SURFACE SPECIFICATION FOR THE LIGO ARM CAVITY MIRRORS

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Introduction : The document presents the surface specifications for the initial LIGO interferometer arm cavity mirrors. The specifications are divided broadly into three spatial scales.

20 - 0.3 cm : The spatial scale (*large*) that primarily determines the cavity field distribution and thereby the interferometer contrast and one component of the diffractive cavity losses. Perturbations on scales 2 cm and shorter have diffracted components that fall outside the mirror radius in the arm cavities.

0.3 - 0.008 cm : The spatial scale (*mid*) that primarily contributes to the scattered light in the LIGO beam tubes producing phase noise through modulation by interaction with the walls and baffles. The mirror perturbation power spectra on these spatial scales are likely to be the same for all the mirrors so that the primary effect in the arm cavities is expected to be cavity loss rather than interferometer contrast defect.

$\leq$ **.008 cm:** The spatial scale (*small*) which in the LIGO contributes primarily diffractive arm cavity loss and to the interferometer contrast defect if the power spectrum on these spatial scales is different for the mirrors in the two arm cavities.

Some of the specifications are inconsistent with each other since they have been arrived at from different considerations. The specification is then determined by the more rigorous condition. A specific example are the allowed higher order Zernike amplitudes, these are larger than the values specified by the surface power spectrum. The inconsistency comes about because different performance criteria have been used. In the case of the Zernike decompositions, the interferometer contrast defect is the driver, while for the power spectrum specification, it is the scattering.

NOTE: A separate issue, not considered in this specification, is the effect from mid and small scale perturbations on the small beam tests that may be performed in cavity ring down or laboratory absorption measurements.

OVERALL LIGO ARM CAVITY PARAMETERS

Optical wavelength: $\lambda \;=\; 5.145 \times 10^{-5}$ cm

Cavity parameters:

Arm cavity length: $L_{\rm arm} \;=\; 4 \times 10^5$ cm

Recycling cavity length: $L_{\rm recyl} \;=\; 1.2 \times 10^3$ cm

Radius of arm cavity front mirror: $R_{\rm front\ arm} \;=\; R_1 \;=\; \infty$ (flat)

Radius of arm cavity back mirror: $R_{\rm back\ arm} \;=\; R_2 \;=\; 6 \times 10^5$ cm

Arm cavity g factor 1: $g_1 \;=\; 1.0$

Arm Cavity g factor 2: $g_2 \;=\; 0.333$

Gaussian spot radius at front: $\omega_0 \;=\; \omega_1 \;=\; 2.15$ cm

Gaussian spot radius at back: $\omega_2 \;=\; 3.73$ cm

Rayleigh range: $z_{\rm r} \;=\; 2.83 \times 10^5$ cm

Radius of recycling mirror: $R_{\rm rcycl} \;=\; 6.64 \times 10^7$ cm (flat)

Recycling cavity g factor: $g \;=\; 1 \;-\; 1.8 \times 10^{-5}$ (unstable cavity)

Optical properties (scattering and losses):

Scattering and absorption loss of surfaces: $A \;\leq\; 1.0 \times 10^{-4}$

BRDF of surfaces: $\frac{dP_{\rm scat}}{d\Omega * P_{\rm inc}} \;\leq\; \frac{1 \times 10^{-6}}{\theta^2}$ sr^{-1}, $\theta \;\leq\; 6 \times 10^{-3}$ radians

Loss coefficient of bulk material: $\alpha \;\leq\; 3 \times 10^{-6}$ cm^{-1}

Contrast defect at antisymmetric port: $1 \;-\; C \;\leq\; 3 \times 10^{-3}$

Approximate rms surface error : $\frac{\sigma_{\rm rms}}{\lambda} \;\leq\; \frac{1}{400}$

Optical Properties (reflectivity and transmission):

Reflectivity of recycling mirror: $R_{\rm recyl} \;=\; 0.96 \;-\; A$

Reflectivity of front arm cavity mirror: $R_{\rm front\ arm} \;=\; 0.97 \;-\; A$

Reflectivity of back arm cavity mirror: $R_{\rm back\ arm} \;=\; 1.0 \;-\; A$

Reflectivity of beam splitter: $R_{\rm beam\ split} \;=\; 0.5 \;-\; A/2$

Transmission of beam splitter: $T_{\rm beam\ split} \;=\; 0.5 \;-\; A/2$

Optics dimensions:

