Gravitational Waves: Theoretical Insight to Measurement

Rainer Weiss, MIT

A Century of General Relativity

Harnack House

Berlin

November 30, 2015

Einstein 1916

SITZUNGSBERICHTE 1916.
XXXIII.
DER
KÖNIGLICH PREUSSISCHEN
AKADEMIE DER WISSENSCHAFTEN.

Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

Einstein 1916

$$\gamma'_{\mu\nu} = \alpha_{\mu\nu} f(x_1 + i x_4) = \alpha_{\mu\nu} f(x - t). \qquad (15)$$

Dabei sind die $\alpha_{\mu\nu}$ Konstante; f ist eine Funktion des Arguments $x - t$. Ist der betrachtete Raum frei von Materie, d. h. verschwinden die $T_{\mu\nu}$, so sind die Gleichungen (6) durch diesen Ansatz erfüllt. Die Gleichungen (4) liefern zwischen den $\alpha_{\mu\nu}$ die Beziehungen

$$\left.\begin{aligned} \alpha_{11} + i\alpha_{14} &= 0 \\ \alpha_{12} + i\alpha_{24} &= 0 \\ \alpha_{13} + i\alpha_{34} &= 0 \\ \alpha_{14} + i\alpha_{44} &= 0 \end{aligned}\right\}. \qquad (16)$$

Von den 10 Konstanten $\alpha_{\mu\nu}$ sind daher nur 6 frei wählbar. Wir können die allgemeinste Welle der betrachteten Art daher aus Wellen von folgenden 6 Typen superponieren

$$\left.\begin{aligned} &\text{a)} \begin{cases} \alpha_{11} + i\alpha_{14} = 0 \\ \alpha_{14} + i\alpha_{44} = 0 \end{cases} \quad \begin{aligned} &\text{b)}\ \alpha_{12} + i\alpha_{24} = 0 \\ &\text{c)}\ \alpha_{13} + i\alpha_{34} = 0 \end{aligned} \quad \begin{aligned} &\text{d)}\ \alpha_{22} \neq 0 \\ &\text{e)}\ \alpha_{23} \neq 0 \\ &\text{f)}\ \alpha_{33} \neq 0 \end{aligned} \end{aligned}\right\}. \qquad (17)$$

$$\text{d)} \quad \frac{1}{i} t_{22} = \frac{f'^2}{4\varkappa} \alpha_{22}^2 = \frac{1}{4\varkappa} \left(\frac{\partial \gamma'_{22}}{\partial t} \right)^2$$

$$\text{e)} \quad \frac{1}{i} t_{23} = \frac{f'^2}{4\varkappa} \alpha_{23}^2 = \frac{1}{4\varkappa} \left(\frac{\partial \gamma'_{23}}{\partial t} \right)^2$$

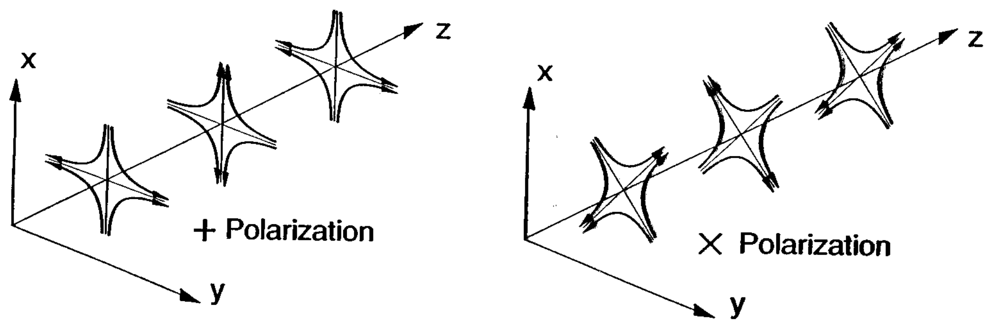
$$\text{f)} \quad \frac{1}{i} t_{33} = \frac{f'^2}{4\varkappa} \alpha_{33}^2 = \frac{1}{4\varkappa} \left(\frac{\partial \gamma'_{33}}{\partial t} \right)^2$$

Es ergibt sich also, daß nur die Wellen des letzten Typs Energie transportieren, und zwar ist der Energietransport einer beliebigen ebenen Welle gegeben durch

$$\mathfrak{I}_x = \frac{1}{i} t_{41} = \frac{1}{4\varkappa} \left[\left(\frac{\partial \gamma'_{22}}{\partial t} \right)^2 + 2 \left(\frac{\partial \gamma'_{23}}{\partial t} \right)^2 + \left(\frac{\partial \gamma'_{33}}{\partial t} \right)^2 \right]. \qquad (18)$$

CONTROVERSIAL GRAVITATIONAL ENERGY – MOMENTUM PSEUDO TENSOR

Einstein 1916



Die in (23), (23a) und (23b) auftretenden Integrale, welche nichts anderes sind als zeitlich variable Trägheitsmomente, nennen wir im folgenden zur Abkürzung J_{22}, J_{33}, J_{23}. Dann ergibt sich für die Intensität $\mathfrak{J}_x$ der Energiestrahlung aus (18)

$$\mathfrak{J}_x = \frac{\varkappa}{64\pi^2 R^2}\left[\left(\frac{\partial^3 J_{22}}{\partial t^3}\right)^2 + 2\left(\frac{\partial^3 J_{23}}{\partial t^3}\right)^2 + \left(\frac{\partial^3 J_{33}}{\partial t^3}\right)^2\right]. \quad (20)$$

SPHERICALLY SYMMETRIC MOTION RADIATES GRAVITATIONAL WAVES

Einstein 2018

Über Gravitationswellen.

Von A. Einstein.

(Vorgelegt am 31. Januar 1918 [s. oben S. 79].)

Die wichtige Frage, wie die Ausbreitung der Gravitationsfelder erfolgt, ist schon vor anderthalb Jahren in einer Akademiearbeit von mir behandelt worden[1]. Da aber meine damalige Darstellung des Gegenstandes nicht genügend durchsichtig und außerdem durch einen bedauerlichen Rechenfehler verunstaltet ist, muß ich hier nochmals auf die Angelegenheit zurückkommen.

Die $\mathfrak{J}_{\sigma\tau}$ sind nach (7a), (22), (24) für die Zeit $t-R$ zu nehmen, also Funktionen von $t-R$, oder bei großem R in der Nähe der x-Achse auch Funktionen von $t-x$. (25), (26) stellen also Gravitationswellen dar, deren Energiefluß längs der x-Achse gemäß (16) die Dichte

$$\frac{t_{41}}{i} = \frac{\varkappa}{64\pi^2 R^2}\left[\left(\frac{\dddot{\mathfrak{J}}_{22}-\dddot{\mathfrak{J}}_{33}}{2}\right)^2 + \dddot{\mathfrak{J}}_{23}^{\,2}\right] \qquad (27)$$

besitzt.

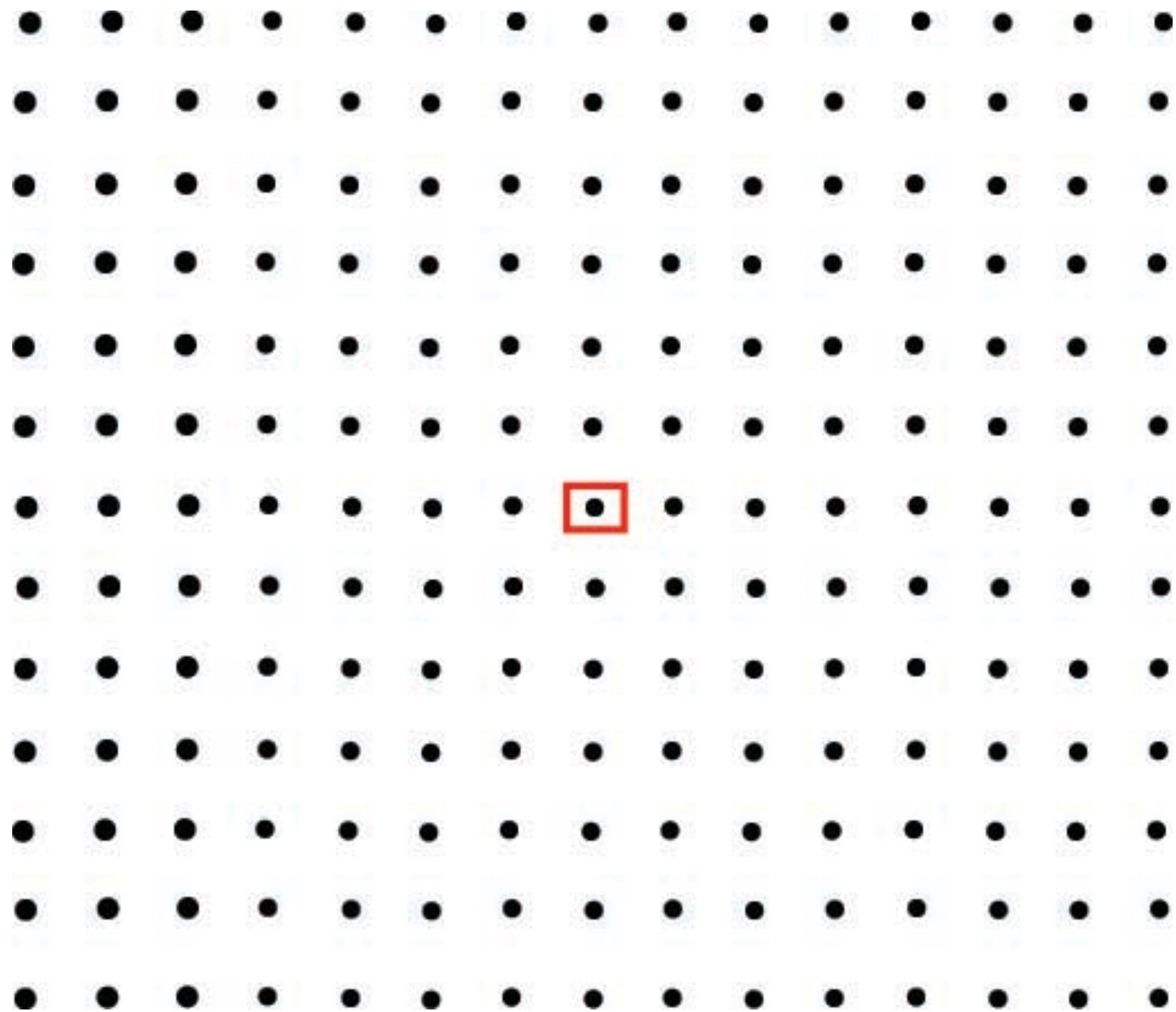
The quadrupole formula a factor of 2 too small

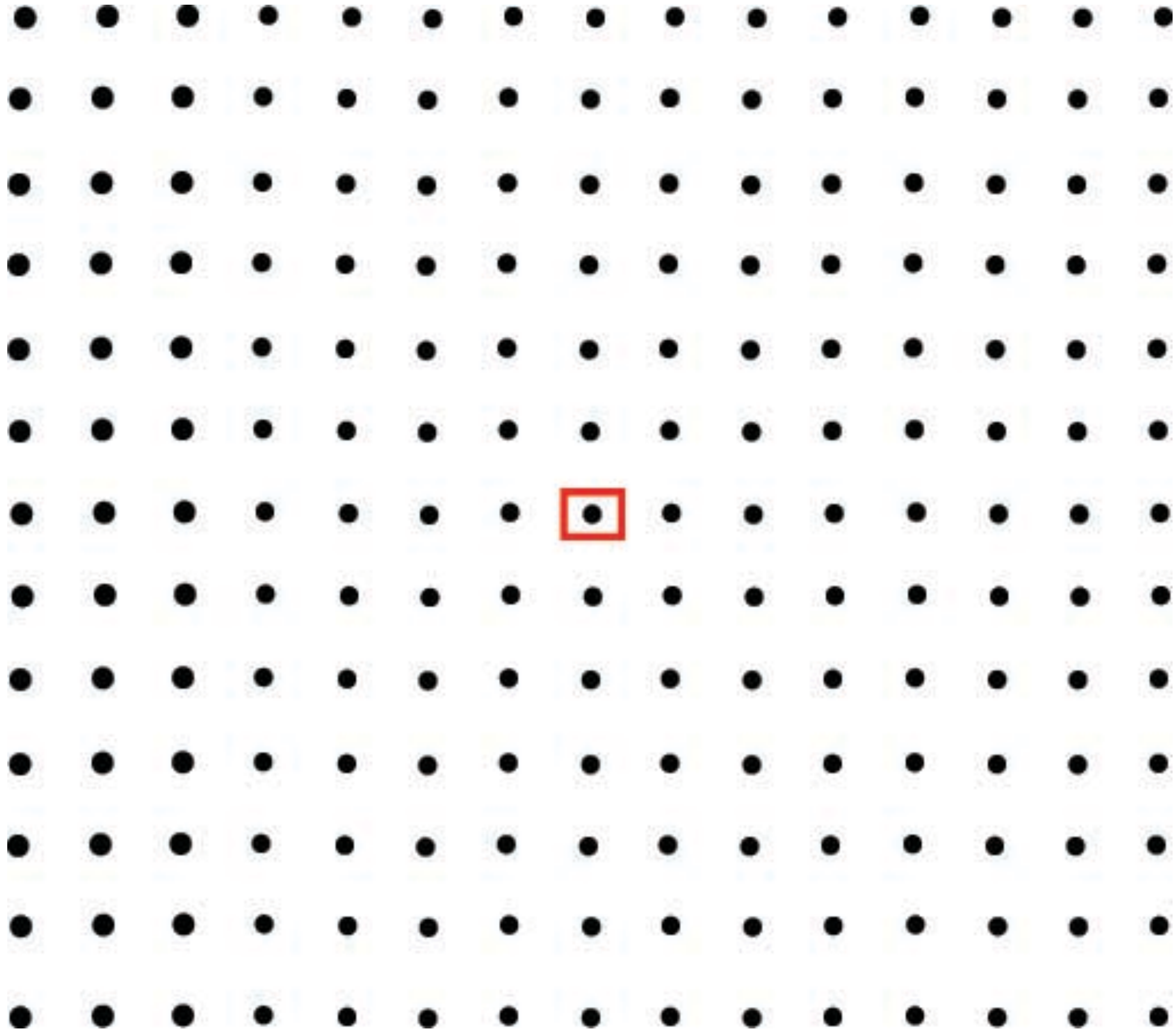
Einstein 1916

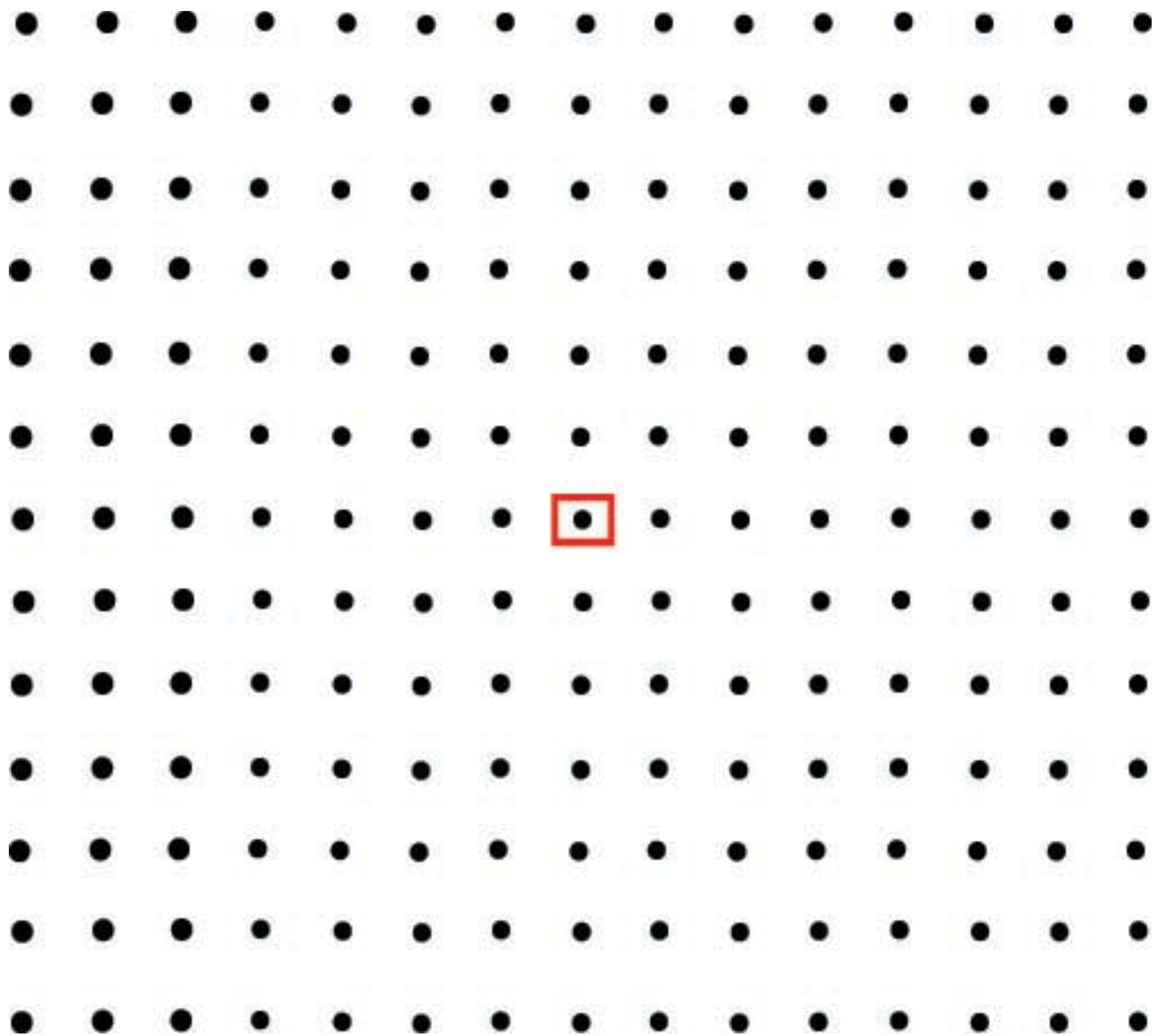
$$A = \frac{\varkappa}{24\pi}\sum_{\alpha\beta}\left(\frac{\partial^3 J_{\alpha\beta}}{\partial t^3}\right)^2. \qquad (21)$$

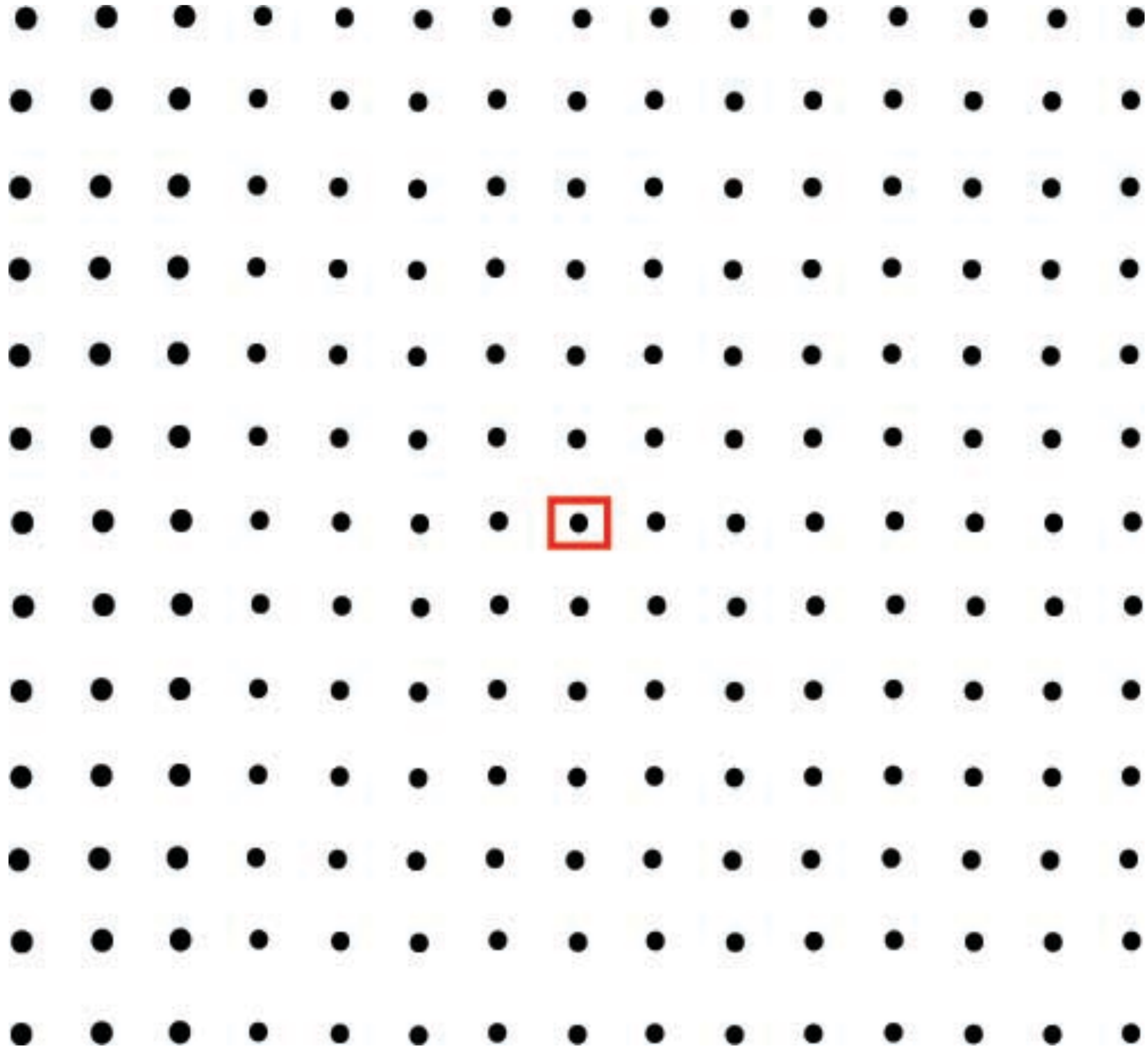
Würde man die Zeit in Sekunden, die Energie in Erg messen, so würde zu diesem Ausdruck der Zahlenfaktor $\frac{1}{c^4}$ hinzutreten. Berücksichtigt man außerdem, daß $\varkappa = 1.87 \cdot 10^{-27}$, so sieht man, daß A in allen nur denkbaren Fällen einen praktisch verschwindenden Wert haben muß.

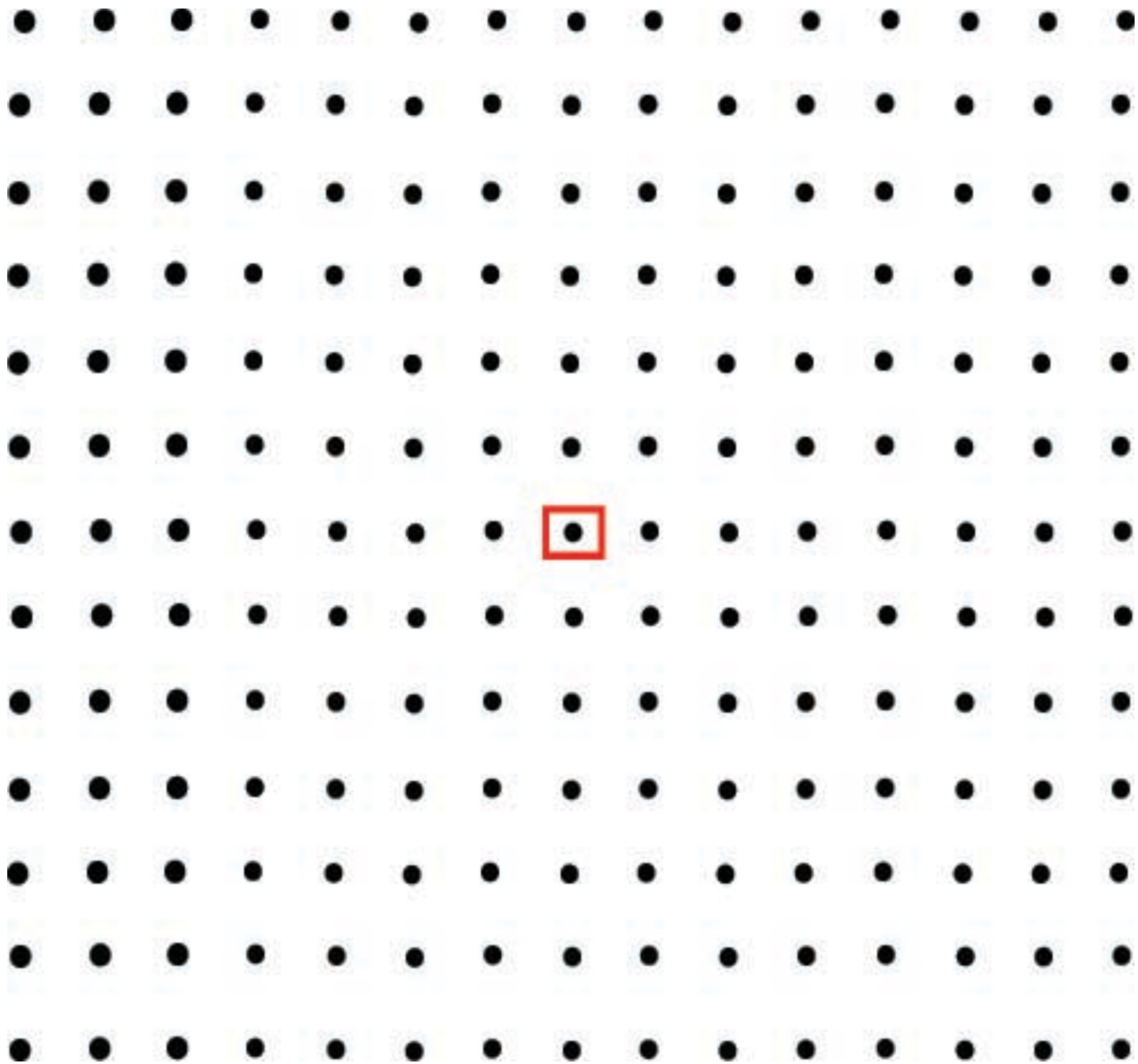
"…..in any case one can think of A will have a practically vanishing value."

