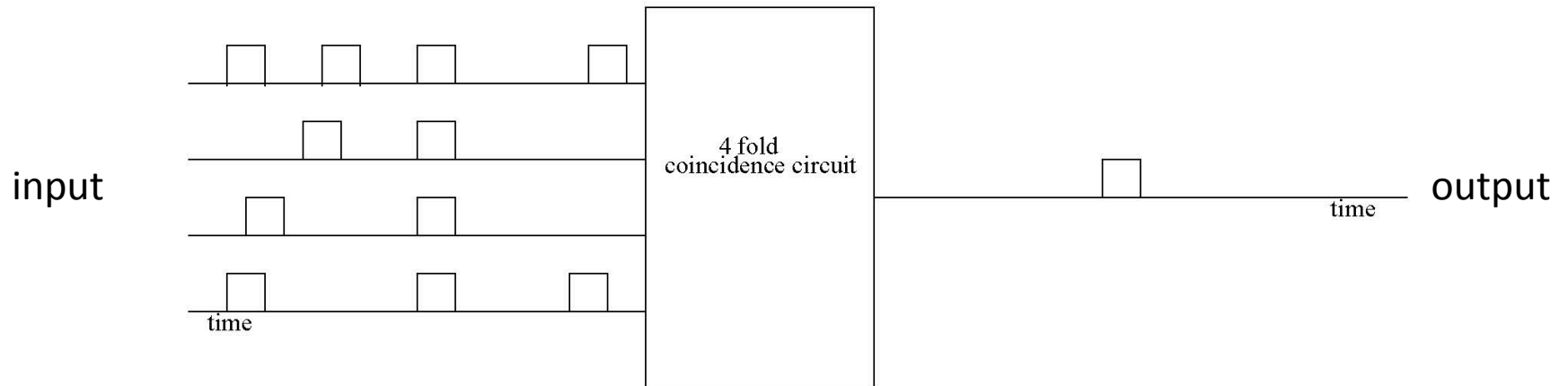


$V_y = \text{count}$

$V_x = mdV \quad m=2$

Schematic multichannel analyser

# Coincidence circuit schematic



Time resolution is  $2 * \text{width of pulse}$



# Electronics





# Junior Physics Laboratory Experiments

8.13 (Fall/Spring)

## **Introductory Experiments**

I. Photoelectric Effect

II. Poisson Statistics

III. Michelson Interferometer

## **Regular Experiments**

1. Compton Scattering

7. The Franck-Hertz Experiment

9. Relativistic Dynamics

12. Pulsed NMR: Spin Echoes

14. The Speed and Mean Life of Cosmic-Ray Muons

15. Rutherford Scattering

17. Optical Emission Spectra of Hydrogenic Atoms

43. Johnson Noise and Shot Noise

46. 21-cm Radio Astrophysics

51. Optical Trapping

8.14 (Spring)

## **Regular Experiments**

6. The Zeeman Effect

11. Optical Pumping of Rb Vapor

12. Pulsed NMR: Spin Echoes

13. Mössbauer Spectroscopy

39. Superconductivity

46. 21-cm Radio Astrophysics

48. Doppler-Free Laser Spectroscopy

49. Quantum Information Processing

51. Optical Trapping

# Acknowledgements

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Ulrich Becker      Professor of Physics