to: Detector commissioning group
from: R. Weiss January 26, 2000
concerning: Estimate for scattering by large optics

The scattering measured from the ITMY reported by Mike Zucker is approximately 10^{-5} /steradian at a scattering angle of 30 milliradians. This note is intended to show that the scattering is not far from what should be expected given the surface characterizations of the large optics substrates. At 30 milliradians the scattering will be dominated by surface perturbations with spatial frequency around 300 cm^{-1} . In this spectral region the spatial spectrum has been determined by Topo scans rather than from the phase front interferometry. The Topo spectra from the CSIRO polished substrates have a $\frac{1}{\nu}$ dependence with a value of typically 10^{-10} waves(6328A)^2 /cm^{-1} . (The GO polished surfaces will have power spectra about 1/10 of this value at these spatial frequencies.)

A reasonable model for the spatial power spectrum of the large optics is the fractal model developed by Church, Takacs and Leonard (SPIE Vol 1165 (1989)). The one dimensional spatial power spectrum is given by

$$S_1(\upsilon) = \frac{A}{(1+(2\pi\upsilon L_{cor})^2)^{\frac{c}{2}}}$$

where ν is the spatial frequency in cm^{-1} , L_{cor} is a correlation length of the surface perturbations in cm, and A is the spatial spectral power density in optical waves^2 / cm^{-1} . c is an experimentally determined power; for the large mirrors c is close to 1, leading to a one dimensional power spectrum that varies as $\frac{1}{\nu}$.

The scattering per solid angle into direction θ due to the surface irregularities associated with this spectrum is given by

$$\frac{dP_{scat}(\theta)}{d\Omega P_{inc}} = \frac{16\pi^{\frac{5}{2}}\Gamma\left(\frac{(c+1)}{2}\right)}{\lambda^2\Gamma\left(\frac{c}{2}\right)} \frac{L_{cor}A}{\left(1+\left(\frac{2\pi\theta L_{cor}}{\lambda}\right)^2\right)^{\frac{(c+1)}{2}}}$$

λ is the optical wavelength in cm and θ is the scattering angle from the specular. The relation is valid for small angle scattering away from the normal of the mirror where the approximate grating equation $\theta = \lambda\upsilon$ applies.

Representative values for the power spectrum parameters are given in the table.

Table 1: Representative parameters for the large optics

λ	1.06 x 10^{-4} cm
c	1
L_{cor}	1 cm
$A(\lambda)$	2 x 10^{-8} $waves^2$ /cm^{-1}

At such large scattering angles as 30 milliradians and correspondingly large spatial frequencies υ = 300 cm^{-1} , the relations between the power spectrum parameters and scattering simplify and are given by

$$\frac{dP_{scat}(\theta)}{d\Omega P_{inc}} = \frac{4A}{\theta^2 L_{cor}} = \frac{8\times10^{-8}}{\theta^2}$$

The scattering estimated at the measurement angle of 30 mrad would be approximately 9 x 10^{-5} / steradian. Note that the scattering is about 1/10 of that we used in the early estimates for beam tube vibration phase noise. The spectrum when integrated gives a 2 Angstrom rms surface deviation . The reduced scattering has come about because of the change in wavelength to 1 micron as well as the excellent polishing done by CSIRO.