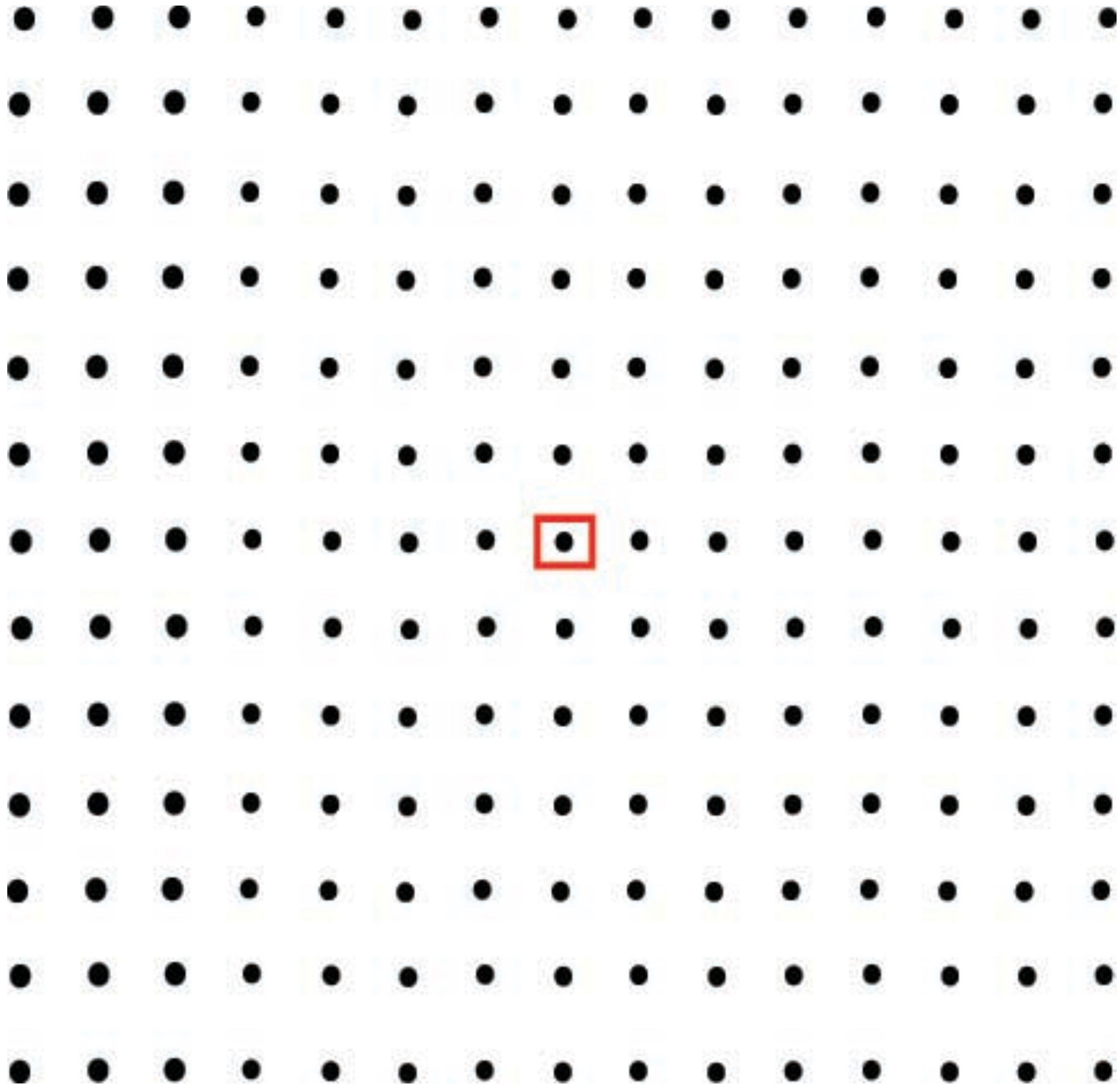


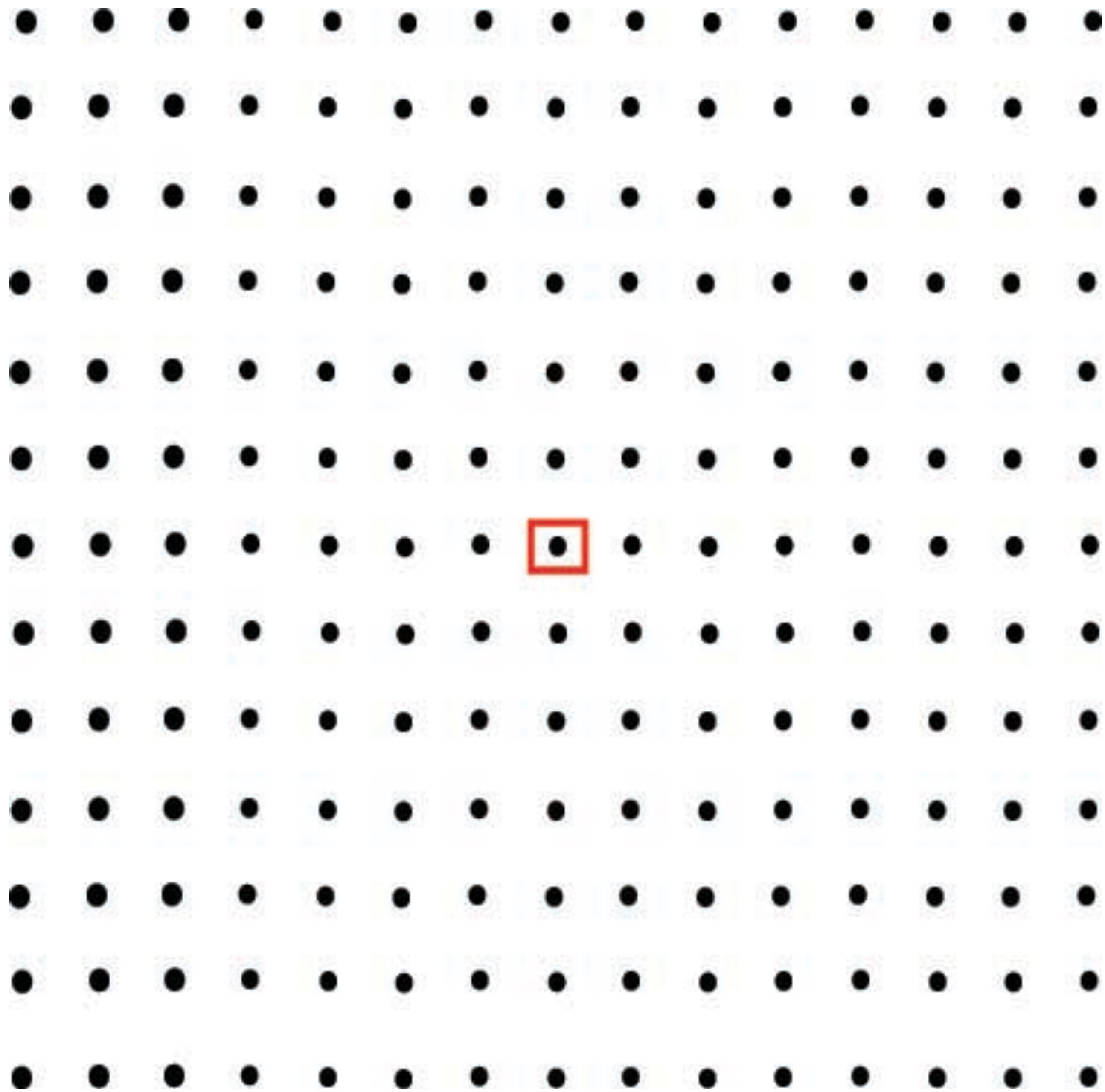


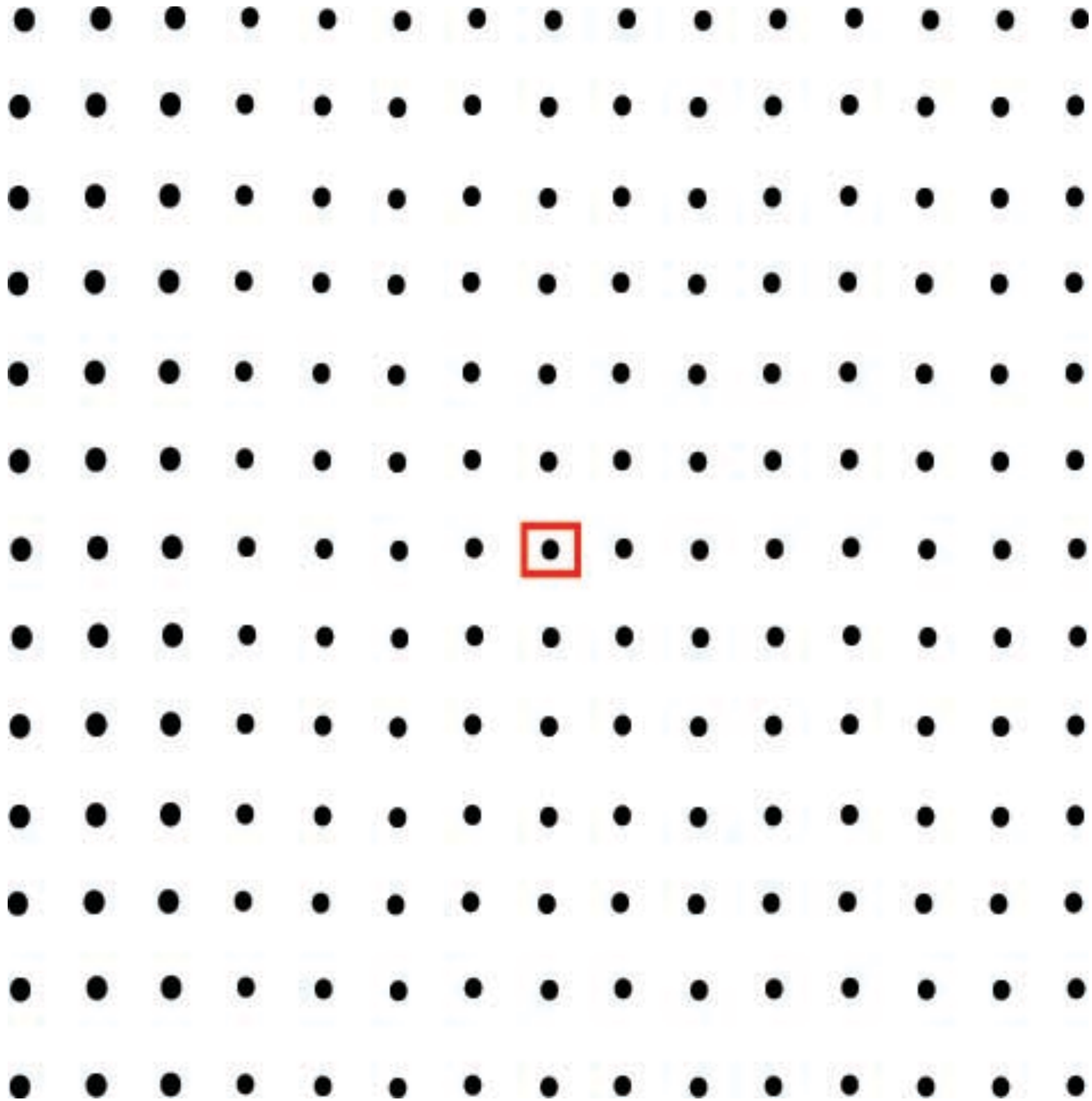


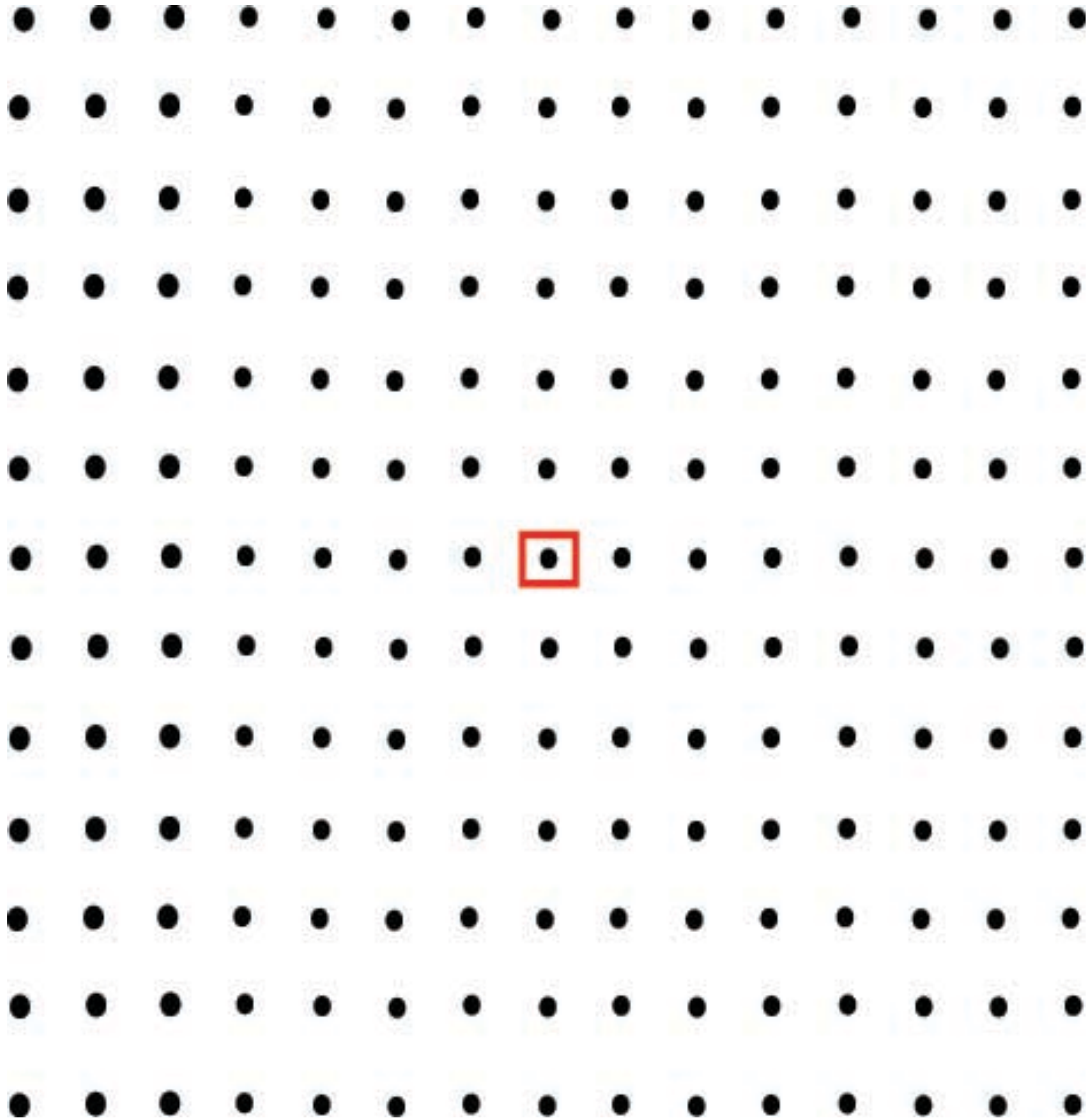


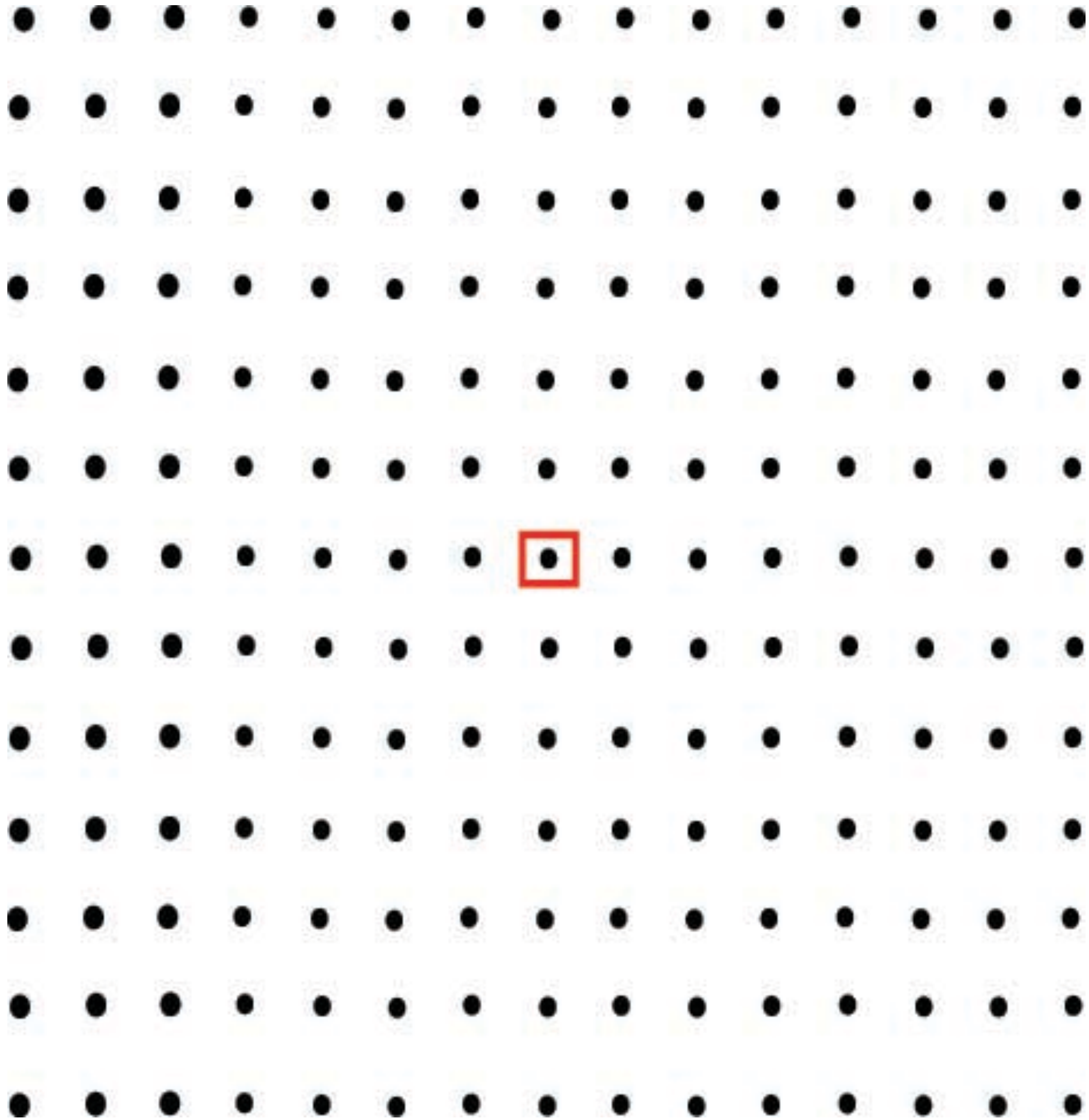


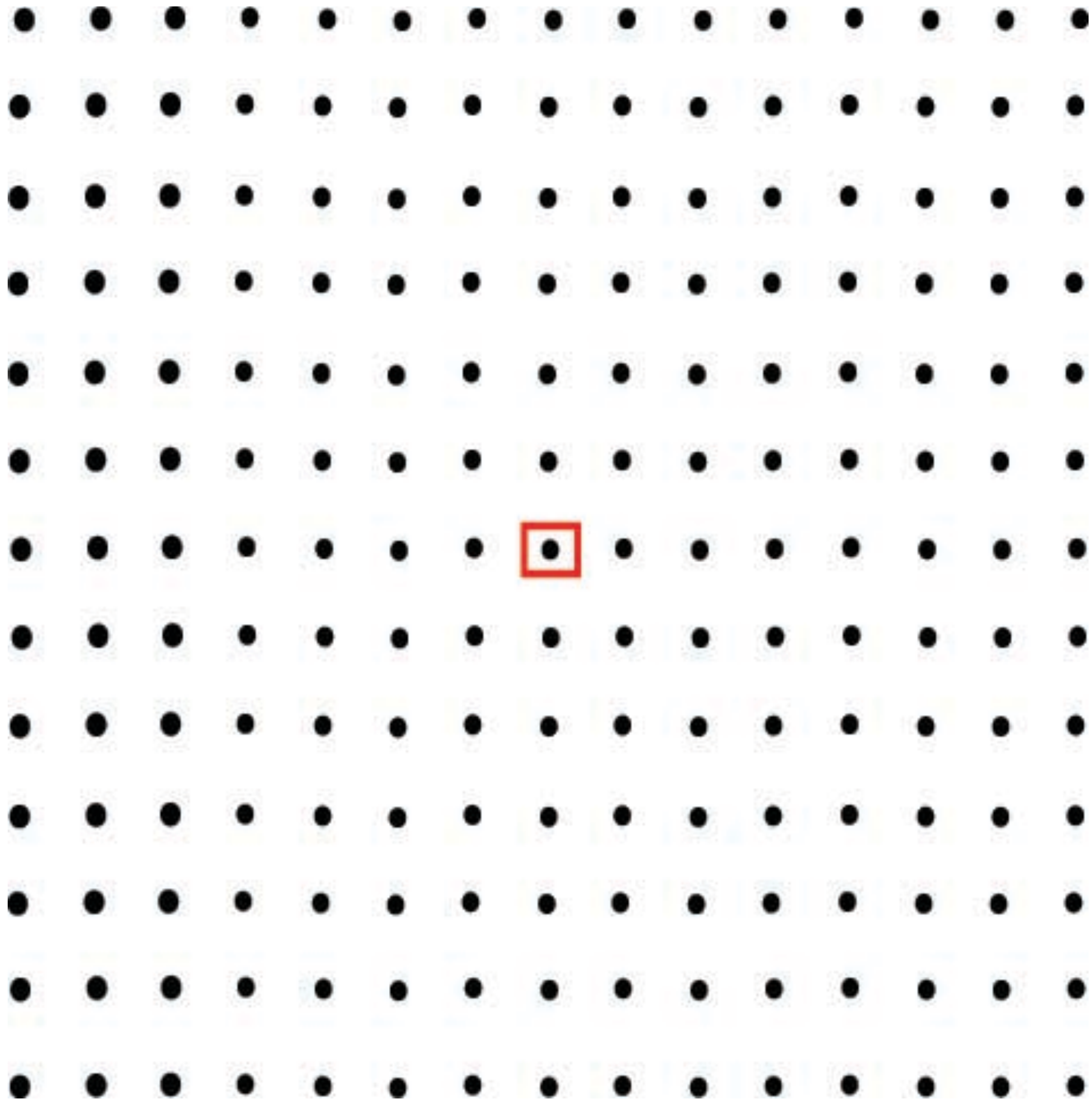


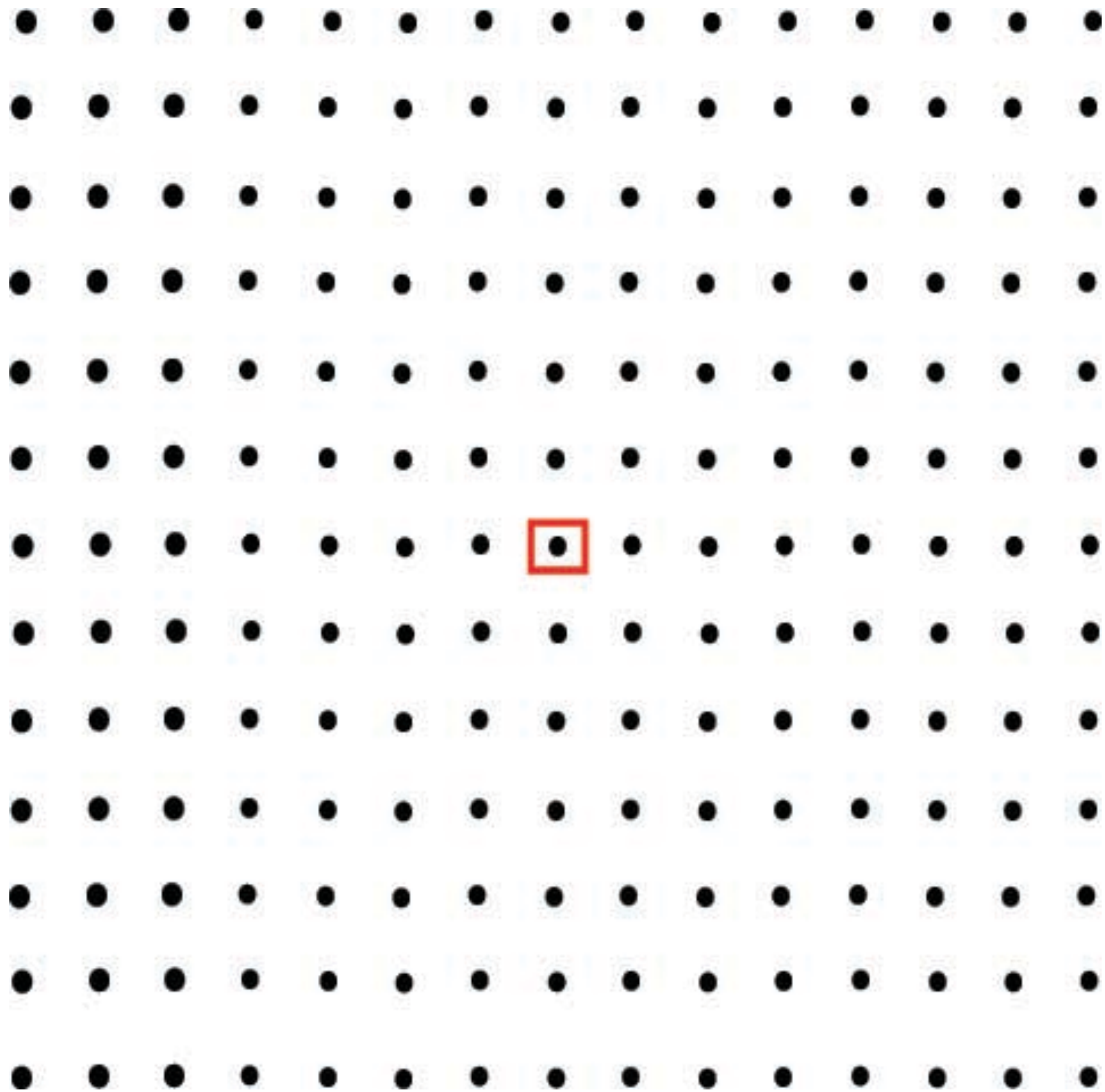


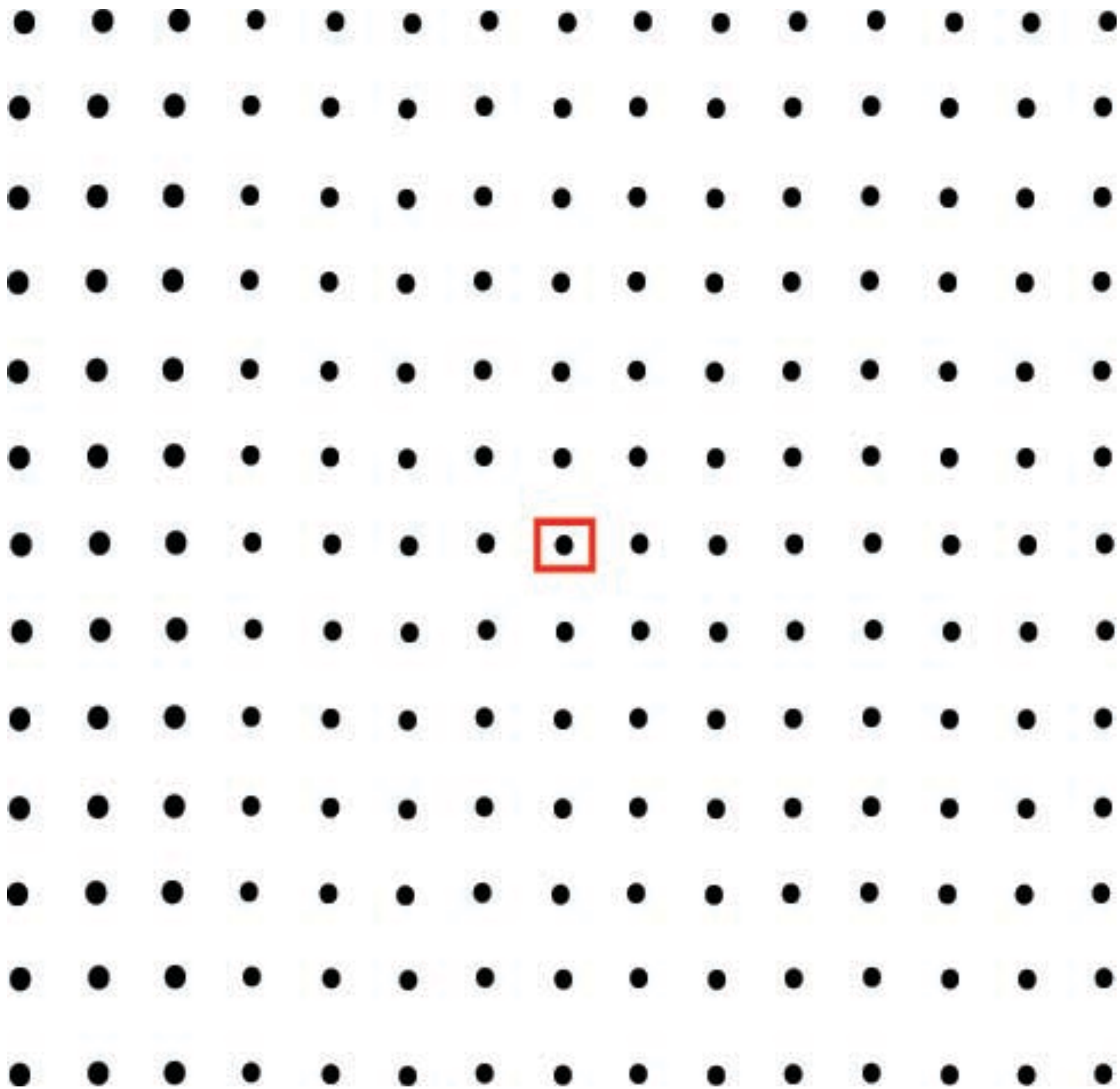


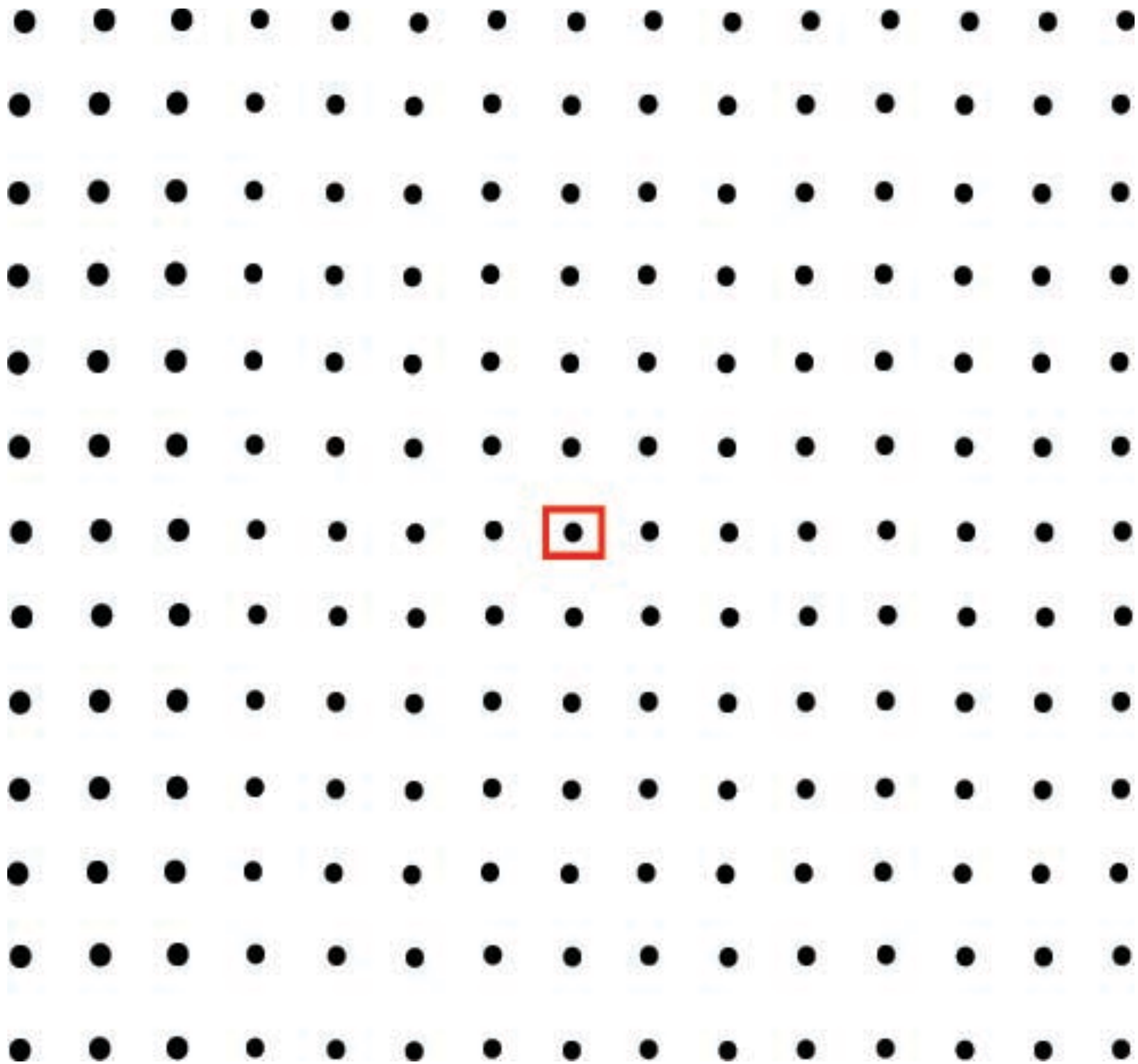


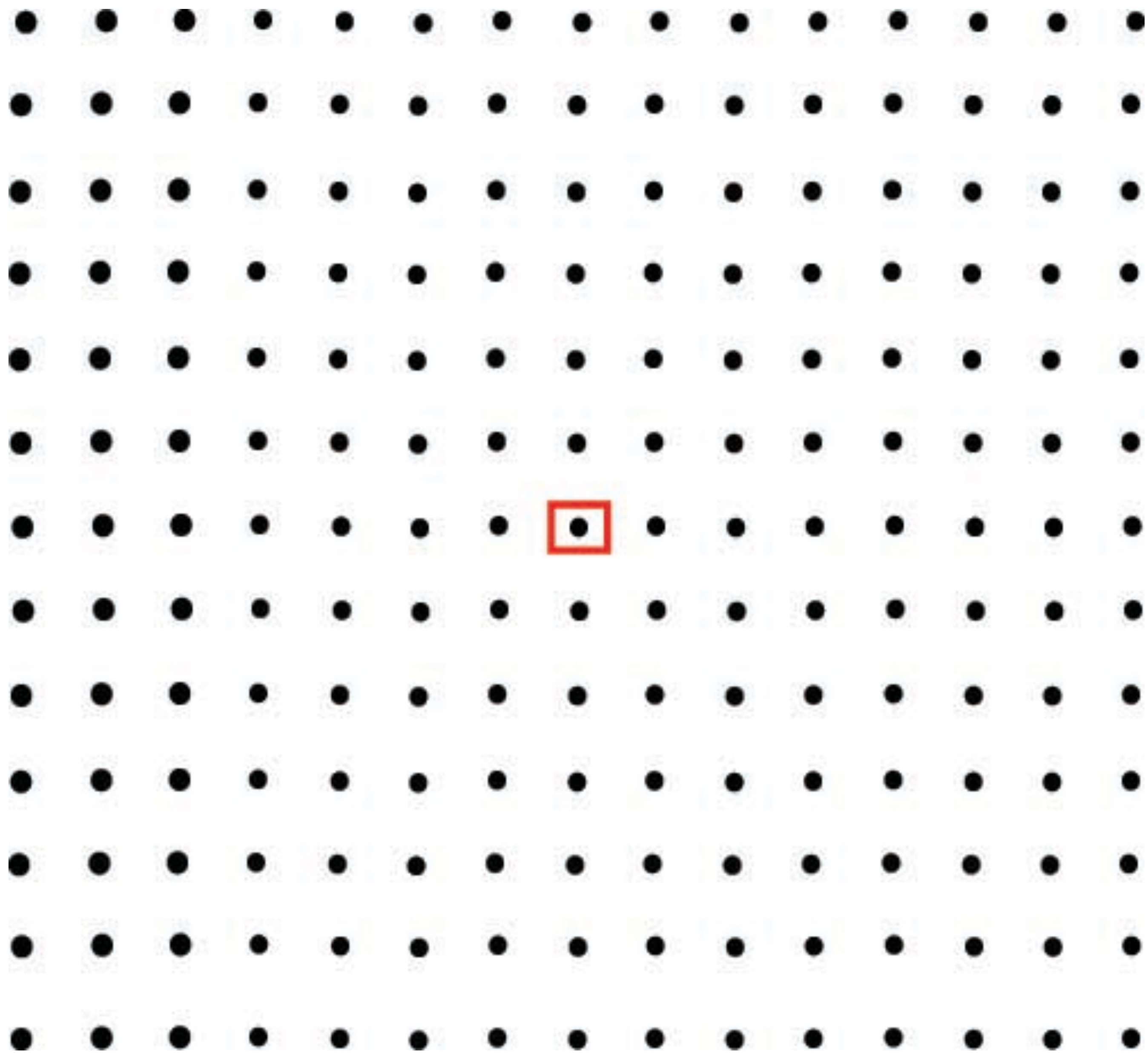


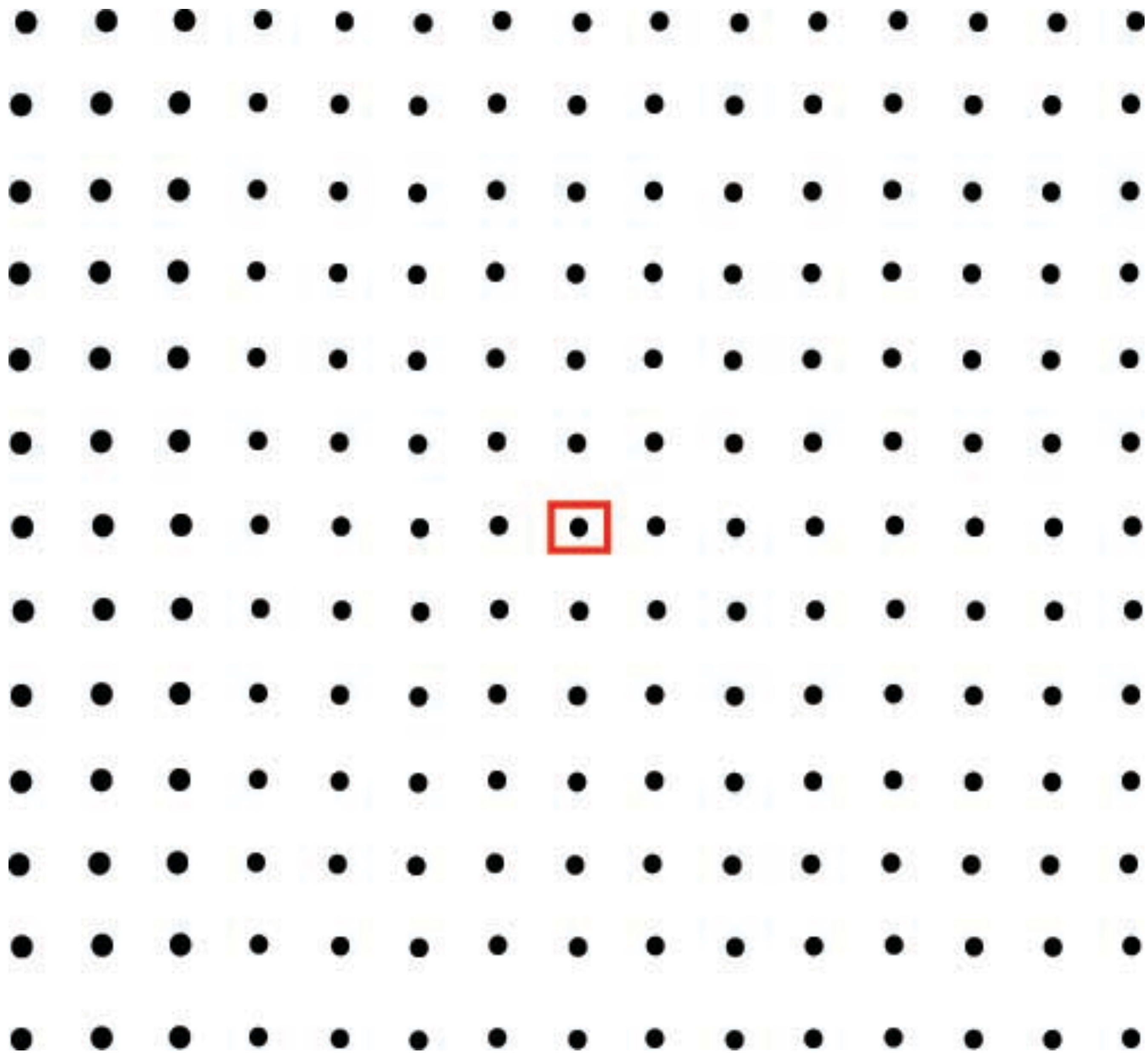


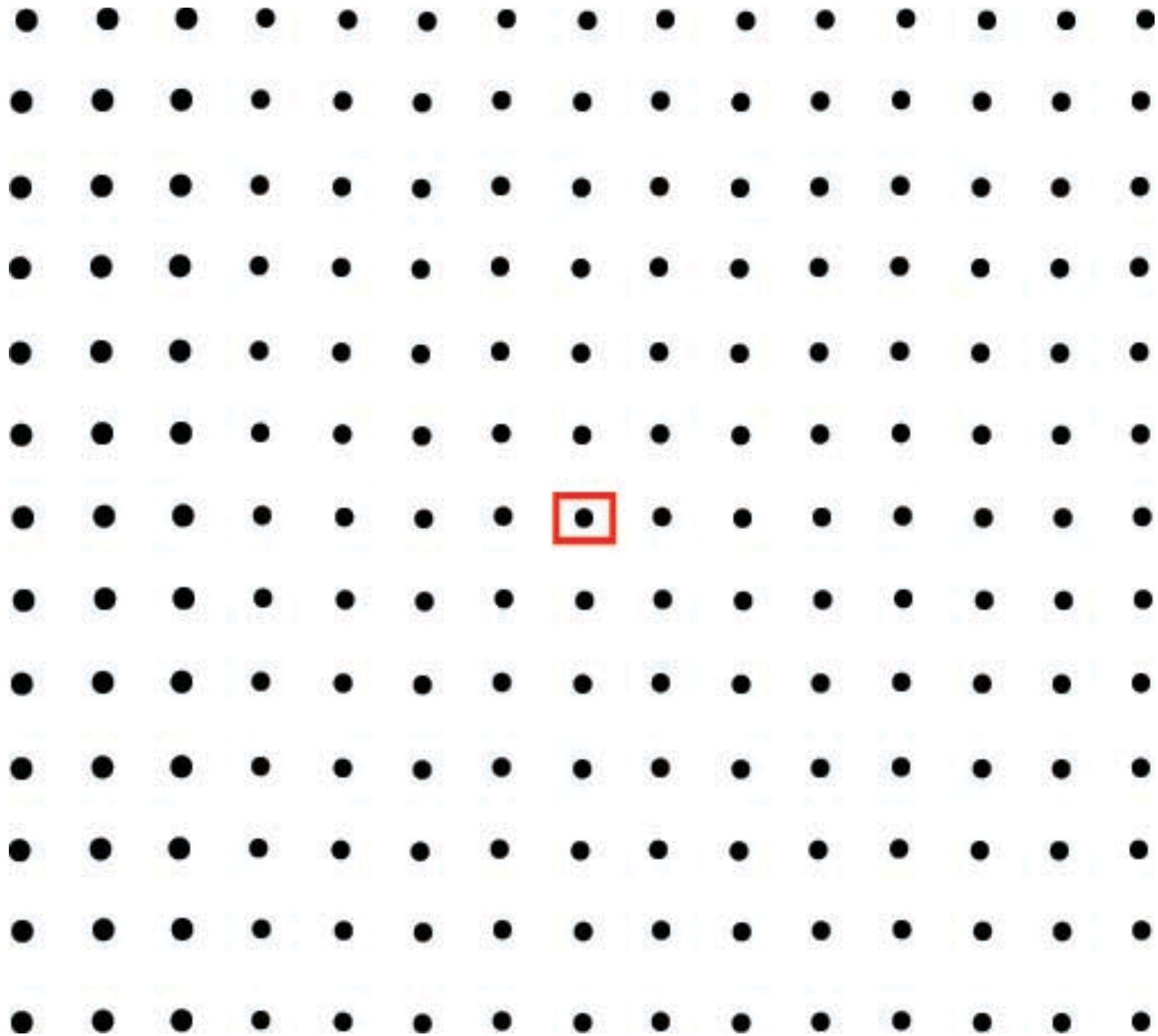


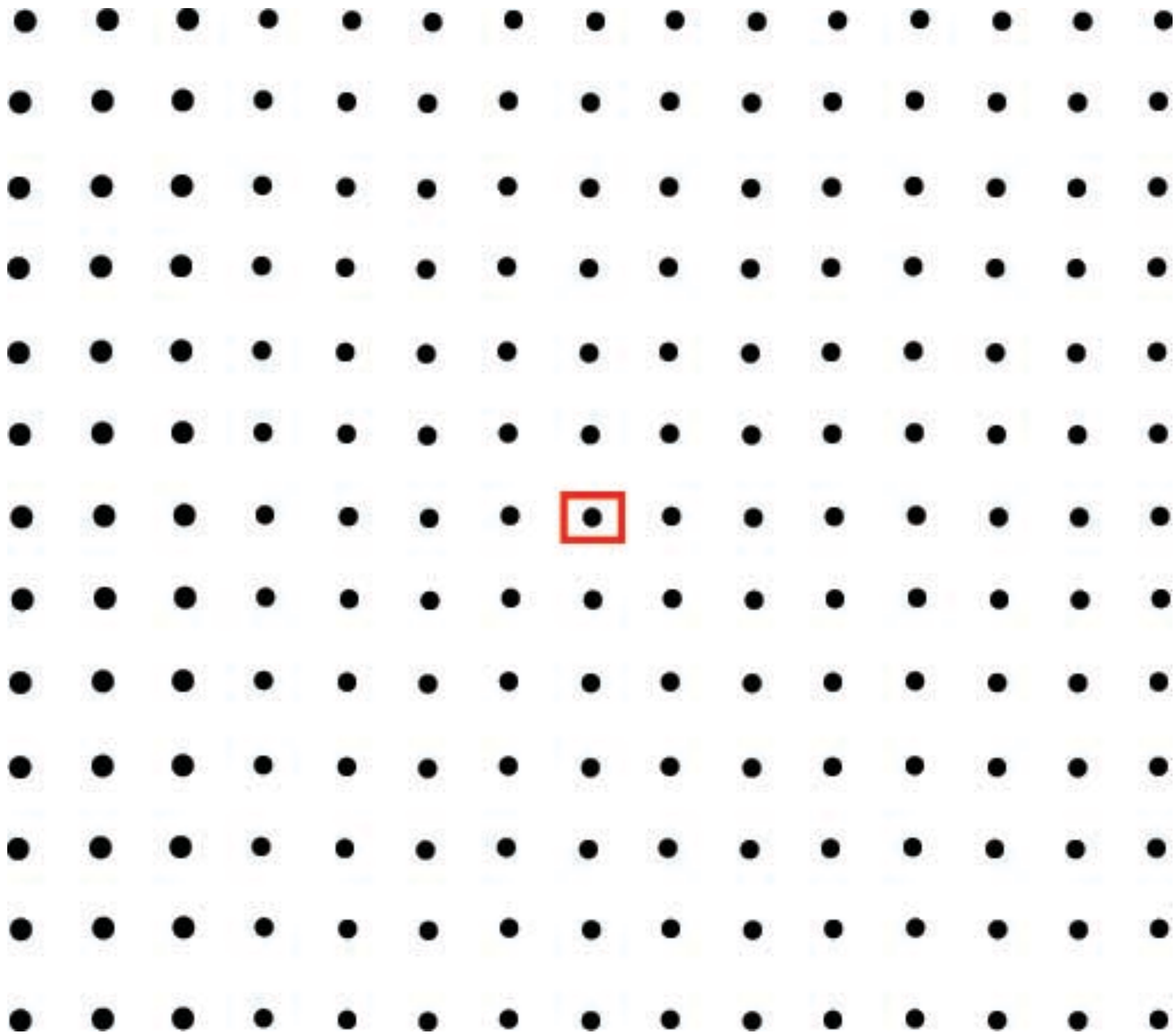


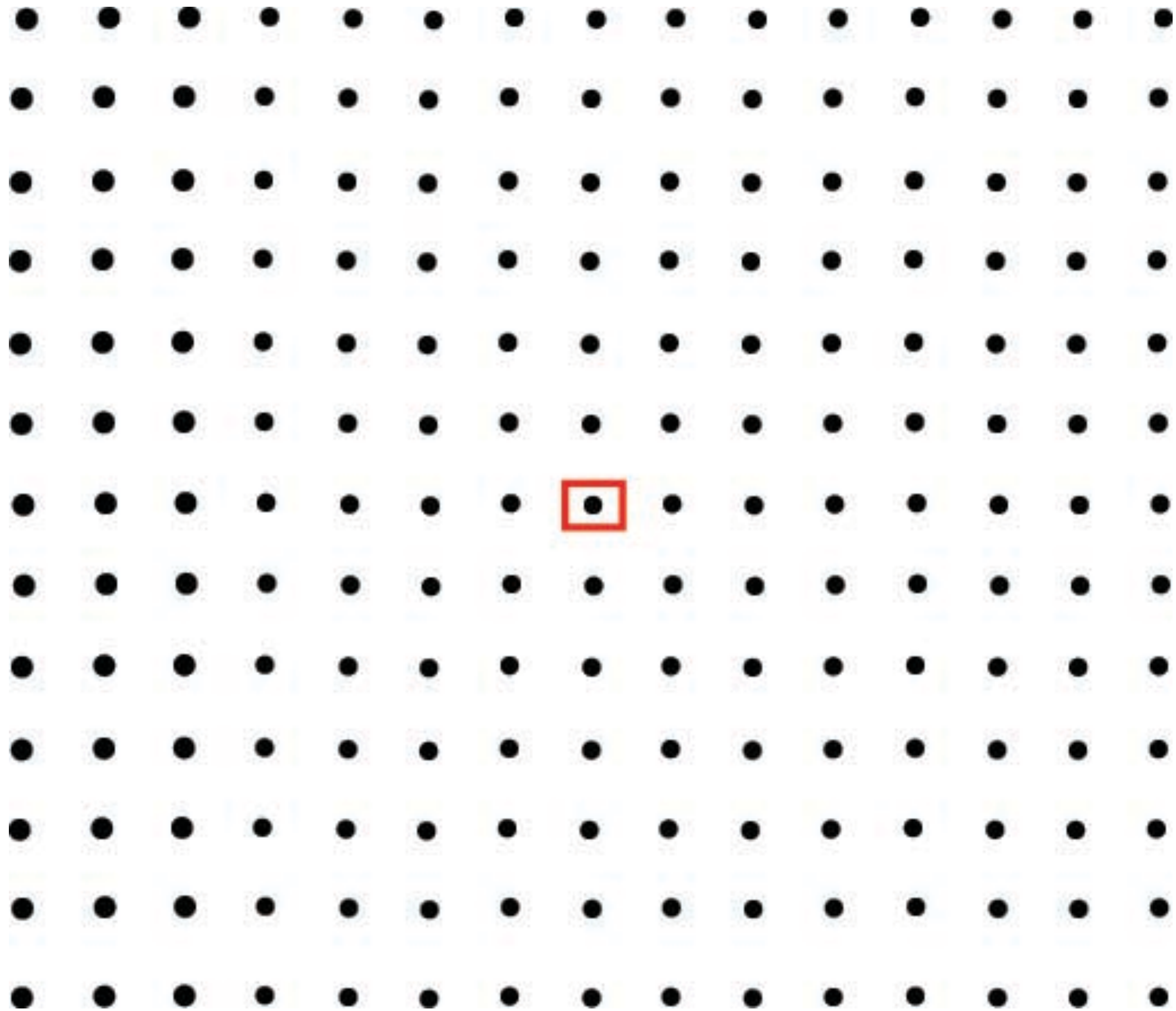


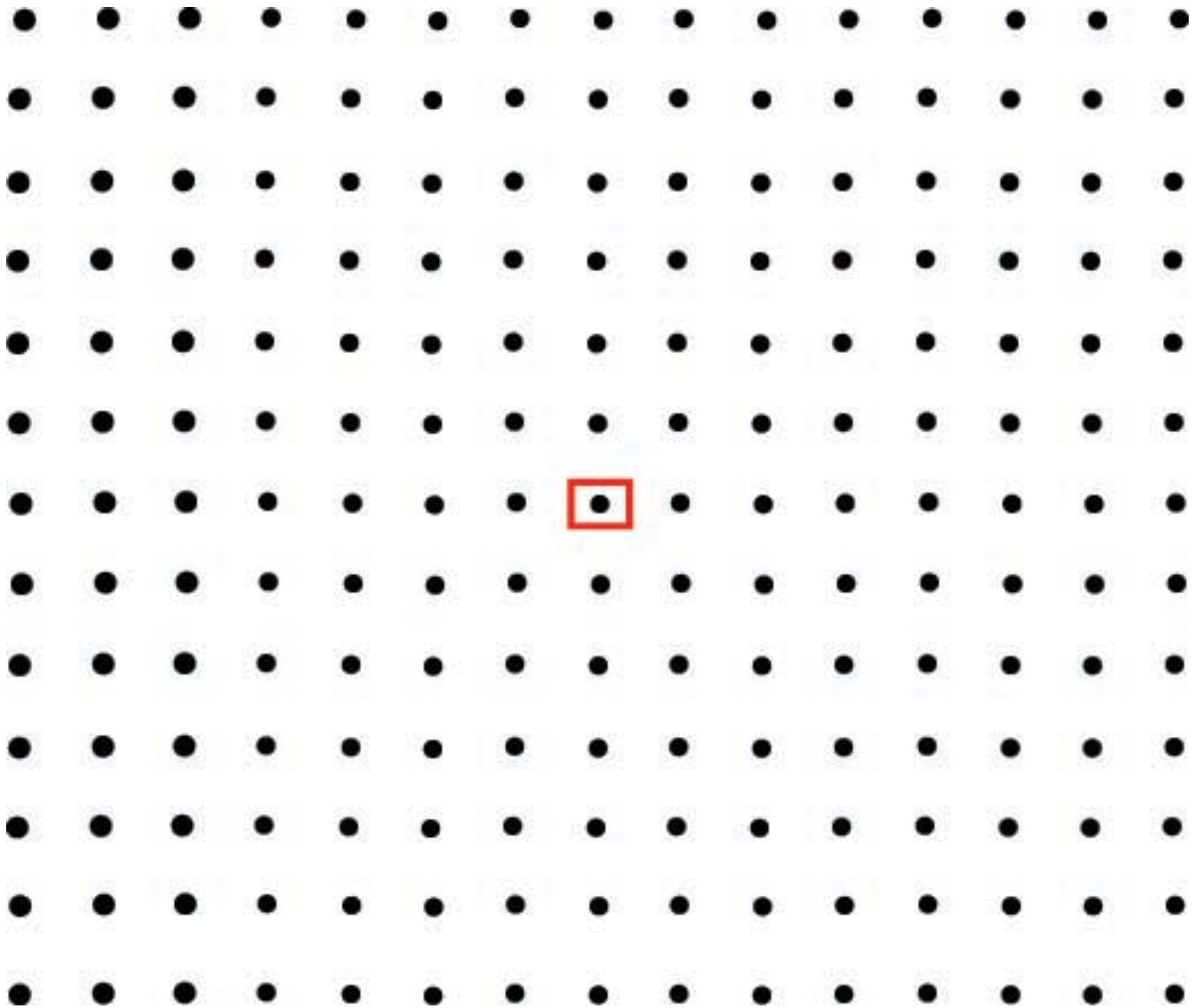


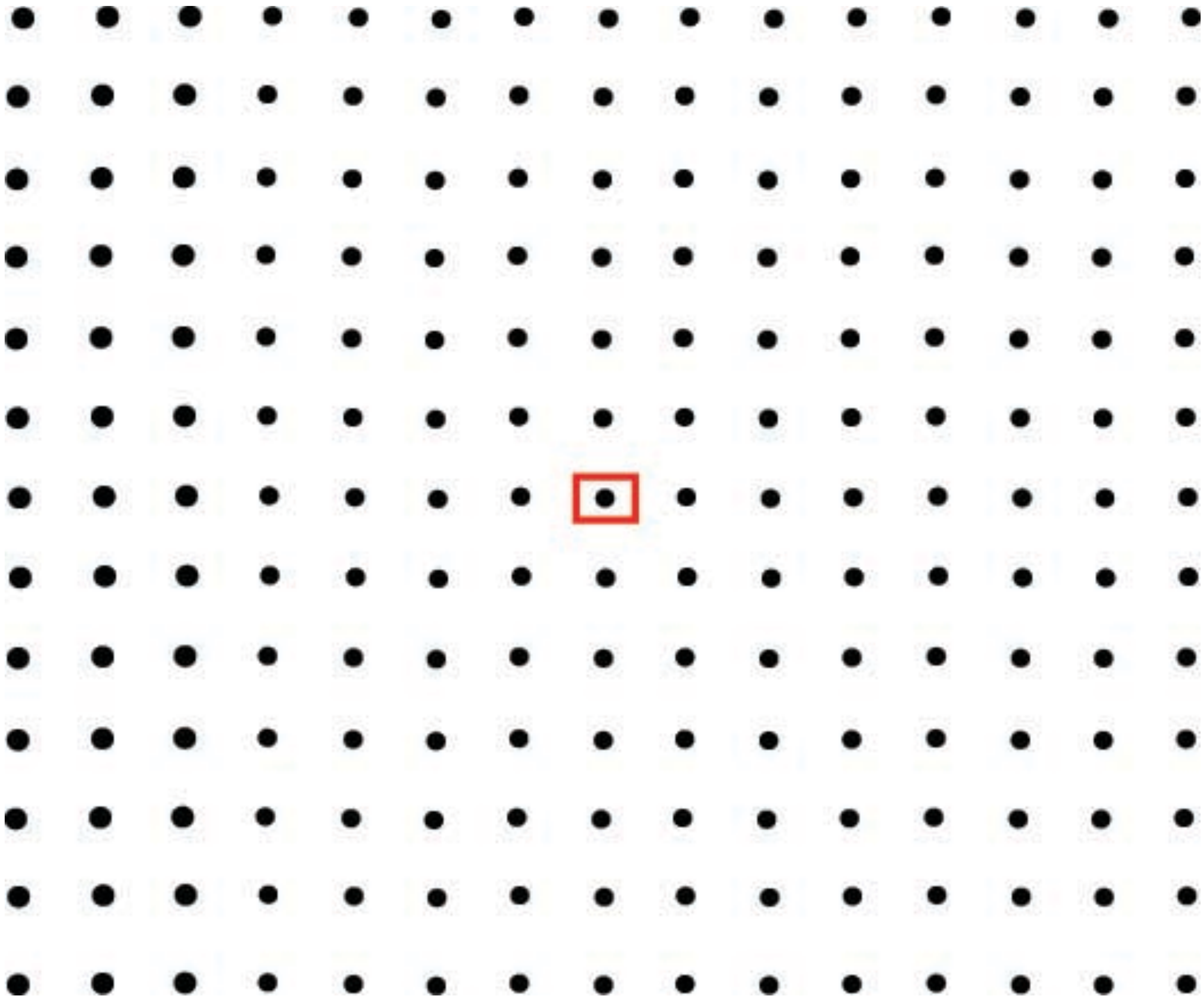


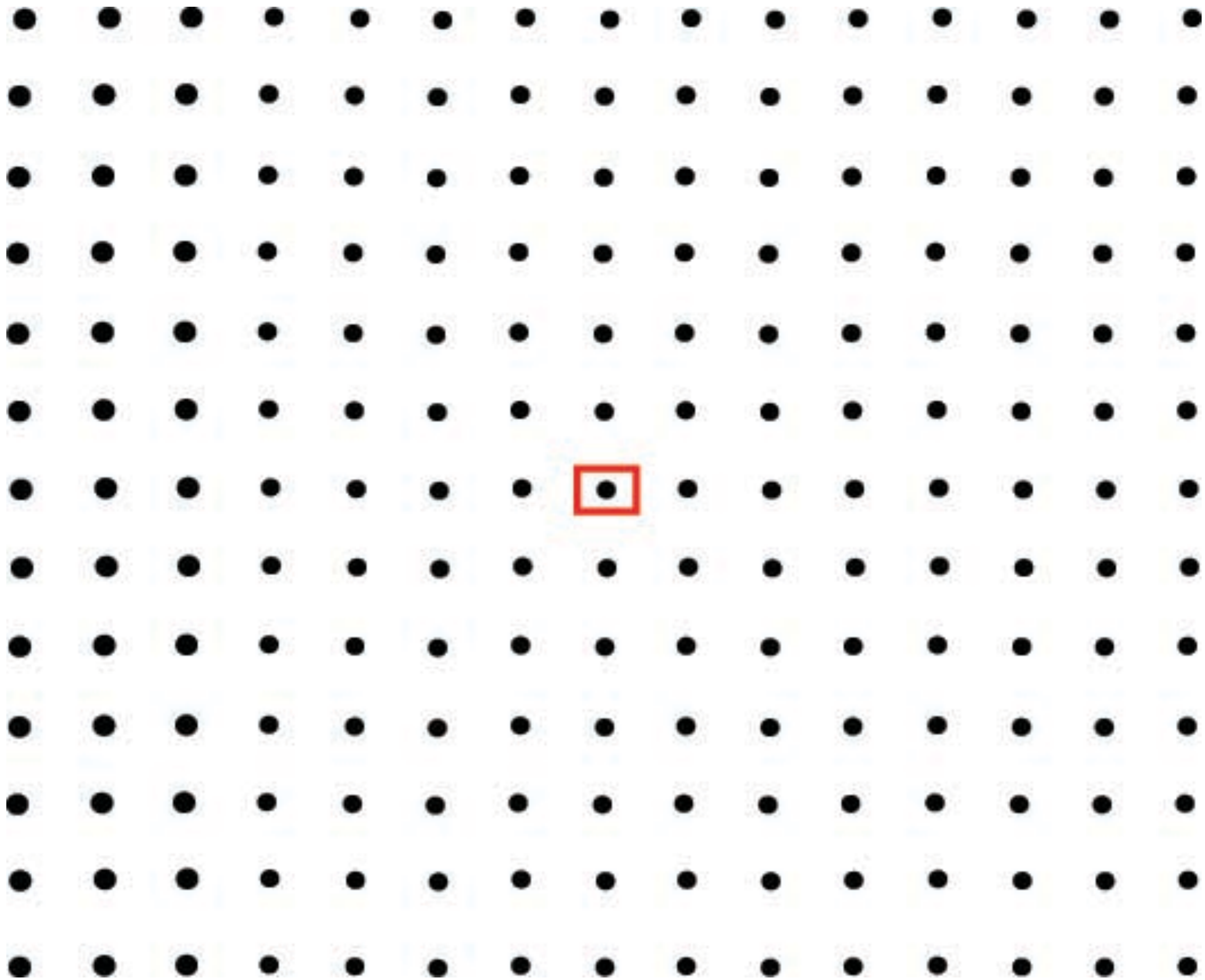


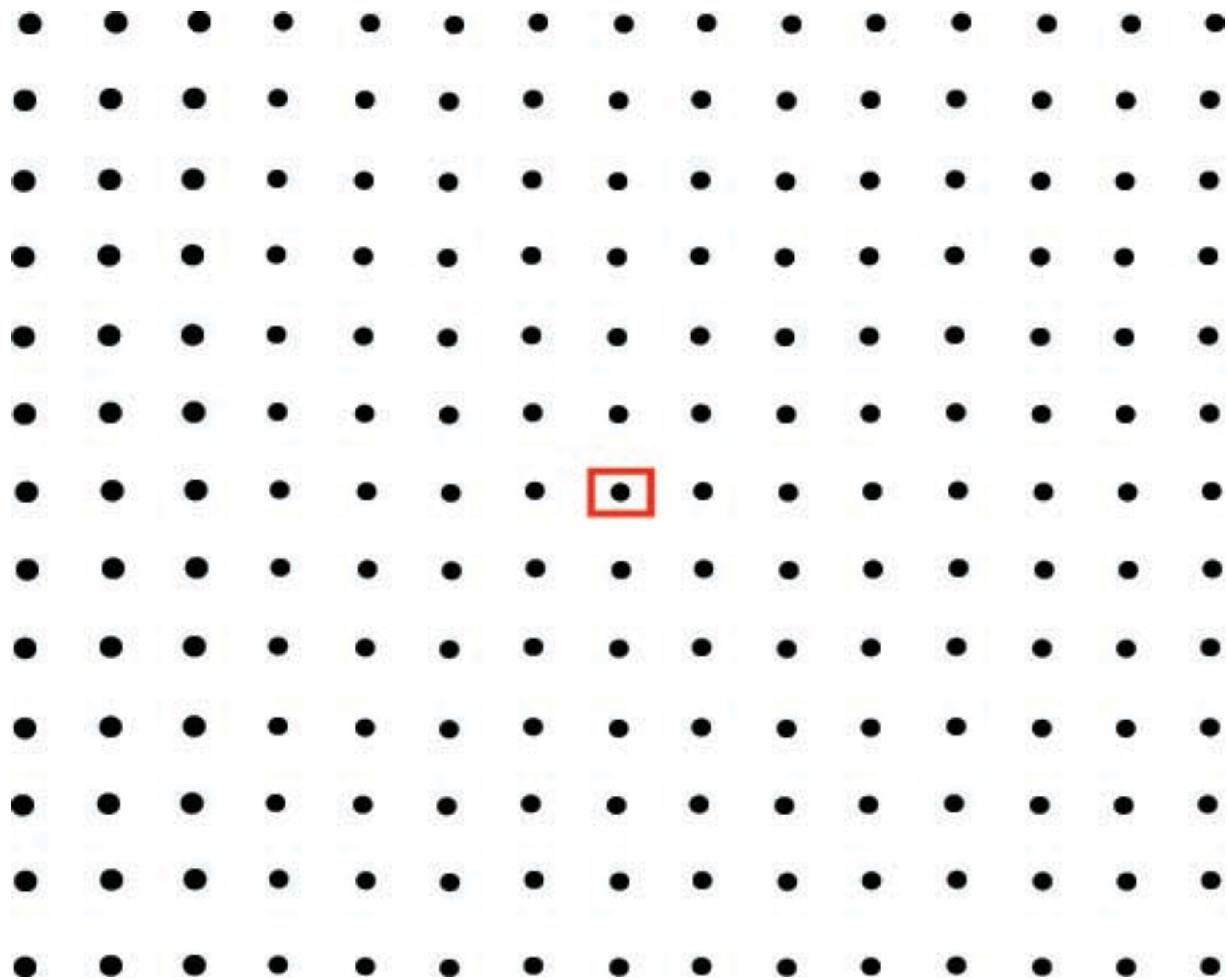


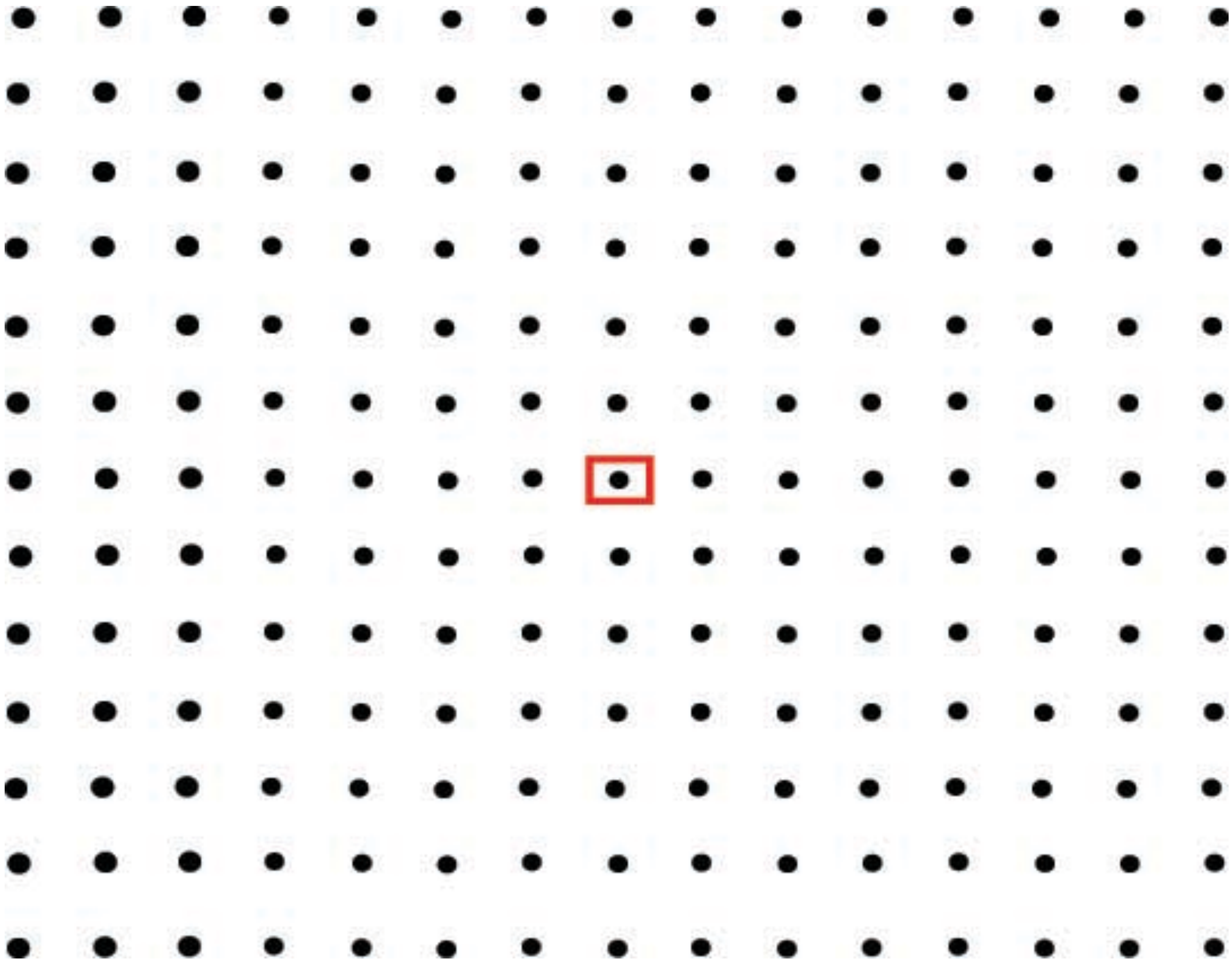


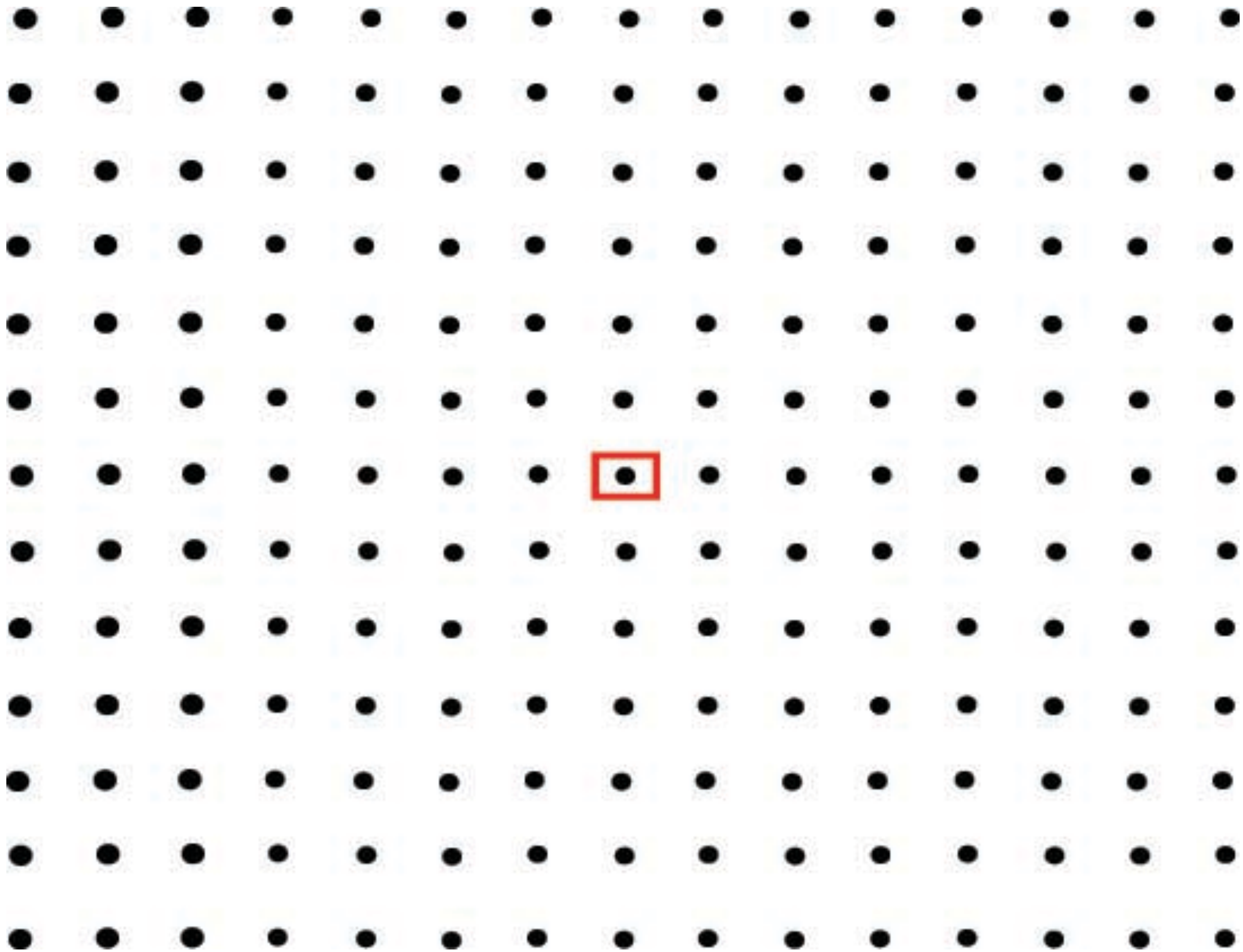


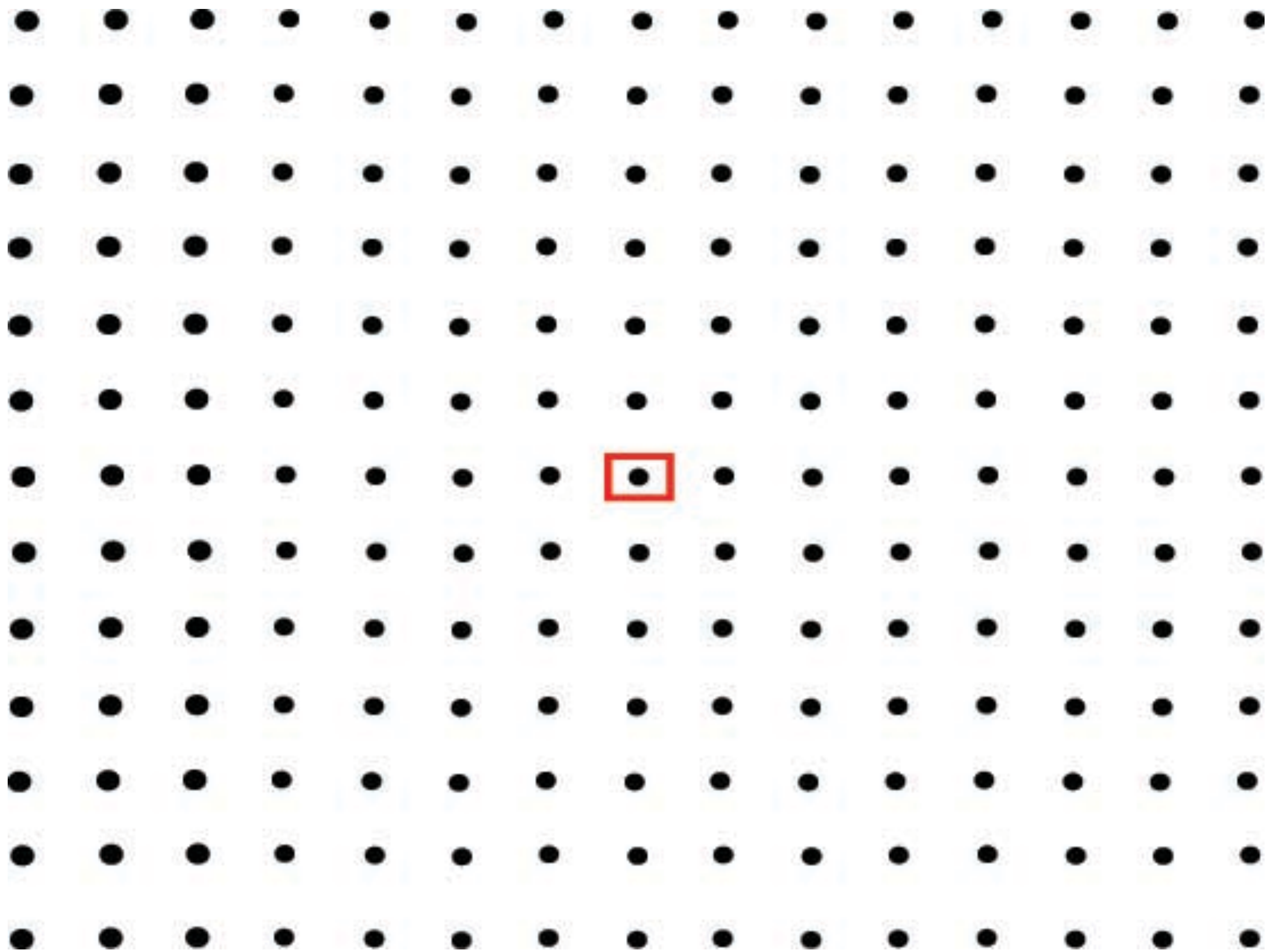


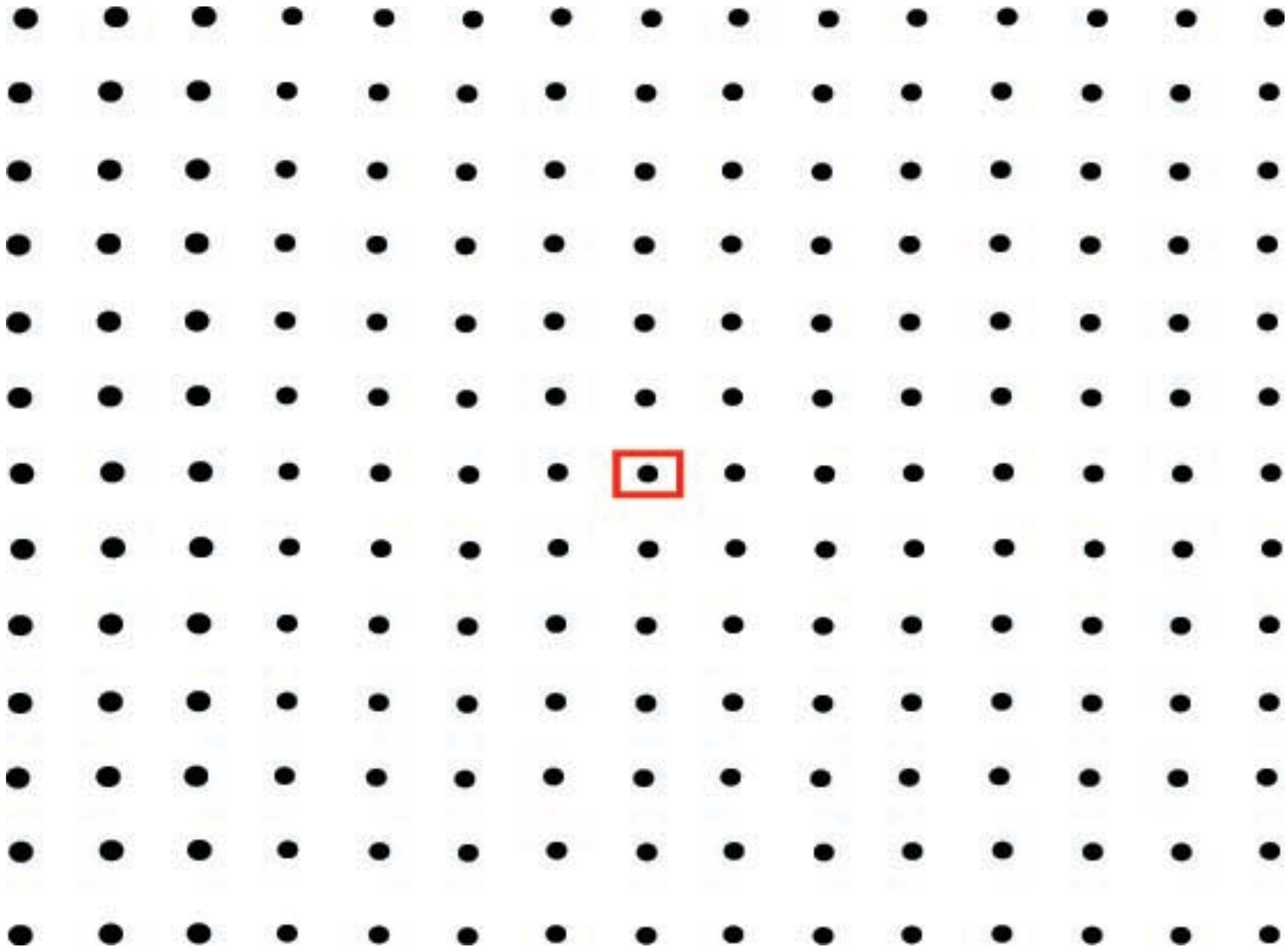


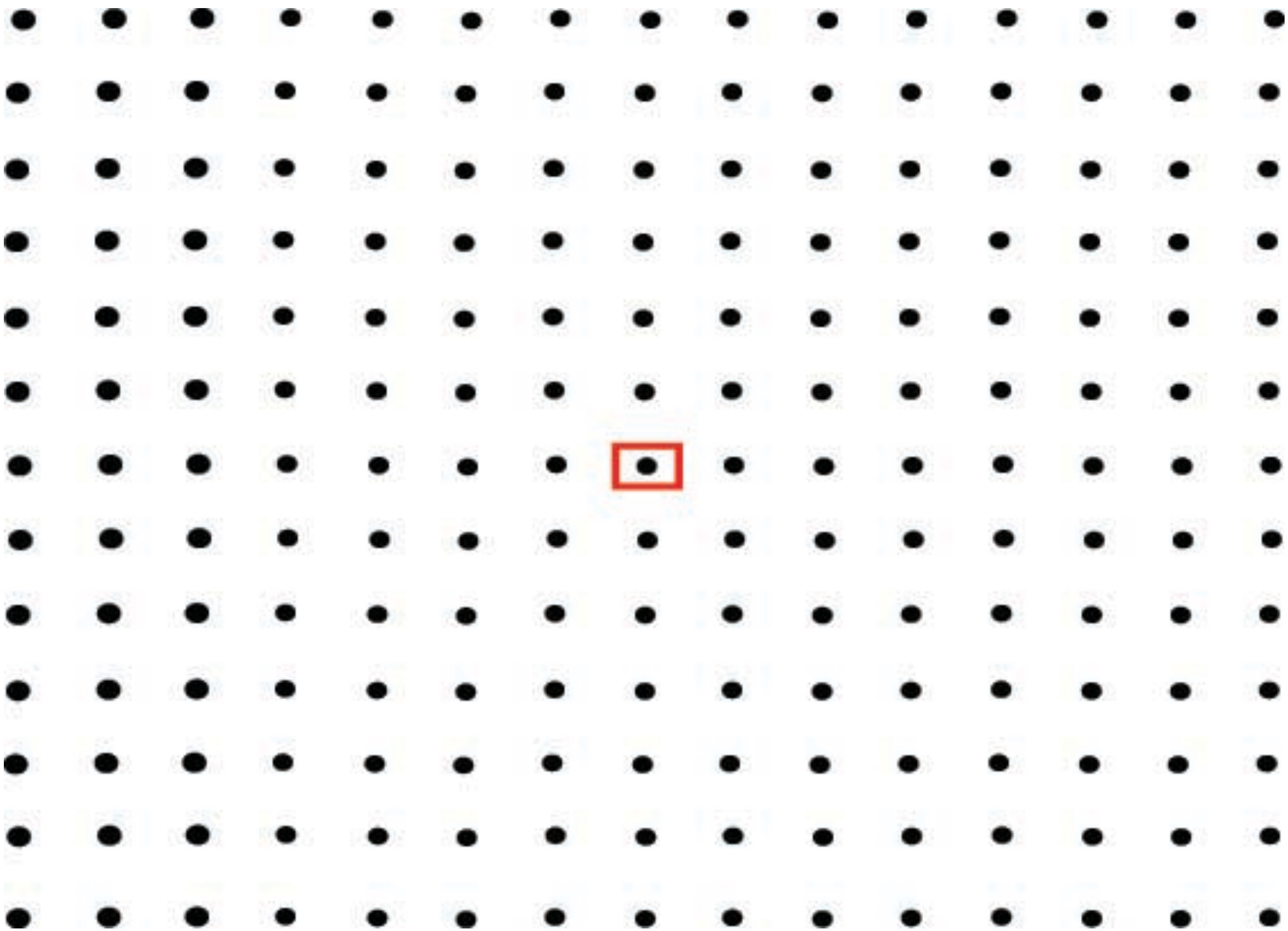


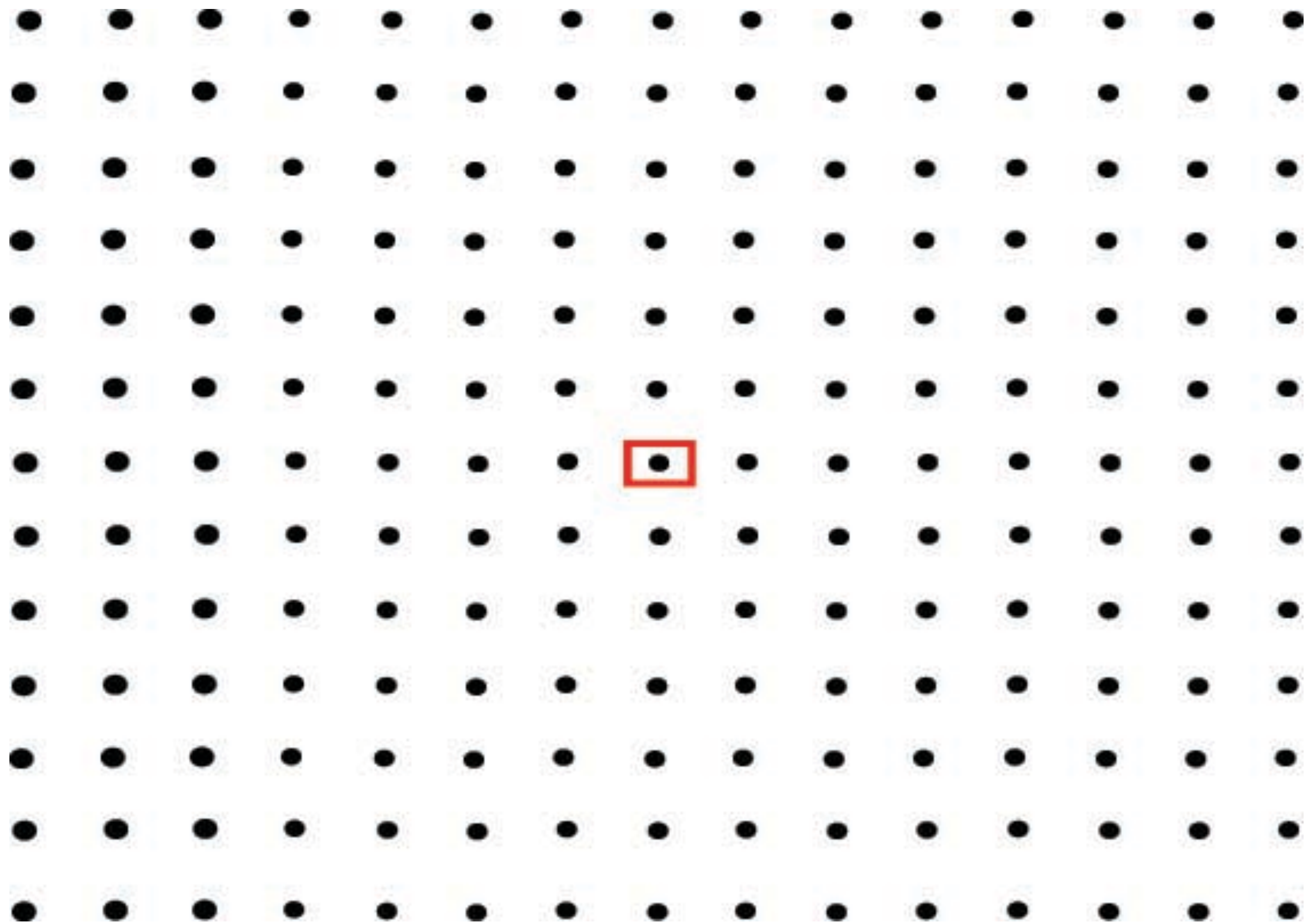


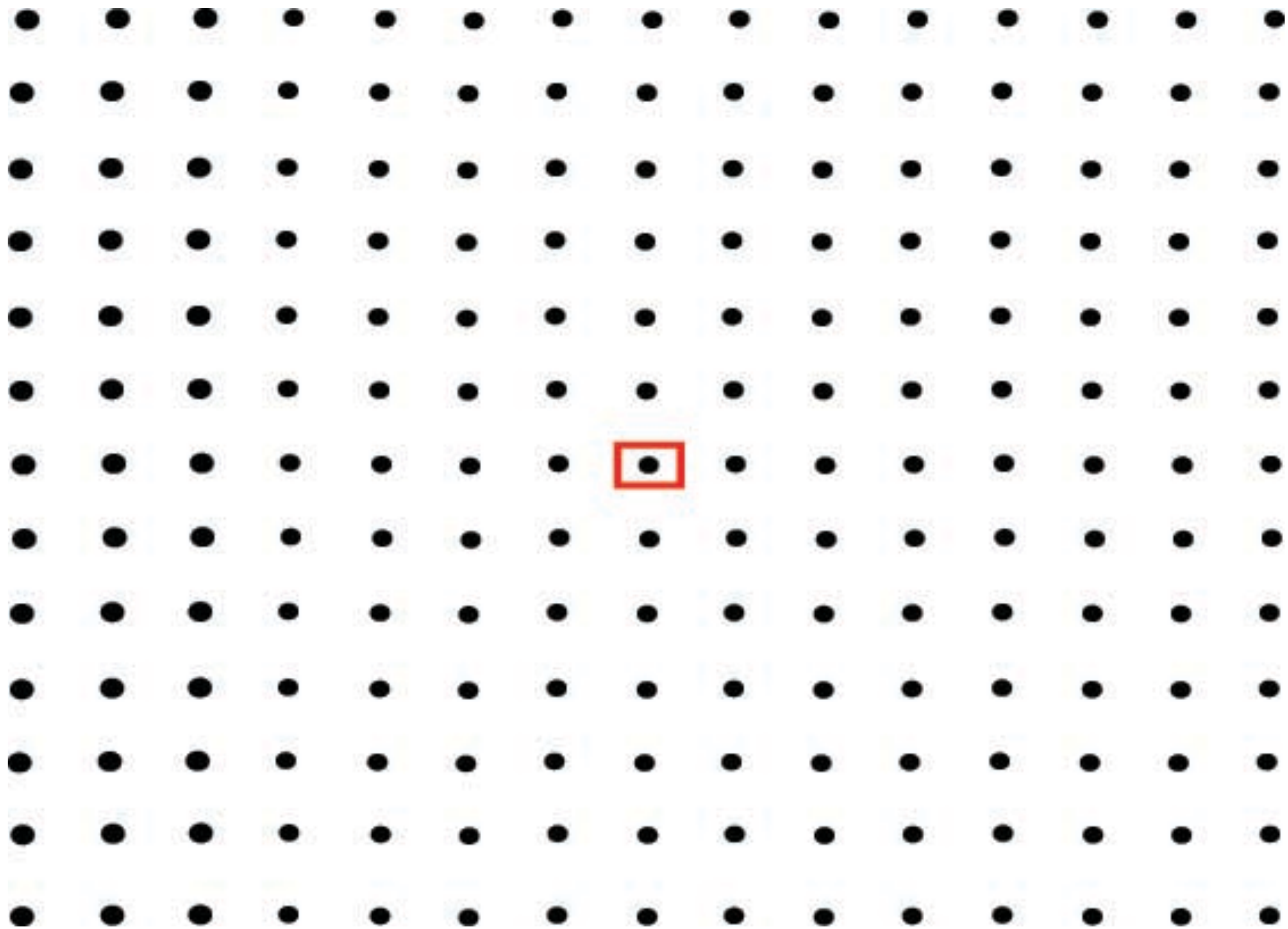


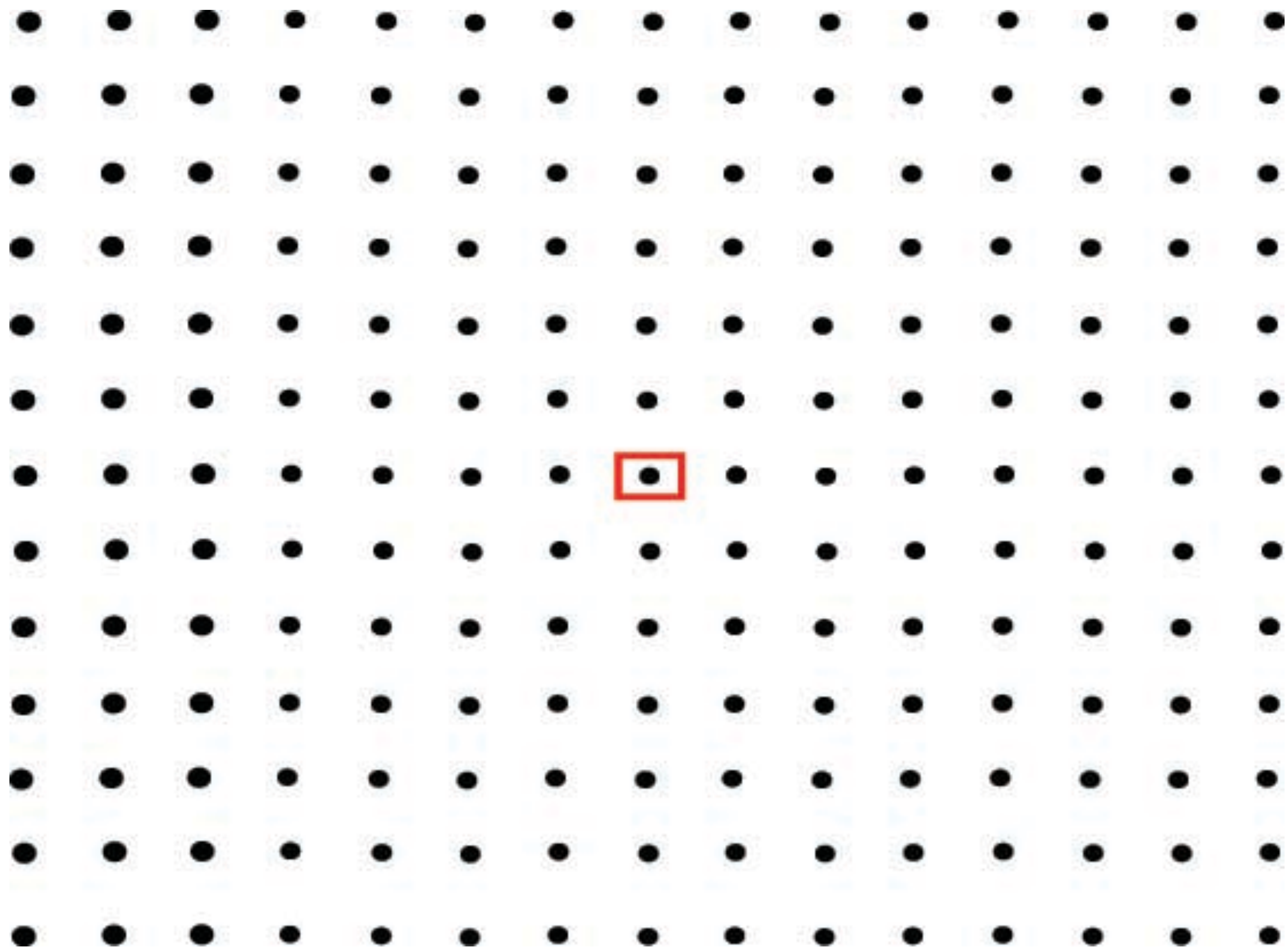


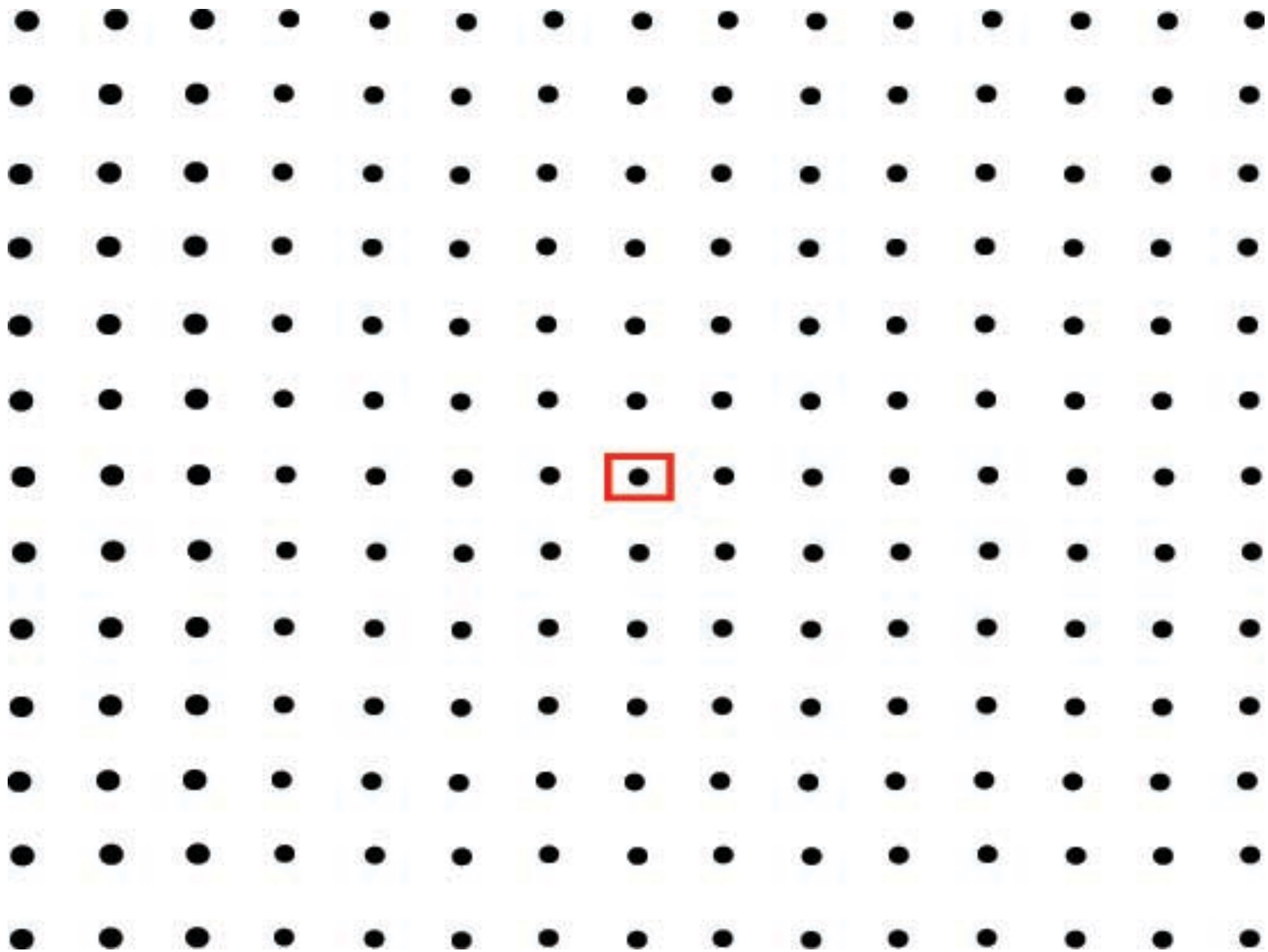


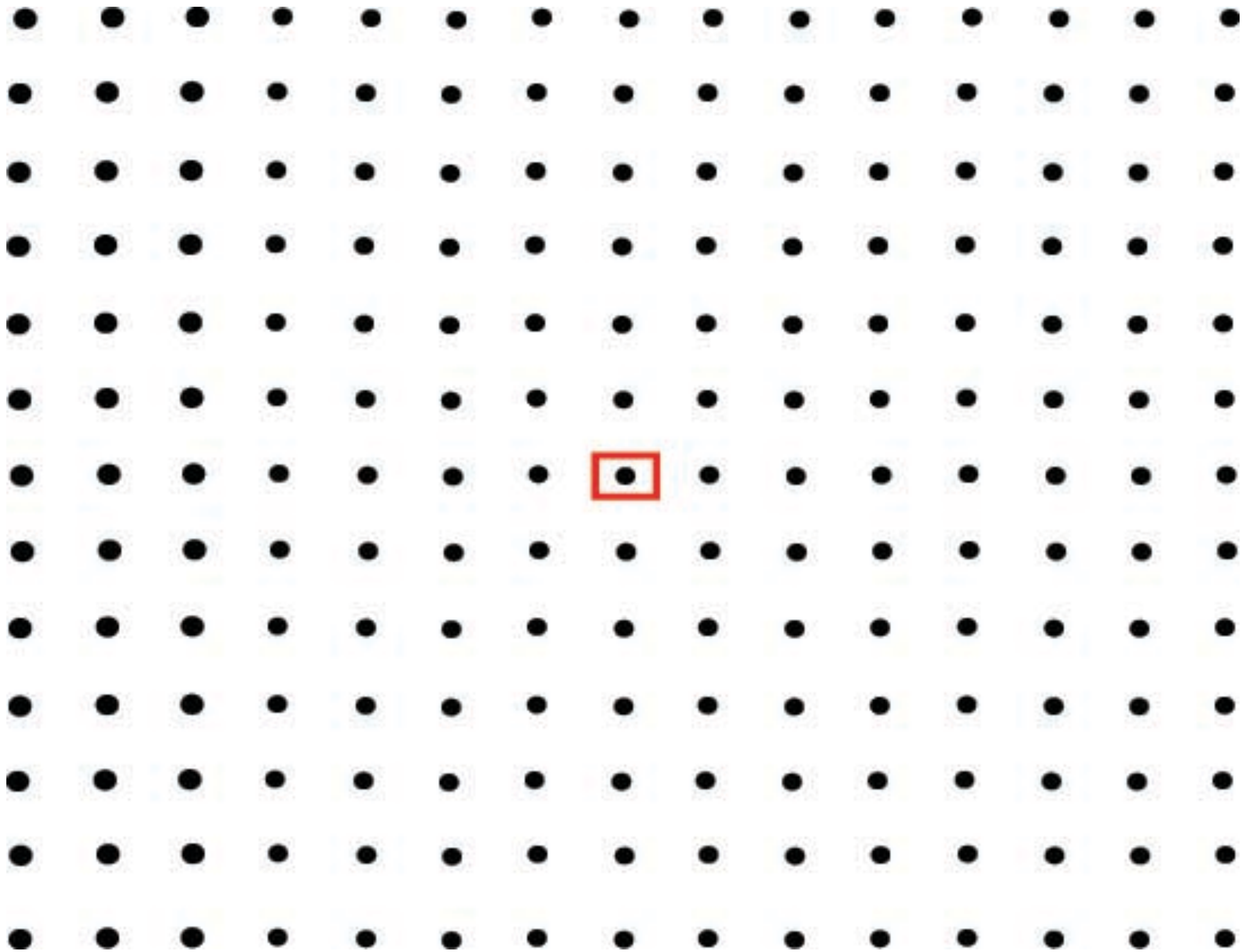


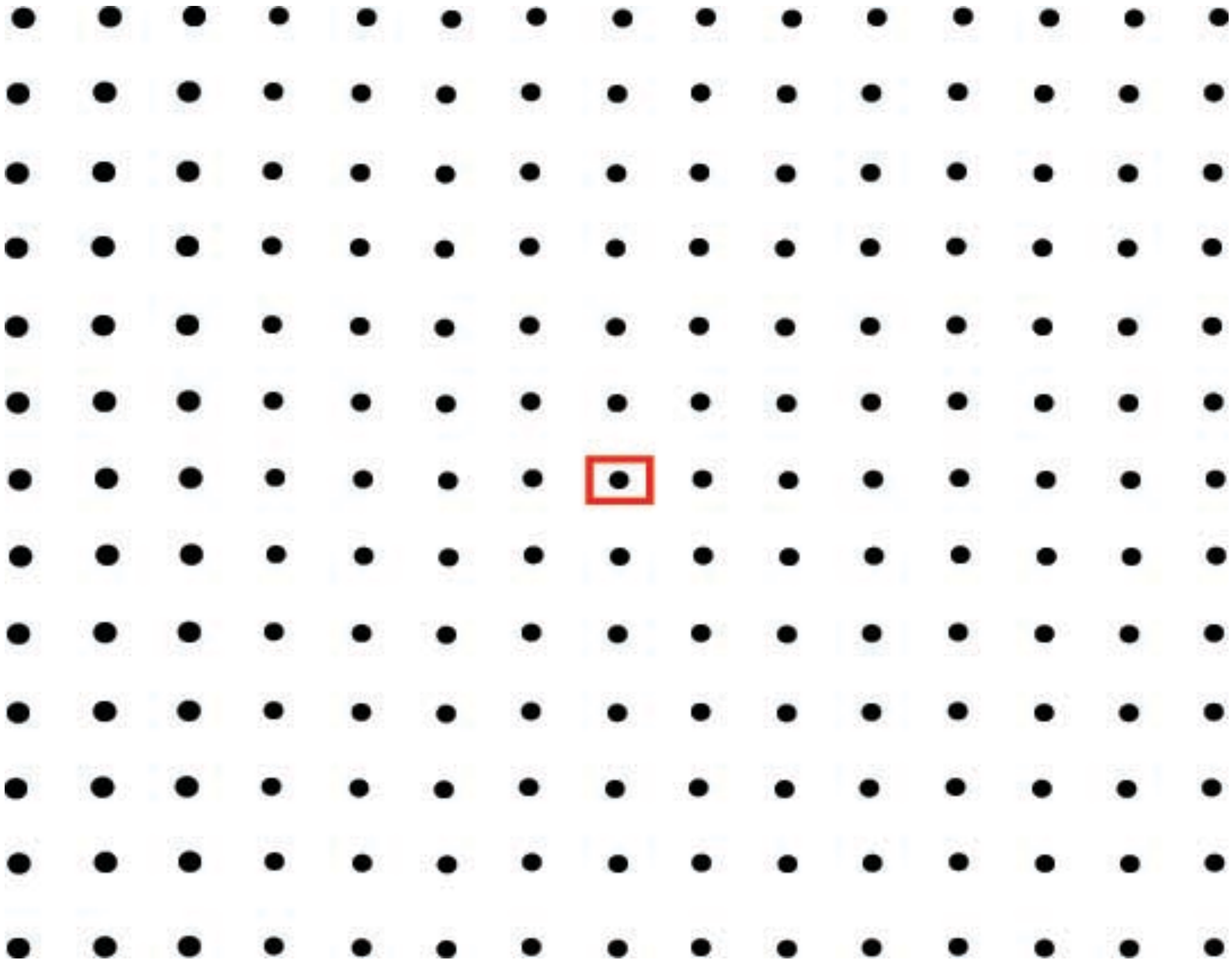


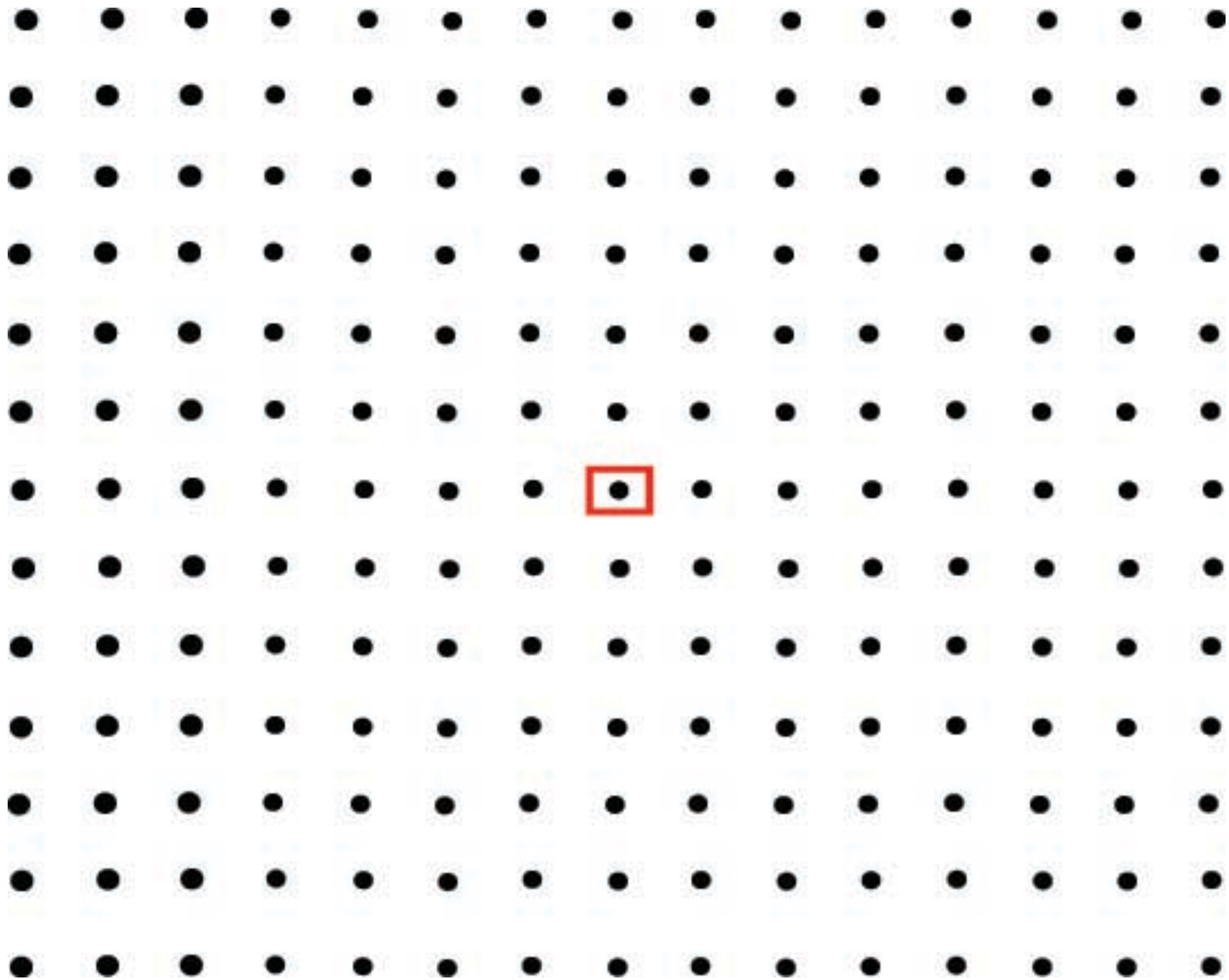


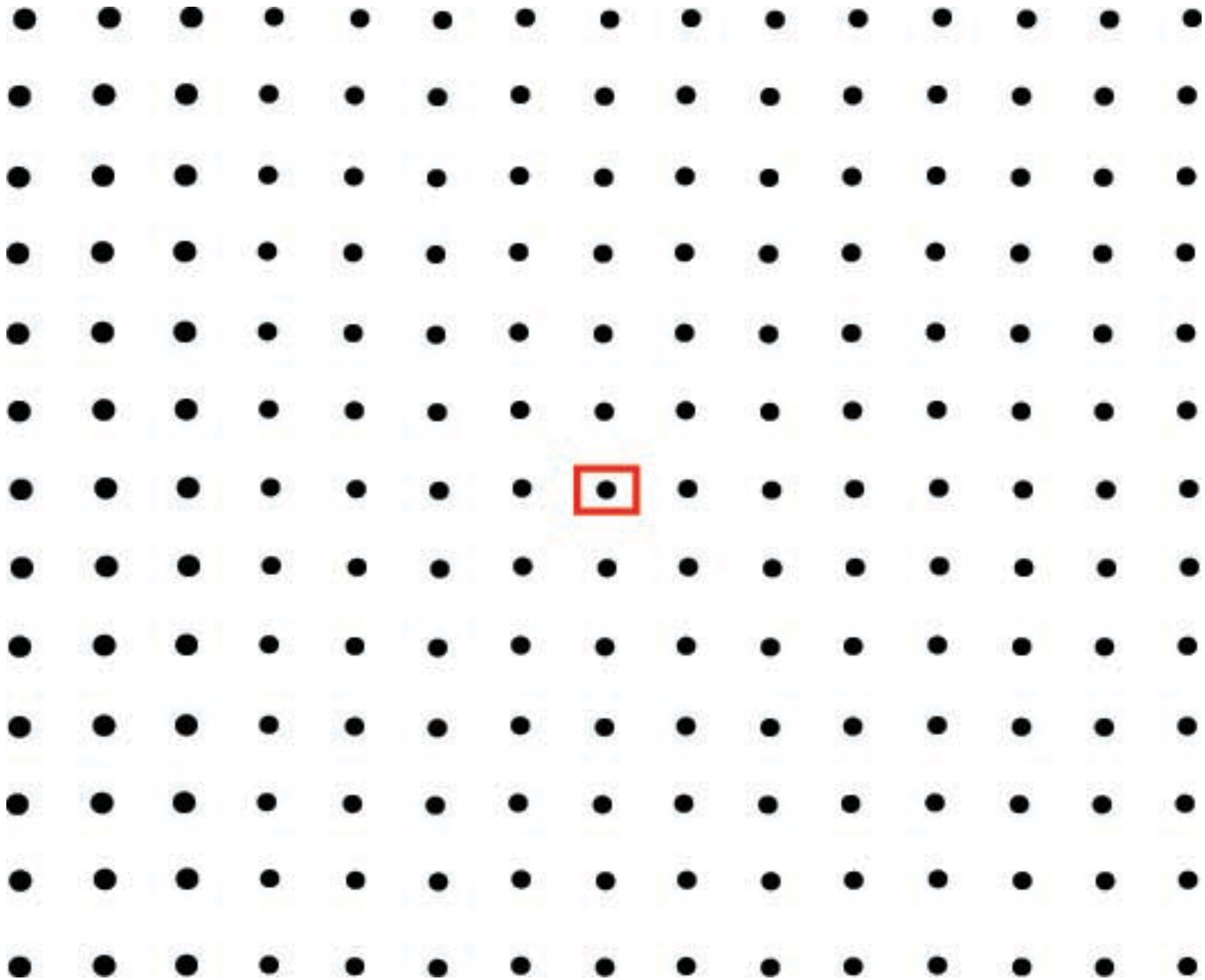


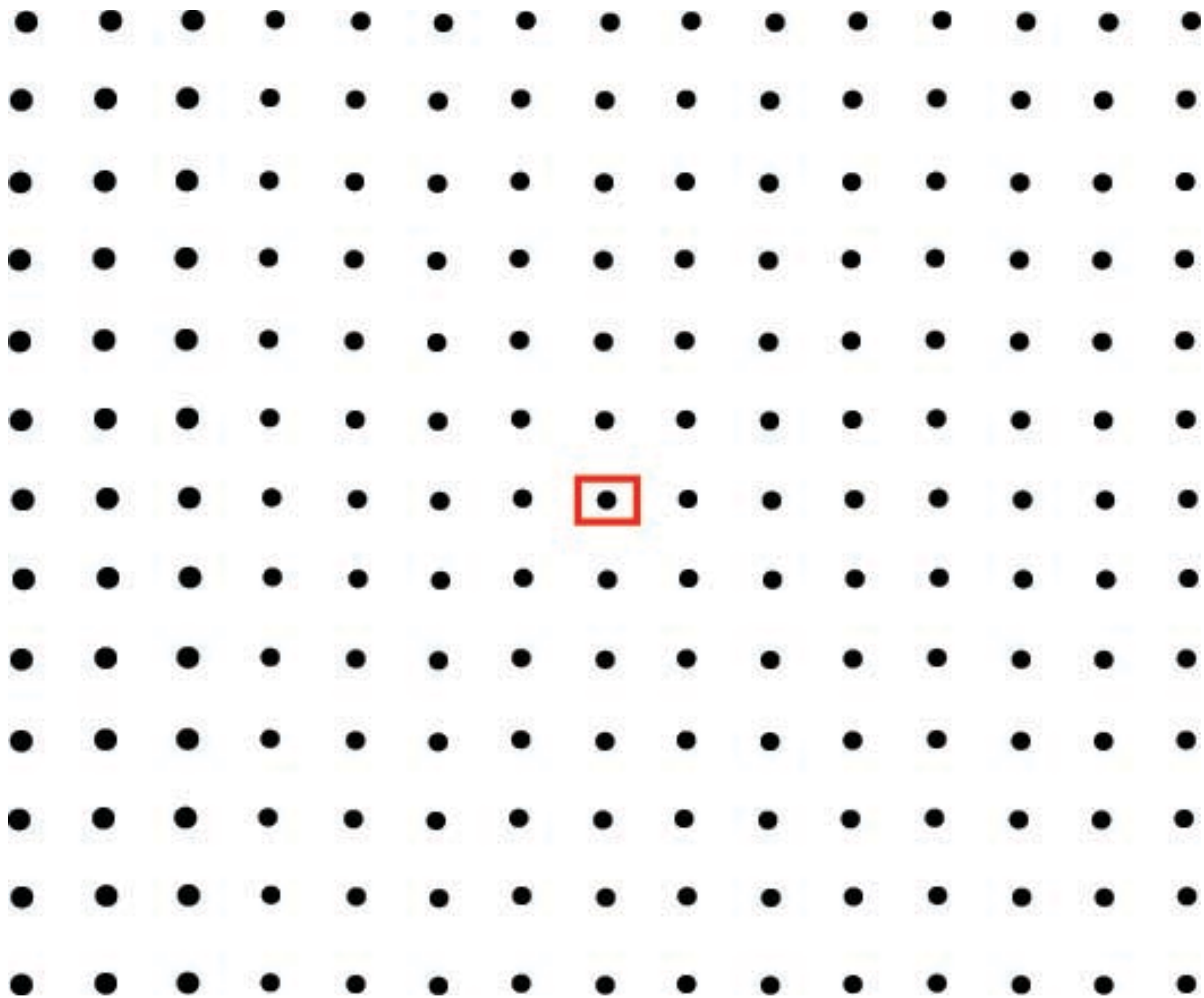