Arm cavity mirror diameter: $D \;=\; 25$ cm

Arm cavity mirror thickness: $t \;=\; 10$ cm

SURFACE SPECIFICATIONS

The specifications are presented several ways:

1. Sums of the squares of the amplitudes of ortho-normalized Zernike functions over an aperture radius of 10 cm.

2. Sums of the squares of the amplitudes of ortho-normalized Zernike functions over a subaperture radius of 5 cm.

3. Sums of the squares of the amplitudes of the ortho-normalized Laguerre - Gauss functions weighted by the lowest order Laguerre - Gauss function (radial and angular index = 0).

4. The 1 dimensional surface power spectrum in units of waves $(5145\text{A})^2\text{cm}$.

5. The 2 dimensional surface power spectrum in units of waves $(5145\text{A})^2\text{cm}^2$.

DEFINITION OF TERMS

Zernike decomposition

The Zernike functions are area normalized and real - use sin and cos as the angular functions.

$$Z_{n,l,+}(r,\theta) = N_{n,l}\ R_{n,l}(r)\ \cos(l\theta)$$
$$Z_{n,l,-}(r,\theta) = N_{n,l}\ R_{n,l}(r)\ \sin(l\theta)$$

The $R_{n,l}$ are the radial Zernike functions.

The Zernike functions are ortho-normal

$$\int_0^R \int_0^{2\pi} Z_{n,l,\pm}(r,\theta)\ Z_{j,q,\pm}(r,\theta)\ rdrd\theta = \delta_{n,j}\delta_{l,q}$$

where R is the aperture radius. The normalization constant is chosen as

$$N_{n,0} = \sqrt{\frac{n+1}{\pi}}$$
$$N_{n,l} = \sqrt{\frac{2(n+1)}{\pi}}$$

The surface, $z(r,\theta)$, is decomposed

$$\frac{z(r,\theta)}{\lambda} = \sum_{0,0}^{n,l,\pm} a_{n,l,\pm}\ Z_{n,l,\pm}(r,\theta)$$

where the amplitude coefficients are defined as

$$a_{n,l,\pm} = \int_0^R \int_0^{2\pi} \frac{z(r,\theta)}{\lambda} Z_{n,l,\pm}(r,\theta) rdrd\theta$$

NOTE: The Zernike functions used in ZYGO interferometer software are not normalized. The functions all have $N_{n,l} = 1$ and are given by a numbering scheme from 1 to 36 that includes the real functions from $n = 0,\ l = 0$ to $n = 7,\ l = 7$.

Surface Power Spectra

The power spectra are parametrized by the prescription given in Church, Takacs and Leonard (SPIE Vol 1165 (1989)) for isotropic fractal surfaces. The one dimensional power spectrum, determined from data taken along a profilometer scan or a line in an interferometric phase map, is represented by

$$S_1(f_x) = \frac{A}{(1 + (2\pi f_x l_{\rm cor})^2)^{c/2}}$$

The one dimensional power spectrum, S_1, and the coefficient A are expressed in units of (waves (5145A))^2 cm. $l_{\rm cor}$ is the surface correlation length in cm. $f(x)$ is the spatial frequency on the surface in cm^{-1} also referred to as wavenumbers. The representation of real surfaces will require different spectral models for the large, mid and small spatial scales.

The isotropic two dimensional power spectrum associated with S_1 is given by

$$S_2(f) \;=\; \frac{\Gamma((c+1)/2)}{\Gamma(c/2)} \frac{\sqrt{\pi} l_{\rm cor} A}{(1 \;+\; (2\pi f l_{\rm cor})^2)^{(c+1)/2}}$$

f is the isotropic spatial frequency $f = \sqrt{f_x^2 + f_y^2}$. $S_2(f)$ is expressed in units of (waves (5145A))^2 cm^2

The mirror BRDF depends on the two dimensional power spectrum and optical wavelength

$$BRDF \;=\; \frac{dP_{\rm scat}}{d\Omega * P_{\rm inc}} \;=\; \frac{16\pi^2}{\lambda^2}\; S_2(f)$$