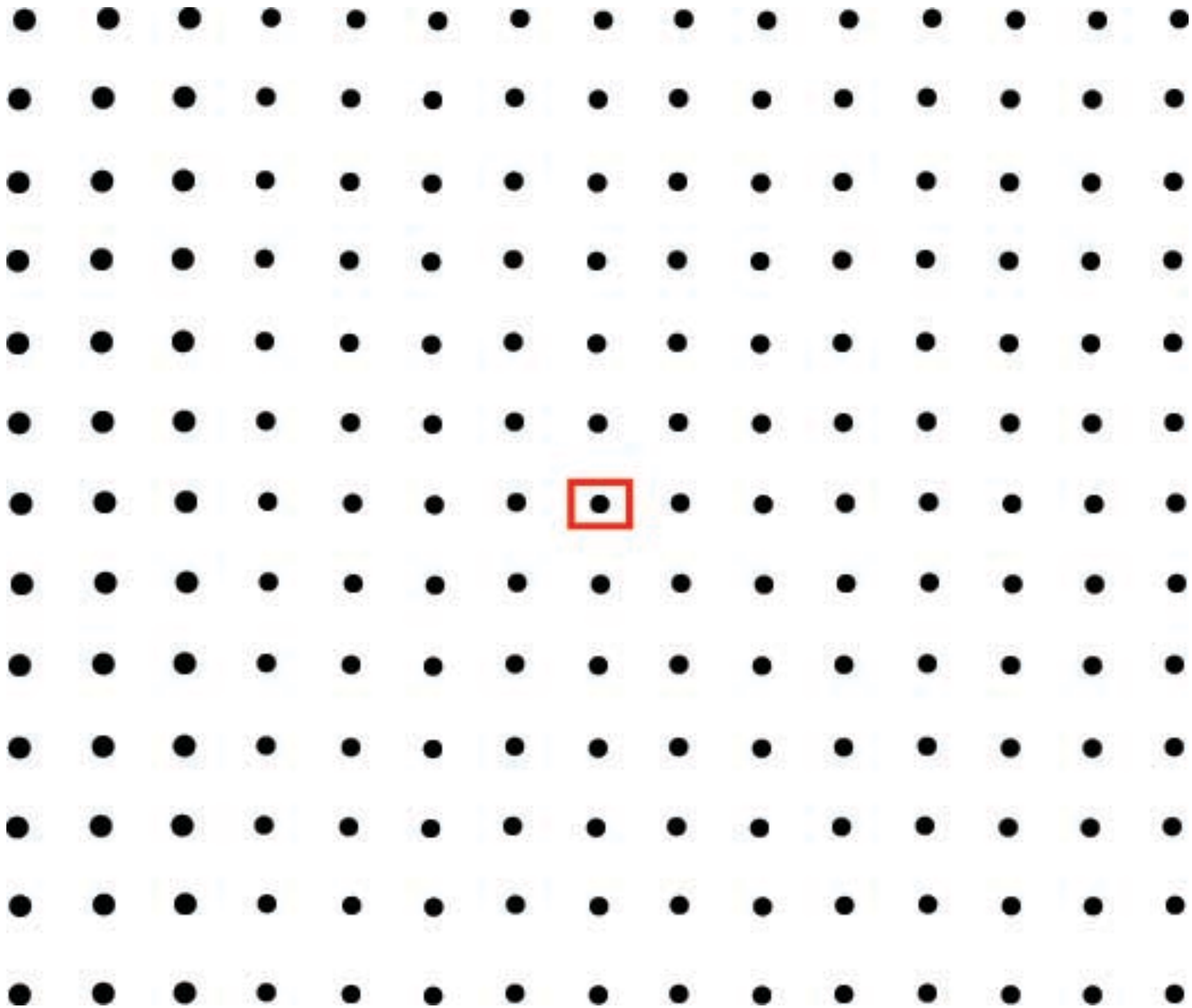












Relations for gravitational waves in modern notation

Intensity: $S_g = \frac{c^3}{16\pi G}\left\langle \dot{h}_+^2 + \dot{h}_x^2 \right\rangle$ pseudo tensor

$$\frac{c^3}{16\pi G} = 7.8\text{x}10^{36}\,\text{erg sec/ cm}^2$$

Power radiated: $P_g = \frac{32Gm^2x_0^4\omega^6}{5c^5}$ quadrupole formula

Relation to estimate GW amplitude: $h \approx \frac{\varphi_{Newton}}{c^2}\frac{v^2}{c^2} = \frac{Gm}{Rc^2}\frac{v^2}{c^2}$

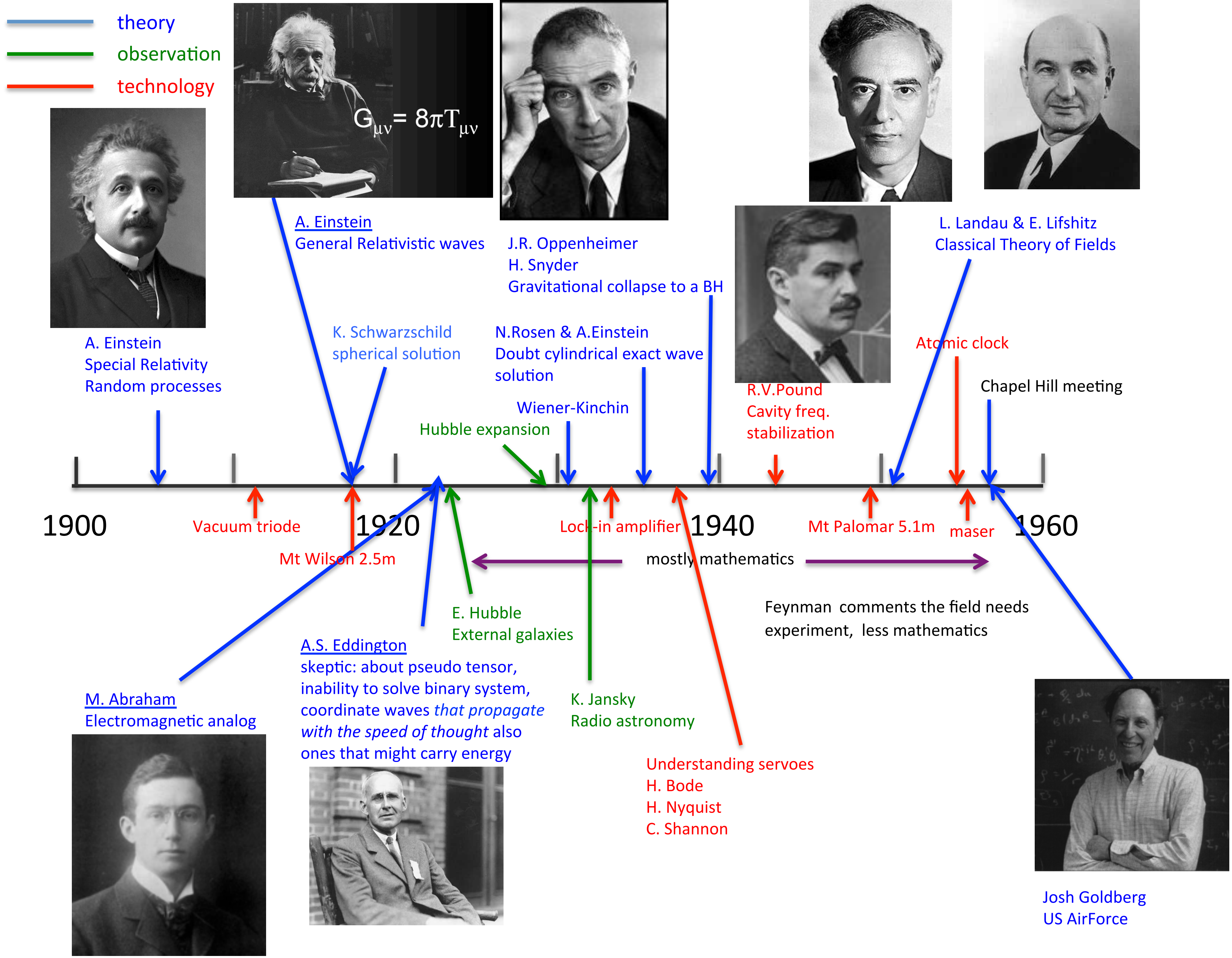
1916 example: binary star system

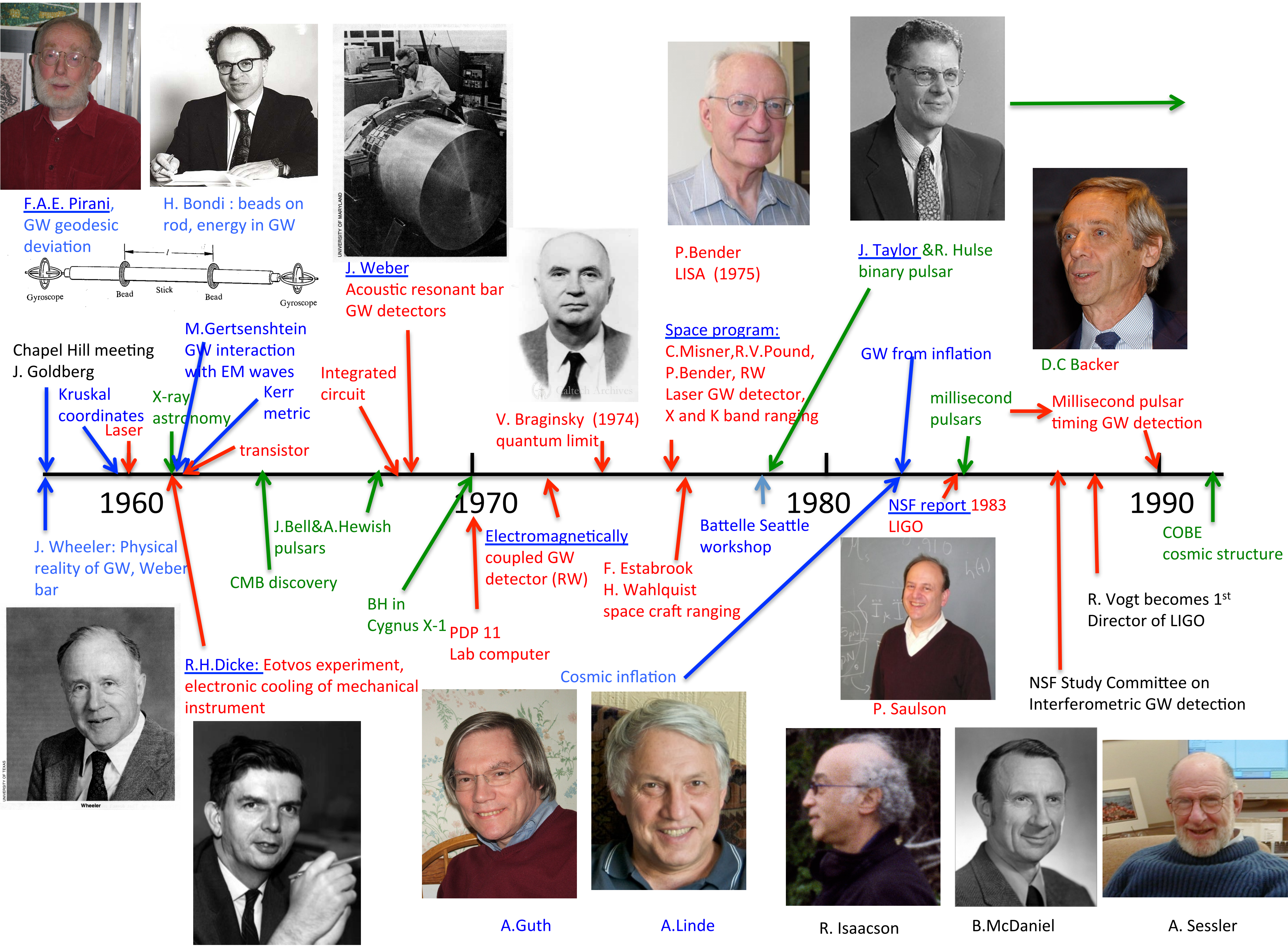
m_1=m_2= 1 solar mass
T_{orbit}= 1 day
R = 10 Kly

h ~ 10^{-23} @ ½ day period

$$Q = \frac{2\pi E_{stored}}{\Delta E_{1period}} \sim 10^{15} \quad \text{decay time} \sim 10^{13}\,\text{years}$$

scaling $h \propto \frac{m^{\frac{5}{3}}}{RT_{orbit}^{\frac{2}{3}}}$ $Q \propto \left(\frac{T_{orbit}}{m}\right)^{\frac{5}{3}}$


theory
observation
technology
$G_{\mu\nu} = 8\pi T_{\mu\nu}$
A. Einstein
Special Relativity
Random processes
A. Einstein
General Relativistic waves
K. Schwarzschild
spherical solution
J.R. Oppenheimer
H. Snyder
Gravitational collapse to a BH
N.Rosen & A.Einstein
Doubt cylindrical exact wave
solution
Wiener-Kinchin
Hubble expansion
R.V.Pound
Cavity freq.
stabilization
L. Landau & E. Lifshitz
Classical Theory of Fields
Atomic clock
Chapel Hill meeting
1900
Vacuum triode
1920
Mt Wilson 2.5m
Lock-in amplifier
1940
Mt Palomar 5.1m
maser
1960
mostly mathematics
Feynman comments the field needs
experiment, less mathematics
E. Hubble
External galaxies
A.S. Eddington
skeptic: about pseudo tensor,
inability to solve binary system,
coordinate waves *that propagate*
with the speed of thought also
ones that might carry energy
M. Abraham
Electromagnetic analog
K. Jansky
Radio astronomy
Understanding servoes
H. Bode
H. Nyquist
C. Shannon
Josh Goldberg
US AirForce



F.A.E. Pirani, GW geodesic deviation
Gyroscope
Bead
Stick
Bead
Gyroscope
H. Bondi : beads on rod, energy in GW
UNIVERSITY OF MARYLAND
J. Weber
Acoustic resonant bar GW detectors
P.Bender
LISA (1975)
J. Taylor &R. Hulse binary pulsar
D.C Backer
Chapel Hill meeting
J. Goldberg
M.Gertsenshtein GW interaction with EM waves
Integrated circuit
Kruskal coordinates
Laser
X-ray astronomy
Kerr metric
transistor
Caltech Archives
V. Braginsky (1974) quantum limit
Space program:
C.Misner,R.V.Pound, P.Bender, RW
Laser GW detector, X and K band ranging
GW from inflation
millisecond pulsars
Millisecond pulsar timing GW detection
1960
1970
1980
1990
J. Wheeler: Physical reality of GW, Weber bar
J.Bell&A.Hewish pulsars
CMB discovery
BH in Cygnus X-1
Electromagnetically coupled GW detector (RW)
PDP 11 Lab computer
F. Estabrook
H. Wahlquist
space craft ranging
Battelle Seattle workshop
NSF report 1983
LIGO
COBE cosmic structure
R. Vogt becomes 1st Director of LIGO
NSF Study Committee on Interferometric GW detection
UNIVERSITY OF TEXAS
Wheeler
R.H.Dicke: Eotvos experiment, electronic cooling of mechanical instrument
Cosmic inflation
P. Saulson
A.Guth
A.Linde
R. Isaacson
B.McDaniel
A. Sessler

Acoustic bar GW Detector groups

R. Garwin

W. Fairbank

E. Amaldi

1965-1975
Room T bars

Bell Labs
Frascati
Glasgow
IBM
Rochester
Max Planck
Rome

A. Tyson

1975-1990+
Cryogenic bars

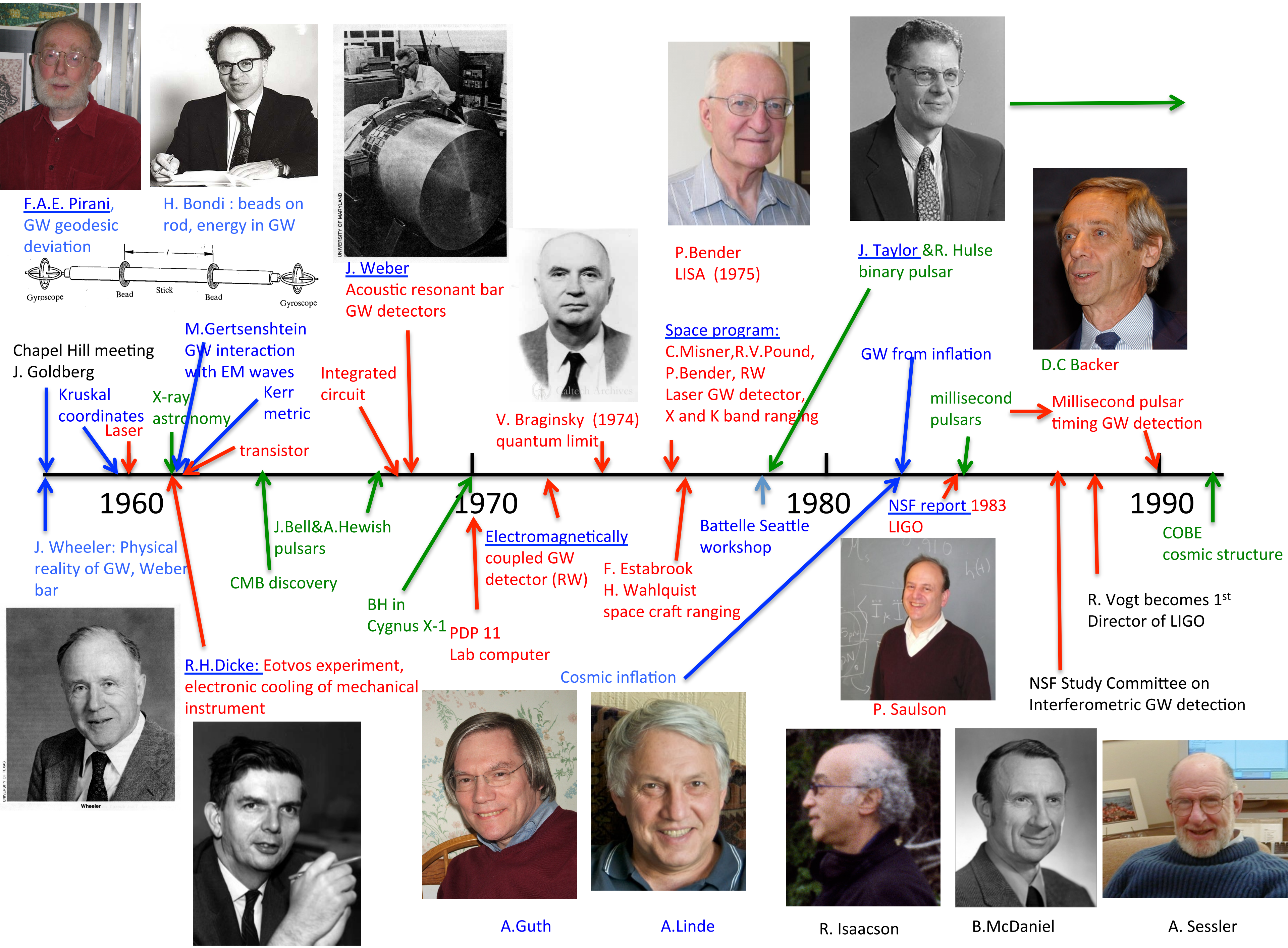
Frascati
Louisiana
Moscow
Perth
Rochester
Stanford

W. Hamilton

2000 ->
Spherical cryogenic detectors

Brazil
Netherlands

P. Michelson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Millisecond pulsar timing GW detection
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1970
1980
1990
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PDP 11
Lab computer
Electromagnetically coupled GW detector (RW)
F. Estabrook
H. Wahlquist
space craft ranging
Battelle Seattle workshop
NSF report 1983
LIGO
COBE
cosmic structure
R. Vogt becomes 1st Director of LIGO
Wheeler
UNIVERSITY OF TEXAS
R.H.Dicke: Eotvos experiment, electronic cooling of mechanical instrument
Cosmic inflation
P. Saulson
NSF Study Committee on Interferometric GW detection
A.Guth
A.Linde
R. Isaacson
B.McDaniel
A. Sessler

Plane gravitational waves

Transverse Plane Wave Solutions with "Electric" and "Magnetic" Terms

Geometric Interpretation

$$ds^2 = g_{ij} dx^i dx^j$$

$$g_{ij} = \eta_{ij} + h_{ij} \qquad \text{weak field}$$

$$\eta_{ij} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ 0 & & -1 & \\ & & & -1 \end{pmatrix} \quad \text{Minkowski Metric of Special Relativity}$$

Gravity Wave Propagating in the x_1 Direction

$$h_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & h_{22} & h_{23} \\ 0 & 0 & h_{32} & h_{33} \end{pmatrix} \quad \text{all } h_{ij} \ll 1$$

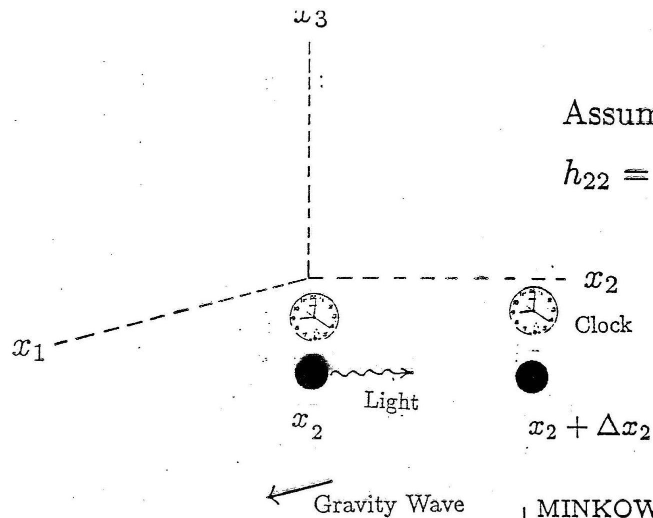
Plane Wave

$$\mathbf{h_{22}} = -\mathbf{h_{33}} \qquad \mathbf{h_{23}} = \mathbf{h_{32}}$$

$+$ **polarization** $\qquad \times$**polarization**

And All Only Function of $x_1 - ct$

Timing light in the gravitational wave



Assume + Polarization

$h_{22} = h \sin(kx_1 - \omega t)$

MINKOWSKI

$$\Delta s^2 = 0 = c^2 \Delta t^2 - \left(1 + h \sin(kx_1 - \omega t)\right) \Delta x_2^2$$

LIGHT RAY

Let $\Delta t \ll \frac{1}{\omega}$ $h \ll 1$

$$c \Delta t \cong \left(1 + \frac{h}{2} \sin(kx_1 - \omega t)\right) \Delta x_2$$

INFERRED DISTANCE BETWEEN POINTS

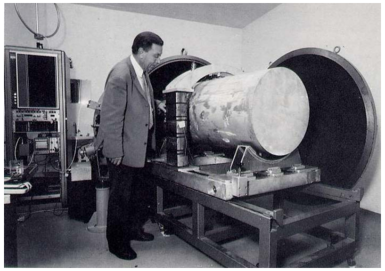
$$\frac{\delta(c \Delta t)}{\Delta x_2} = \frac{h}{2} \sin(kx_1 - \omega t)$$

Time Dependent **Strain**

$$\frac{\Delta l}{l} = \frac{h}{2}$$

The Measurable Quantity

Initial interferometric GW detector groups late 1970's



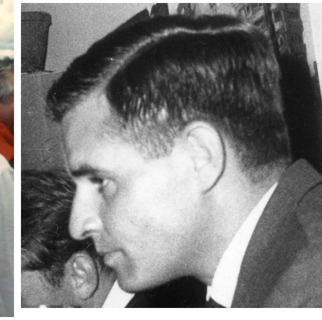
H. Billing

L. Schnupp

K. Maischberger

W.Winkler

R. Schilling



A. Rudiger

Max Planck Garching

Glasgow

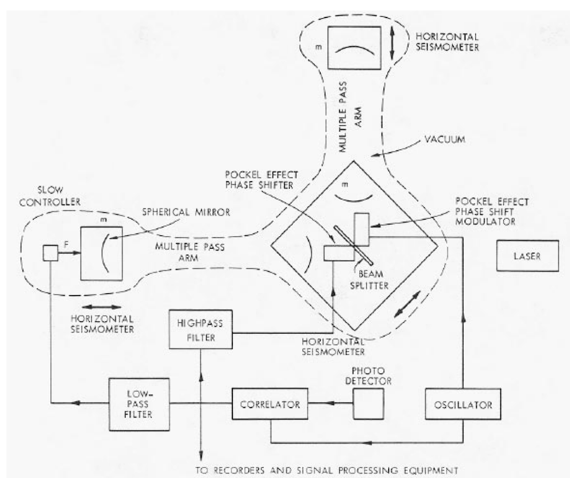
R. Drever

J. Hough

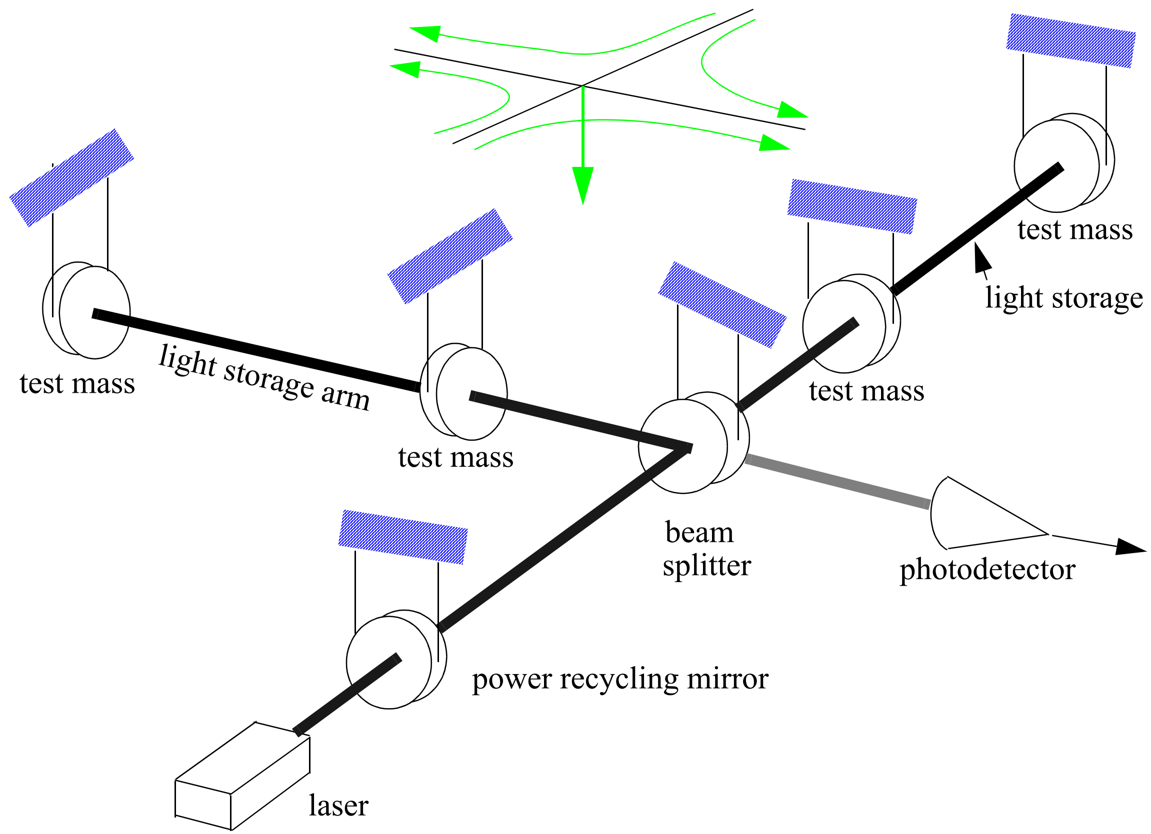
B. Meers

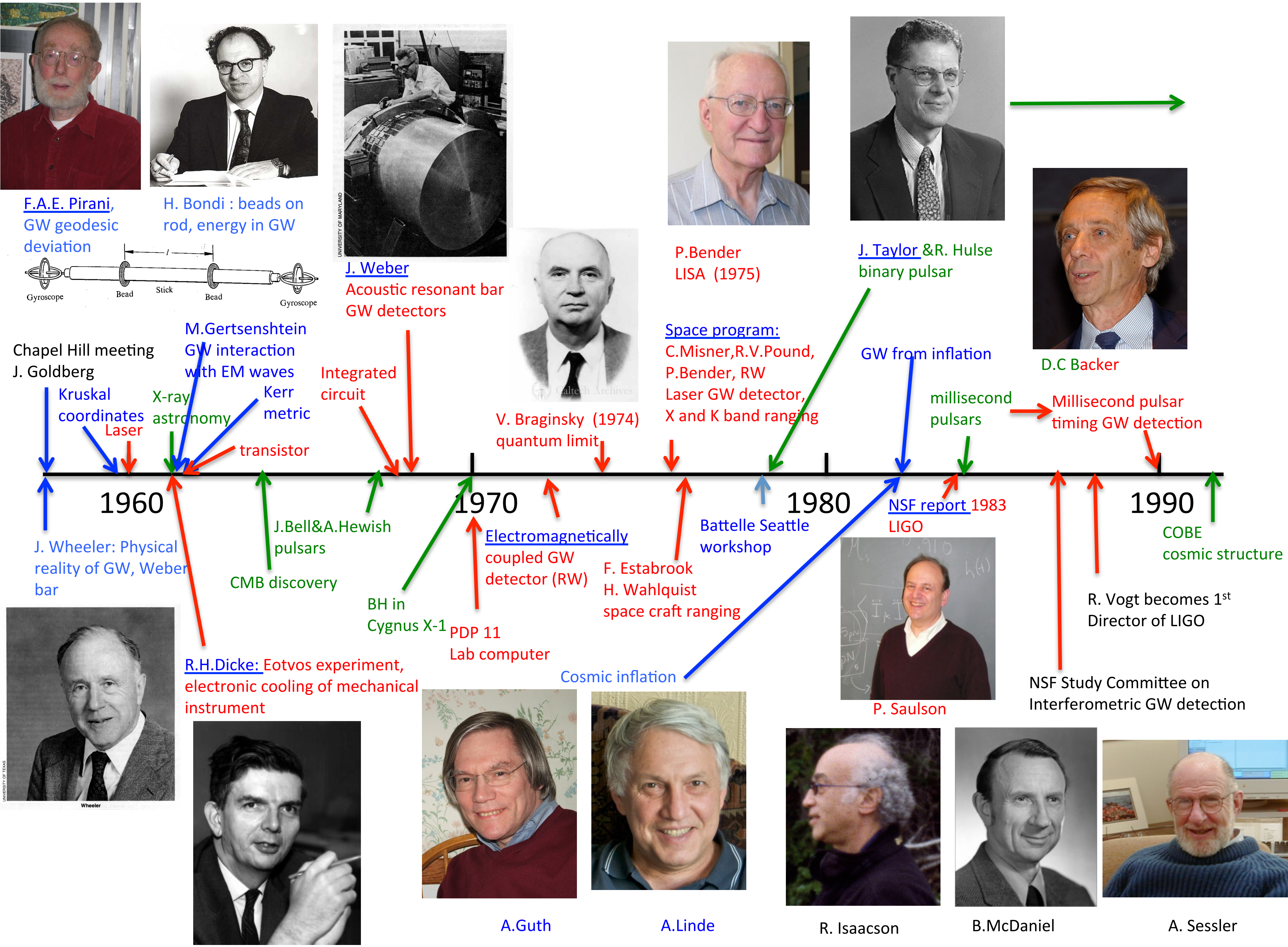
H. Ward

MIT



J. Livas , D.H. Shoemaker, D. Dewey


test mass
light storage arm
test mass
test mass
test mass
light storage arm
beam
splitter
photodetector
power recycling mirror
laser



F.A.E. Pirani, GW geodesic deviation
Gyroscope
Bead
Stick
Bead
Gyroscope
H. Bondi : beads on rod, energy in GW
UNIVERSITY OF MARYLAND
J. Weber
Acoustic resonant bar GW detectors
Caltech Archives
P.Bender
LISA (1975)
J. Taylor &R. Hulse
binary pulsar
D.C Backer
Chapel Hill meeting
J. Goldberg
Kruskal coordinates
Laser
M.Gertsenshtein
GW interaction with EM waves
X-ray astronomy
Kerr metric
transistor
Integrated circuit
V. Braginsky (1974)
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Space program:
C.Misner,R.V.Pound,
P.Bender, RW
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GW from inflation
millisecond pulsars
Millisecond pulsar timing GW detection
1960
1970
1980
1990
J. Wheeler: Physical reality of GW, Weber bar
CMB discovery
J.Bell&A.Hewish pulsars
BH in Cygnus X-1
Electromagnetically coupled GW detector (RW)
PDP 11
Lab computer
F. Estabrook
H. Wahlquist
space craft ranging
Battelle Seattle workshop
NSF report 1983
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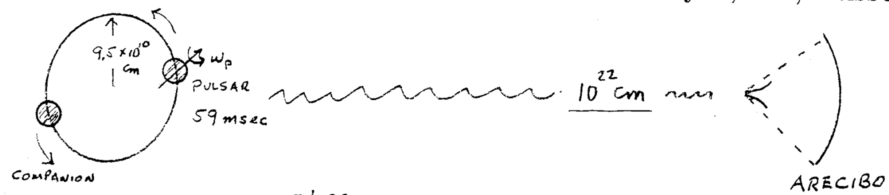
COBE
cosmic structure
R. Vogt becomes 1st Director of LIGO
Wheeler
UNIVERSITY OF TEXAS
R.H.Dicke: Eotvos experiment, electronic cooling of mechanical instrument
Cosmic inflation
P. Saulson
NSF Study Committee on Interferometric GW detection
A.Guth
A.Linde
R. Isaacson
B.McDaniel
A. Sessler

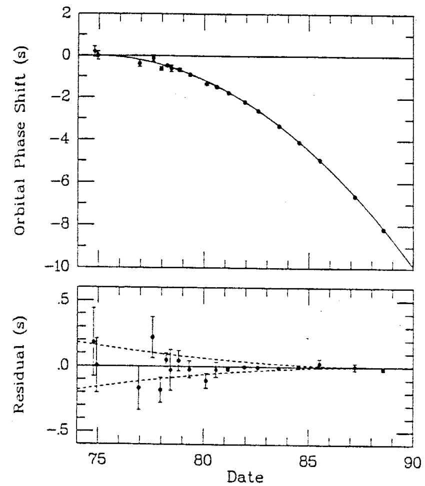
The Binary Pulsar PSR 1913 + 16

Taylor, J.H., Weisberg, J.M.



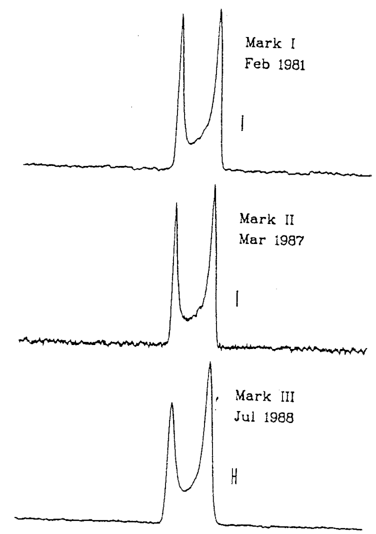
ORBITAL PERIOD ~ 7.75 hrs

$e \sim 0.6$ $\quad \frac{V_{ORB}}{c} \sim 10^{-3}$ $\quad \frac{GM}{2ac^2} \sim 10^{-6}$

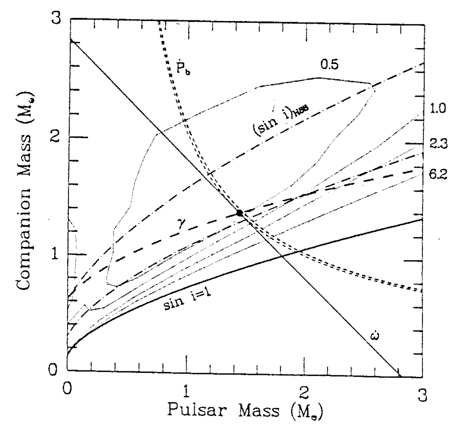


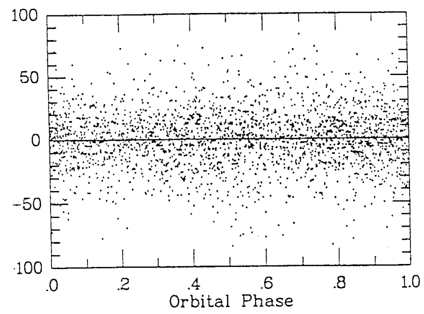
Energy loss to gravitational radiation

$$P = G(\dddot{Q})^2/45c^5$$

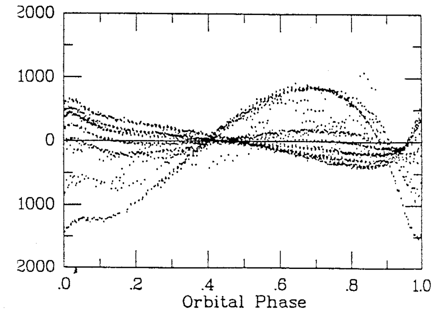


Detected Pulsar pulses



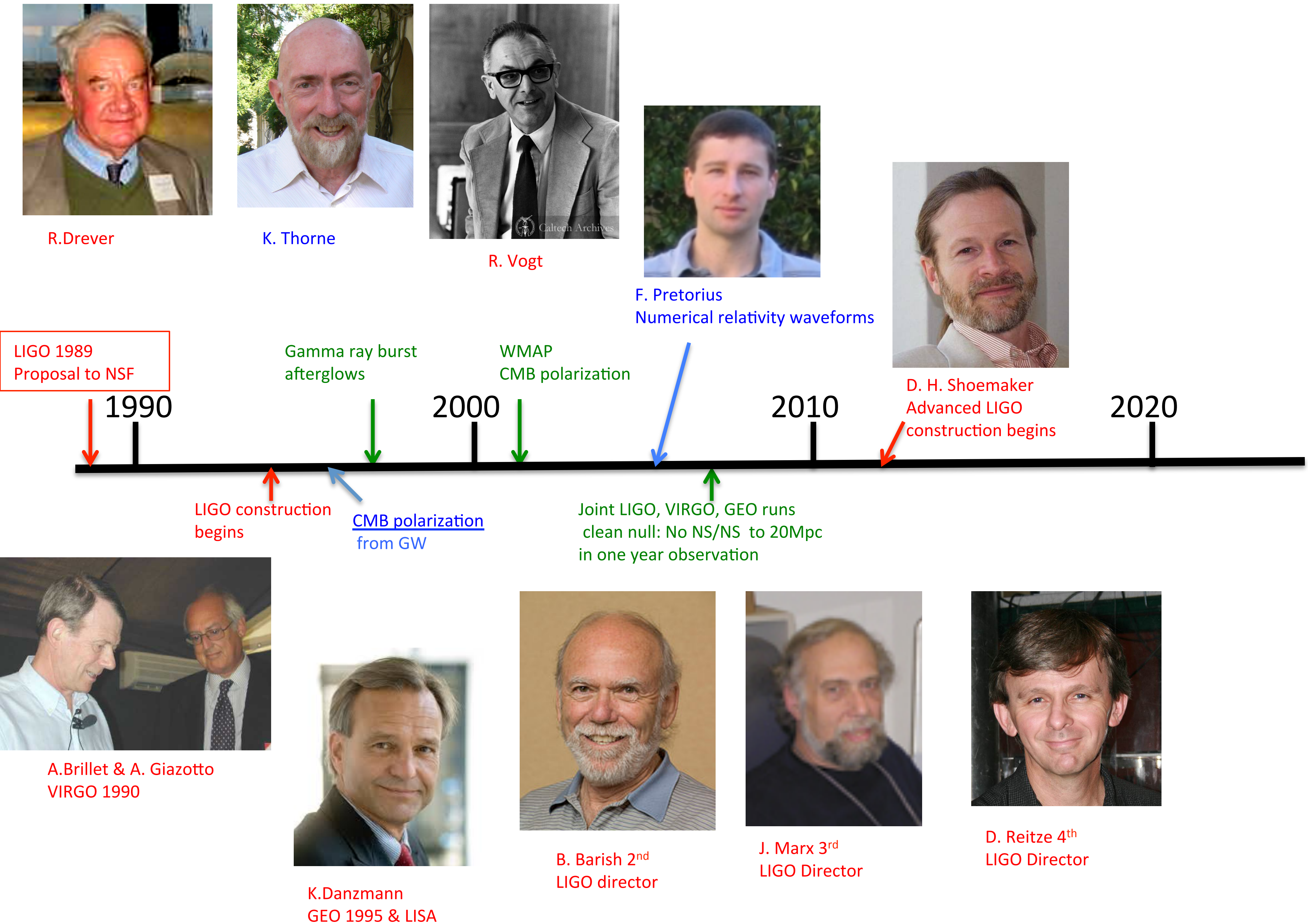


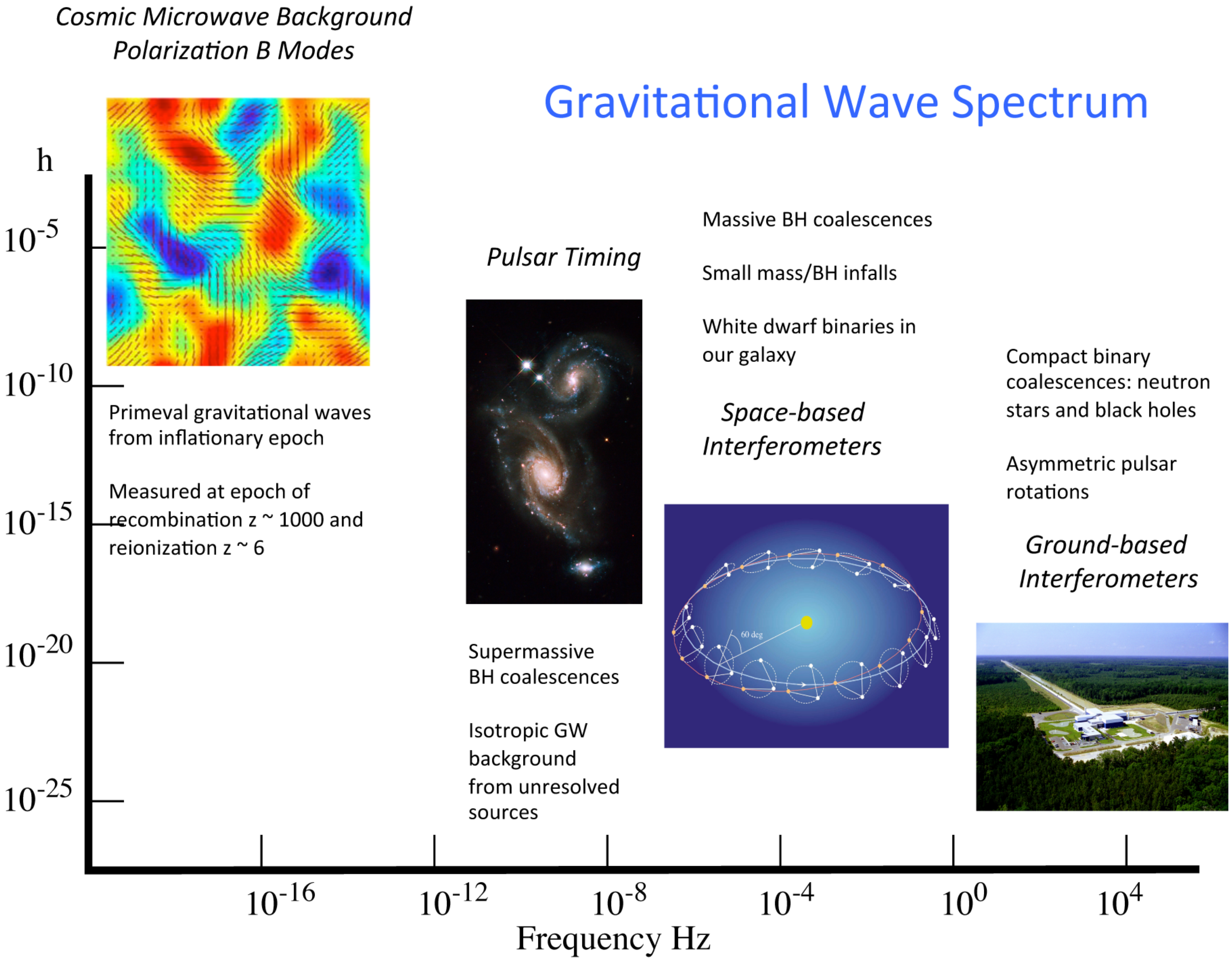
Post fit residuals



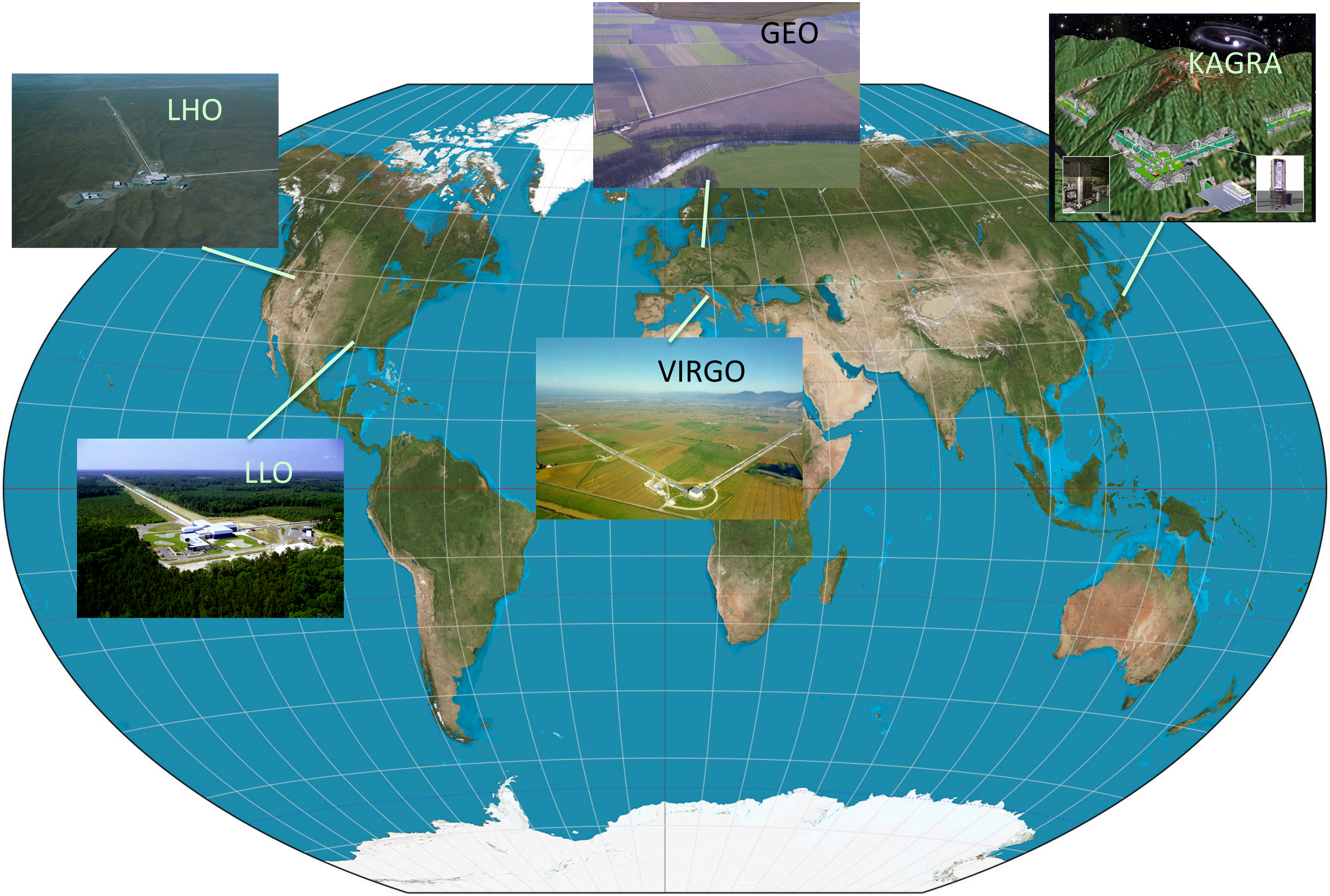
Post fit residuals if $\gamma \rightarrow 0$

Taylor, J.H., Weisberg, J.M.


R.Drever
K. Thorne
R. Vogt
F. Pretorius
Numerical relativity waveforms
LIGO 1989
Proposal to NSF
Gamma ray burst
afterglows
WMAP
CMB polarization
D. H. Shoemaker
Advanced LIGO
construction begins
1990
2000
2010
2020
LIGO construction
begins
CMB polarization
from GW
Joint LIGO, VIRGO, GEO runs
clean null: No NS/NS to 20Mpc
in one year observation
A.Brillet & A. Giazotto
VIRGO 1990
K.Danzmann
GEO 1995 & LISA
B. Barish 2nd
LIGO director
J. Marx 3rd
LIGO Director
D. Reitze 4th
LIGO Director

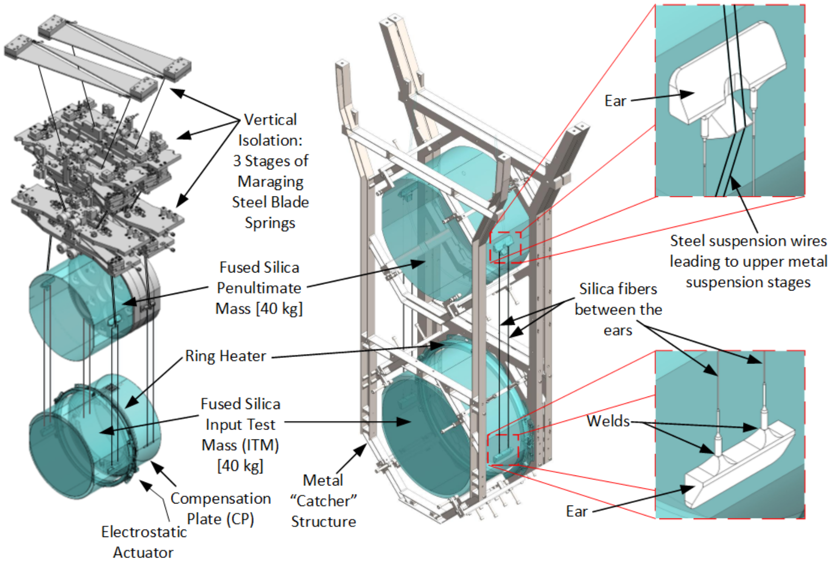


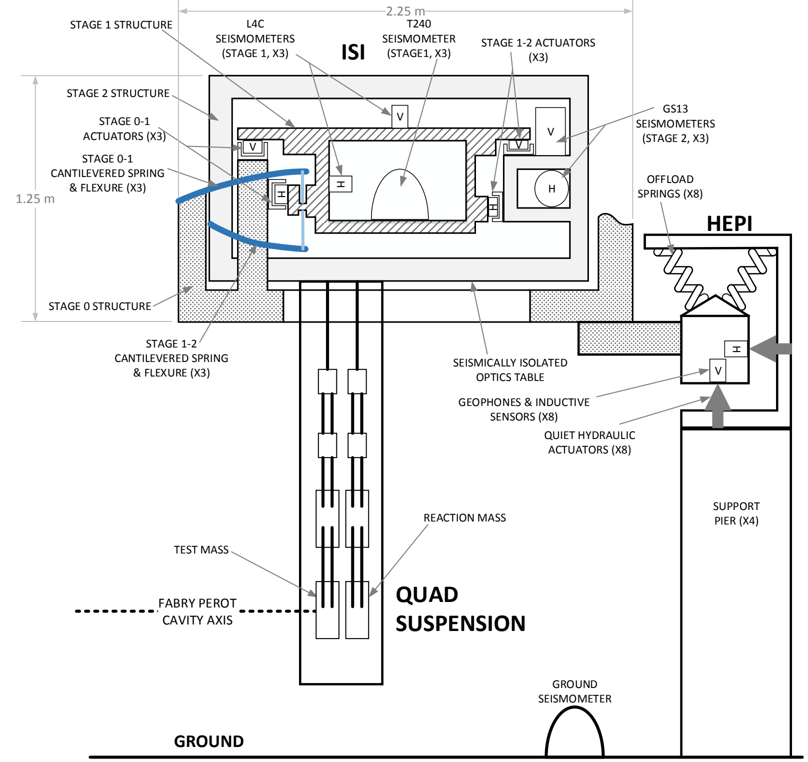
Gravitational Wave Spectrum
Cosmic Microwave Background
Polarization B Modes
h
10^-5
10^-10
10^-15
10^-20
10^-25
Primeval gravitational waves
from inflationary epoch
Measured at epoch of
recombination z ~ 1000 and
reionization z ~ 6
Pulsar Timing
Supermassive
BH coalescences
Isotropic GW
background
from unresolved
sources
Massive BH coalescences
Small mass/BH infalls
White dwarf binaries in
our galaxy
Space-based
Interferometers
60 deg
Compact binary
coalescences: neutron
stars and black holes
Asymmetric pulsar
rotations
Ground-based
Interferometers
10^-16
10^-12
10^-8
10^-4
10^0
10^4
Frequency Hz


GEO
KAGRA
LHO
VIRGO
LLO

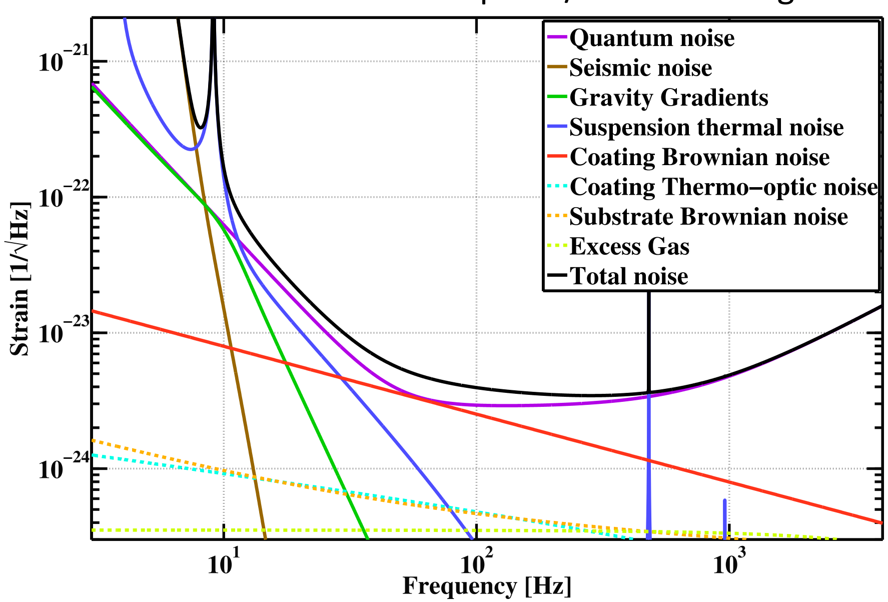
Classes of sources and searches

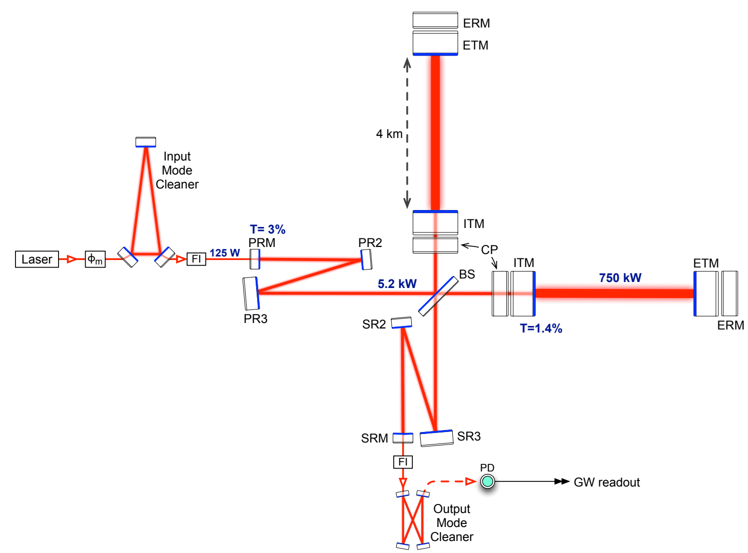
- Compact binary inspiral: template search
  - BH/BH
  - NS/NS and BH/NS
- Low duty cycle transients: wavelets,T/f clusters
  - Supernova
  - BH normal modes
  - Unknown types of sources
- Triggered searches
  - Gamma ray bursts
  - EM transients
- Periodic CW sources
  - Pulsars
  - Low mass x-ray binaries (quasi periodic)
- Stochastic background
  - Cosmological isotropic background
  - Foreground sources : gravitational wave radiometry

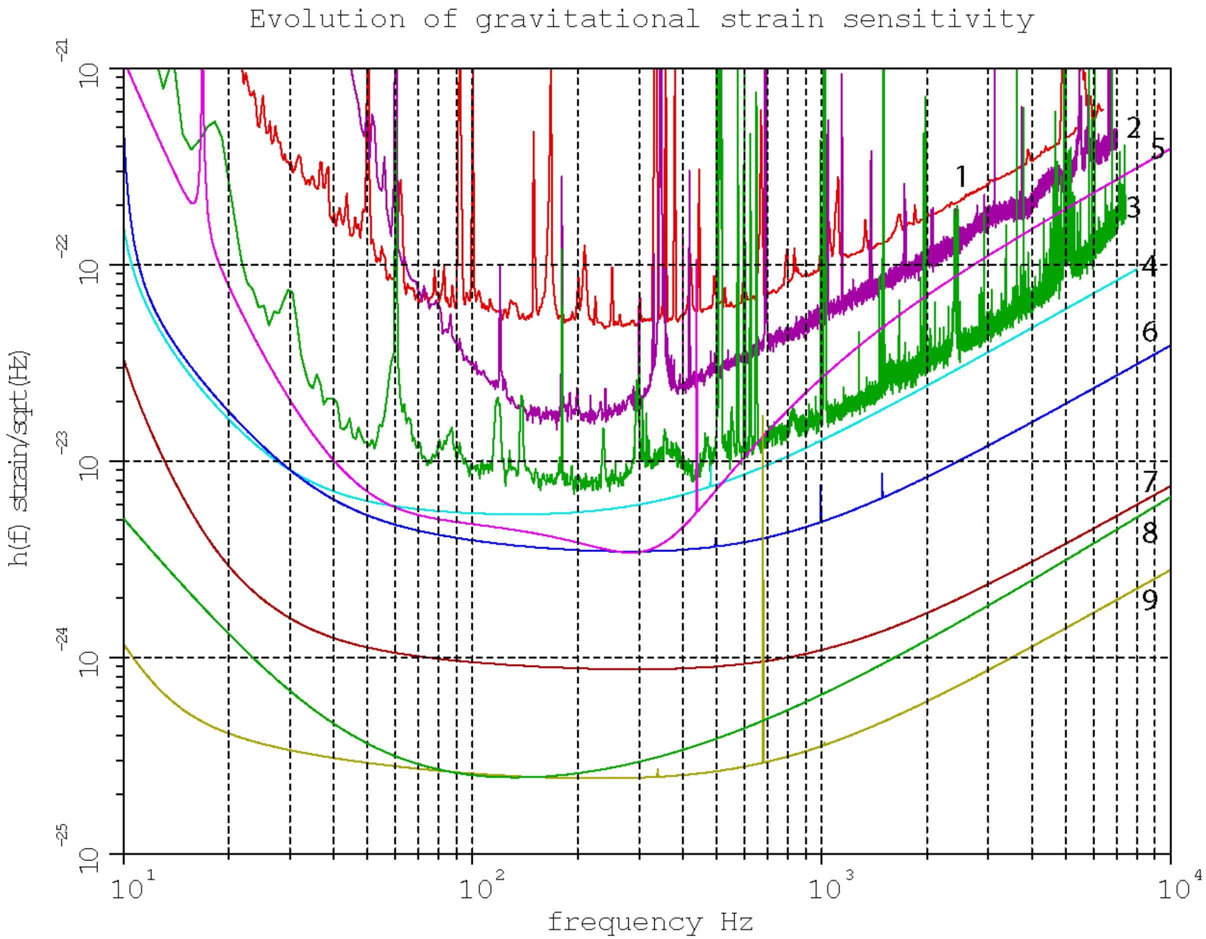




Advanced LIGO 190Mpc NS/NS noise budget



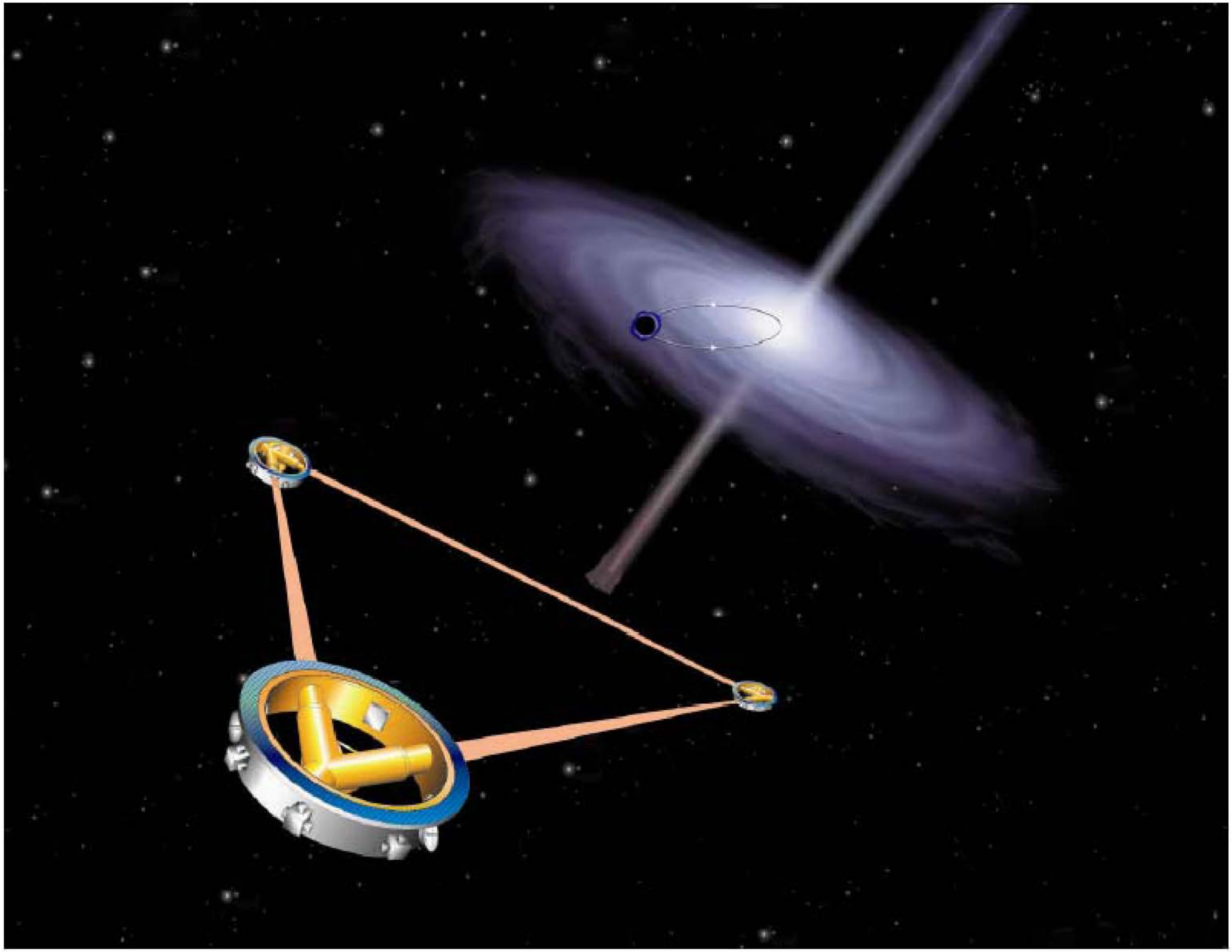




1 VIRGO 2009
2 Enhanced LIGO 2009
3 Advanced LIGO 65Mpc NS/NS 2015
4 Advanced LIGO 150Mpc NS/NS Low Power
5 Advanced VIRGO
6 Advanced LIGO 190Mpc NS/NS High Power
7 4km "Voyager" example 600Mpc NS/NS
8 Einstein telescope B
9 40km "Cosmic Explorer" example

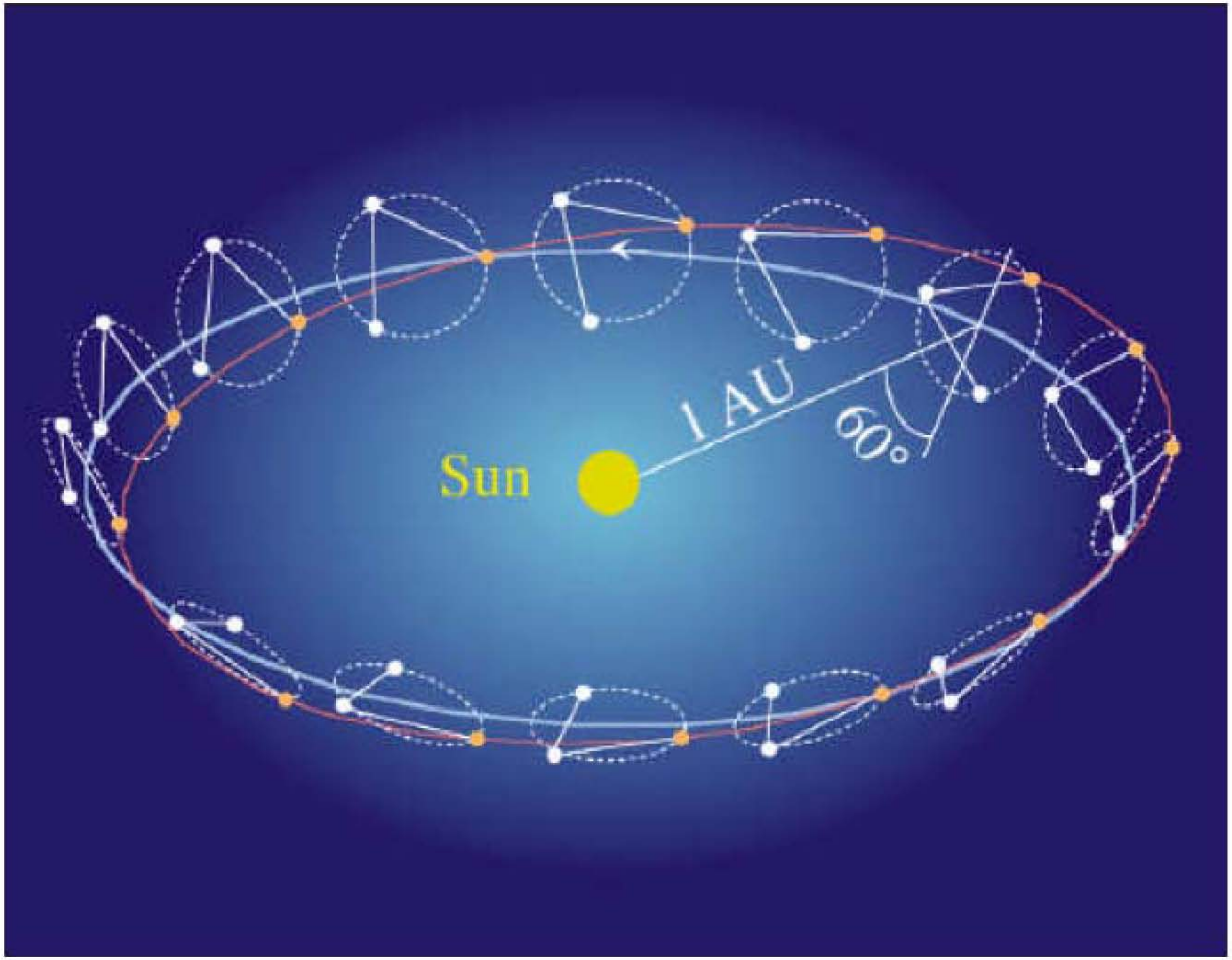
Epoch	Estimated Run Duration	$E_{GW} = 10^{-2} M_{\odot} c^2$ Burst Range (Mpc) LIGO	Virgo	BNS Range (Mpc) LIGO	Virgo	Number of BNS Detections	% BNS Localized within 5 $\deg^2$	20 $\deg^2$
2015	3 months	40 – 60	–	40 – 80	–	0.0004 – 3	–	–
2016–17	6 months	60 – 75	20 – 40	80 – 120	20 – 60	0.006 – 20	2	5 – 12
2017–18	9 months	75 – 90	40 – 50	120 – 170	60 – 85	0.04 – 100	1 – 2	10 – 12
2019+	(per year)	105	40 – 80	200	65 – 130	0.2 – 200	3 – 8	8 – 28
2022+ (India)	(per year)	105	80	200	130	0.4 – 400	17	48

Mission Concept

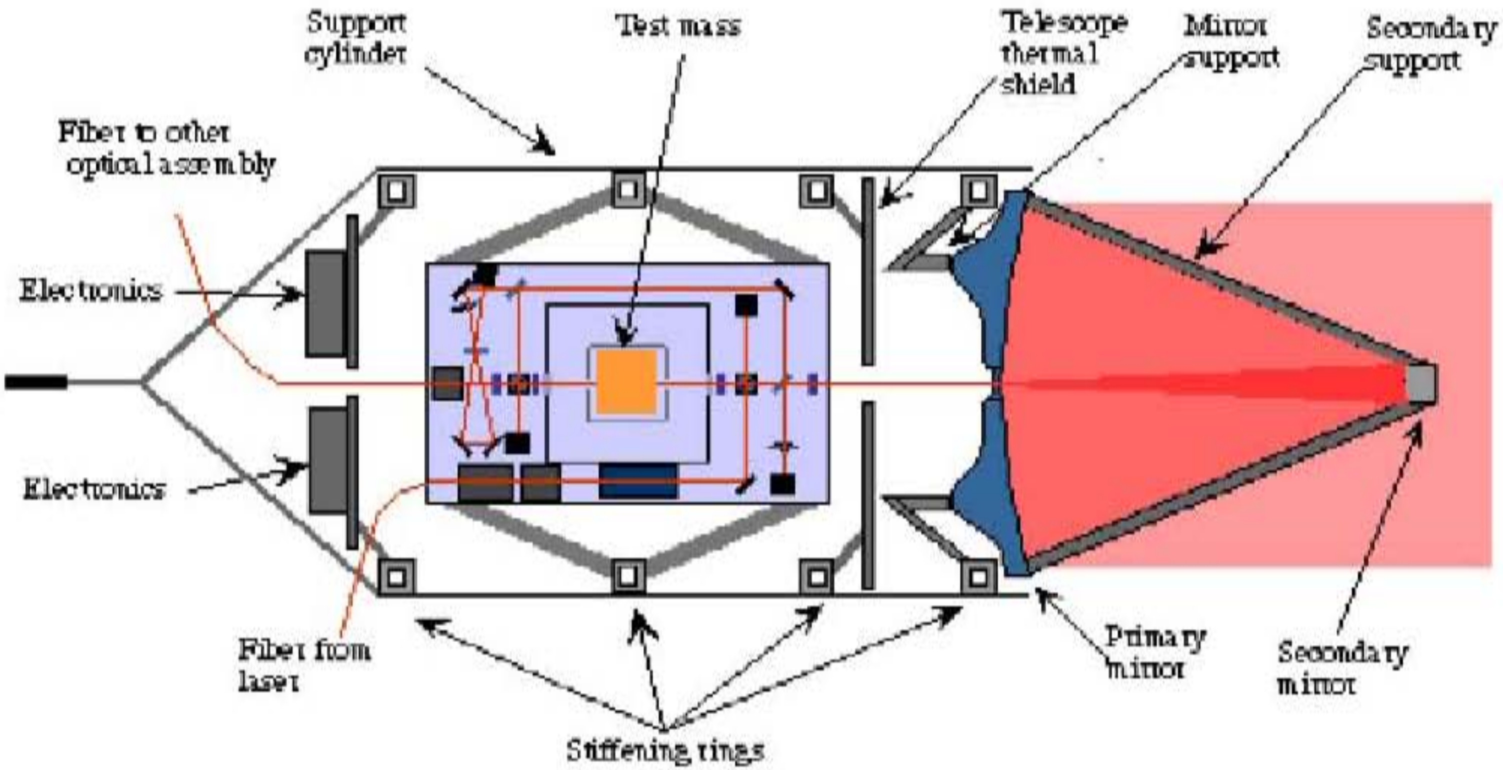


Spacecraft Orbits

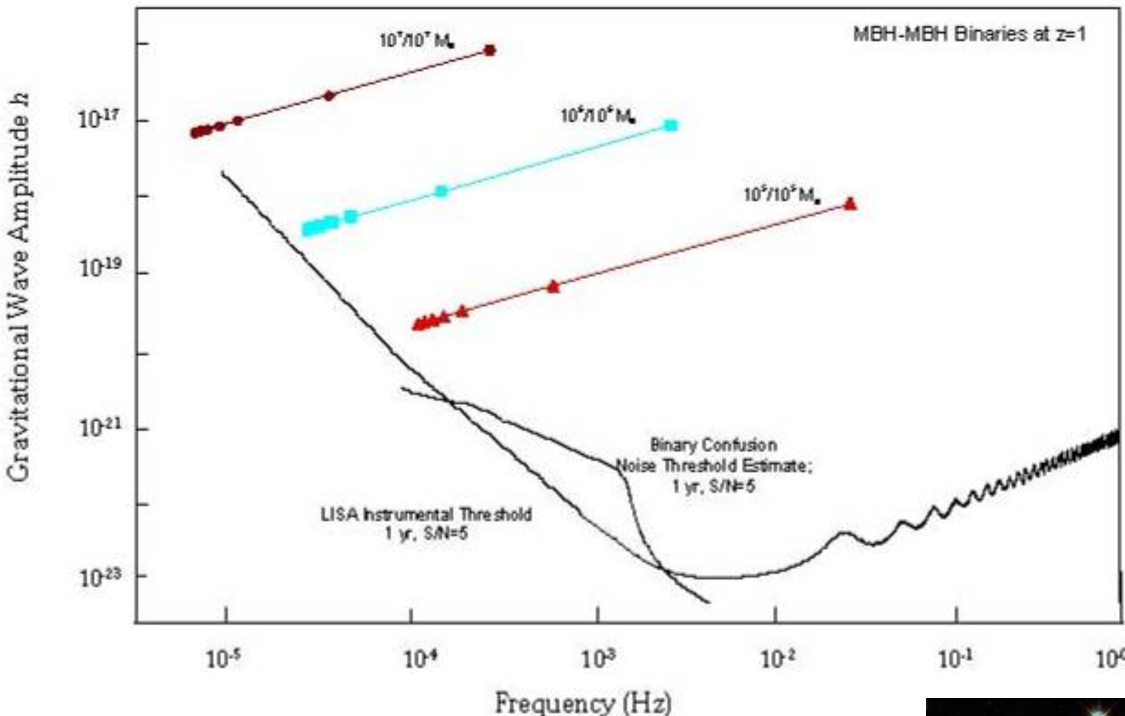
- Spacecraft orbits evolve under gravitational forces only
- Spacecraft fly "drag-free" to shield proof masses from non-gravitational forces

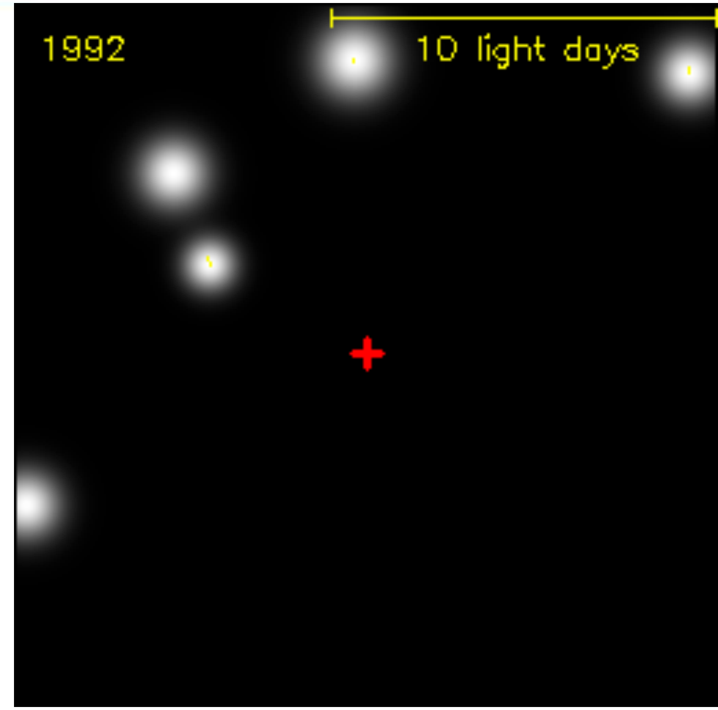


Optical System



Massive Black Holes in Merging Galaxies

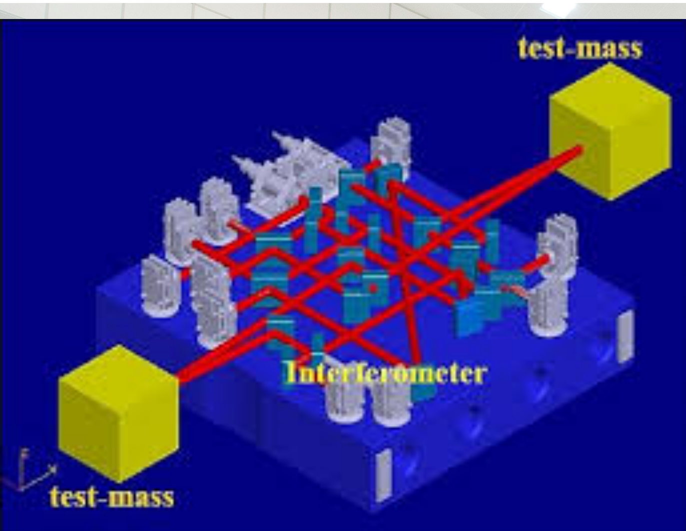


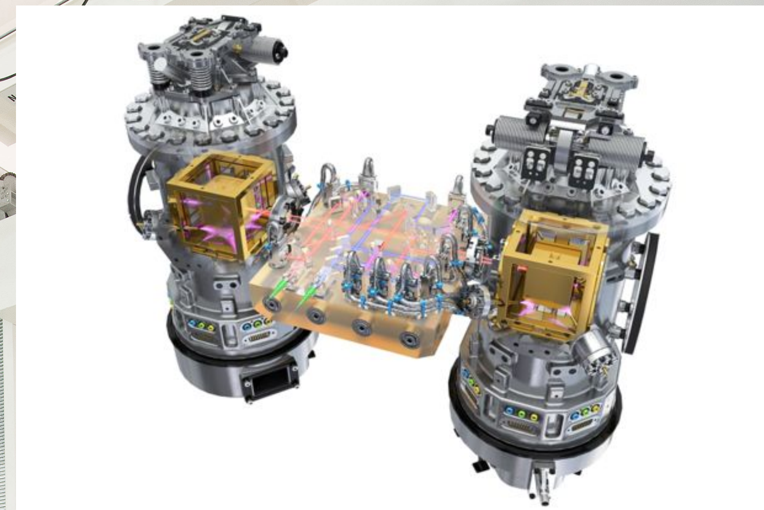


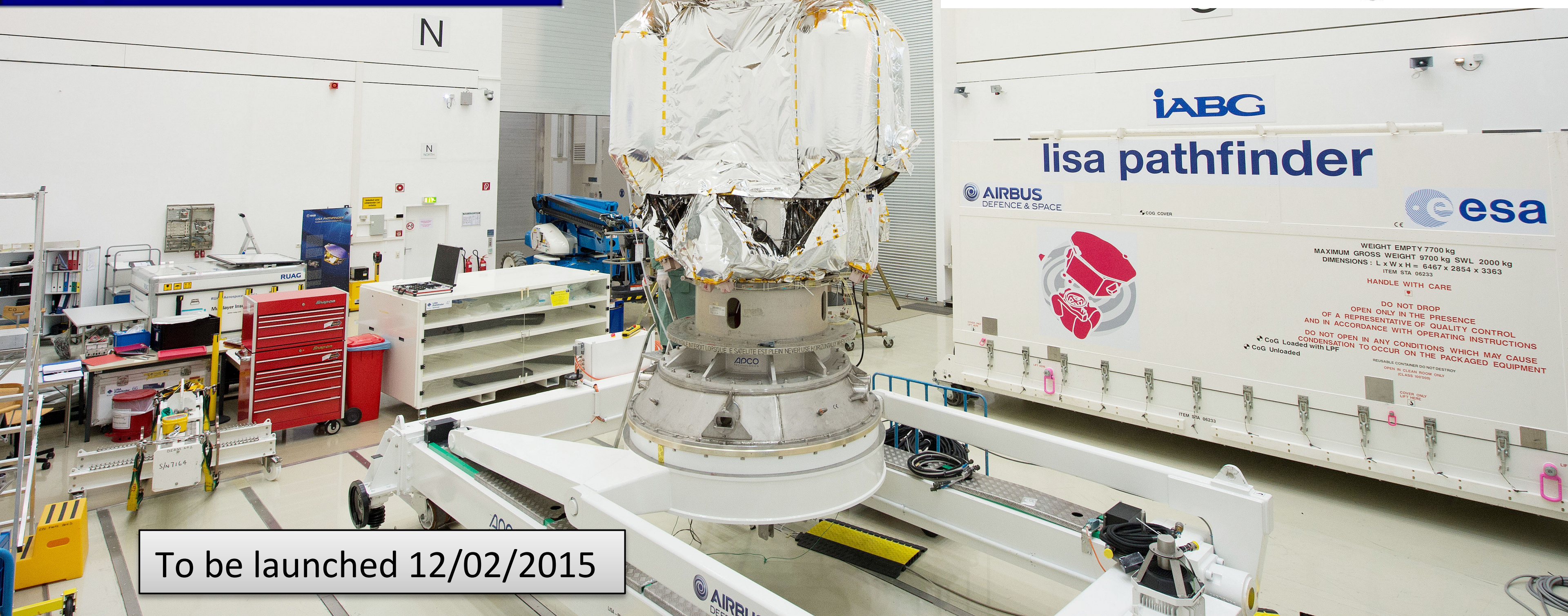
R. Genzel

Hubble Space Telescope

LISA Pathfinder







To be launched 12/02/2015

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LIGO Scientific Collaboration

LIGO

LSC

Trinity University

University of Maryland 1856

Andrews University

Washington State University

California State University Fullerton

indigo

ANU The Australian National University

Montana State University

American University Washington, DC

University of the West of Scotland UWS

Montclair State University

R·I·T

ICTP SAIFR

Tsinghua University

Michigan

University of Strathclyde

University of Glasgow

CITA–ICAT

Whitman College

LSU

NASA Goddard Space Flight Center

UTB TSC

University of Cambridge

The University of Western Australia

The University of Mississippi

The University of Adelaide Australia

The University of Melbourne

Southern University Agricultural & Mechanical College

University of Minnesota

Embry-Riddle Aeronautical University

University of Washington

GEO 600

Universitat de les Illes Balears

Georgia Tech

Cardiff University

Charles Sturt University

UF University of Florida

UW Milwaukee University of Wisconsin

ACIGA

University of Southampton

Korean Gravitational-Wave Group

University of Rochester

Leibniz Universität Hannover

Penn State 1855

William Smith College

Science & Technology Facilities Council Rutherford Appleton Laboratory