The grating relations couple the scattering angle and surface spatial frequency. At angles where $\theta \approx \sin(\theta)$, the spatial frequency, optical wavelength and the scattering angle are related as

$$\theta \;\approx \lambda f$$

so that the BRDF can be expresed in terms of the scattering angle (incident beam assumed at normal incidence to the surface) by

$$BRDF \;=\; \frac{dP_{\rm scat}(\theta)}{d\Omega * P_{\rm inc}} \;=\; \frac{16\pi^{5/2}}{\lambda^2} \frac{\Gamma((c+1)/2)}{\Gamma(c/2)} \frac{l_{\rm cor} A}{(1 \;+\; (2\pi\theta l_{\rm cor}/\lambda)^2)^{(c+1)/2}}$$

The one and two dimensional power spectra are designed to give the same surface variance in waves^2

$$\frac{\sigma^2}{\lambda^2} \;=\; \int_0^\infty S_1(f_x) df_x \;=\; 2\pi \int_0^\infty S_2(f) f df \;=\; \frac{A}{2\sqrt{\pi}(c-1) l_{\rm cor}} \frac{\Gamma((c+1)/2)}{\Gamma(c/2)}$$

The total scattering loss by the surface is related to the surface variance

$$\frac{P_{\rm total\ scat}}{P_{\rm inc}} \;=\; 16\pi^2 \frac{\sigma^2}{\lambda^2}$$

NOTE: The one dimensional power spectra in some commercial software is given in units of microns^3 and the spatial frequencies are given in microns^{-1}. The two dimensional power spectra are given in units of microns^4. The conversion of the power spectra used in these specifications to those using microns as the basis are the following:

$$S_1(\mu^{-1}) \;=\; 2.65 \times 10^3 \; S_1({\rm cm}^{-1})$$

$$S_2(\mu^{-1}) \;=\; 2.65 \times 10^7 \; S_2(\mathrm{cm}^{-1})$$

Gaussian Weighted Laguerre - Gauss Decomposition

A direct but unconventional specification for resonant cavity performance is the decomposition of the mirror surface into weighted Laguerre - Gauss functions. These quantities provide a measure of the amount of excitation into higher order cavity modes by the mirror surface irregularities when illuminated by the lowest order cavity mode (assumed to be at a waist on the surface of the mirror). The real Laguerre - Gauss functions are

$$LG_{p,m,+}(r,\theta) \;=\; \frac{M_{p,m}}{w_0}(\frac{\sqrt{2}r}{w_0})^m L_p^m(\frac{2r^2}{w_0^2})e^{-r^2/w_0^2}\cos(m\theta)$$

$$LG_{p,m,-}(r,\theta) \;=\; \frac{M_{p,m}}{w_0}(\frac{\sqrt{2}r}{w_0})^m L_p^m(\frac{2r^2}{w_0^2})e^{-r^2/w_0^2}\sin(m\theta)$$

where w_0 is the Gaussian waist radius and $L_p^m(\frac{2r^2}{w_0})$ are the Laguerre polynomials. The Laguerre-Gauss functions are ortho-normal

$$\int_0^\infty \int_0^{2\pi} LG_{p,m,\pm}(r,\theta)\; LG_{j,q,\pm}(r,\theta)\; rdrd\theta \;=\; \delta_{p,j}\delta_{m,q}$$

when the normalization constant is chosen as

$$M_{p,0} \;=\; (\frac{2p!}{\pi((m+p)!)^3})^{1/2}$$

$$M_{p,m} \;=\; (\frac{4p!}{\pi((m+p)!)^3})^{1/2}$$

The Gaussian weighted decomposition components are defined as

$$b_{p,m,0,0,\pm} \;=\; \int_0^\infty \int_0^{2\pi} LG_{p,m,\pm}(r,\theta)\frac{z(r,\theta)}{\lambda}LG_{0,0}(r,\theta)rdrd\theta$$

The interpretation of $b_{p,m,0,0,\pm}$ requires some care since even a perfect surface of finite radius will give non vanishing values due to the diffraction loss by the finite aperture. With the parameters of the LIGO cavities and mirror diameters, values of $b^2_{p,m,0,0,\pm} \;\leq\; 1\times10^{-7}$ on the spherical mirror and $b^2_{p,m,0,0,\pm} \;\leq\; 2\times10^{-15}$ on the flat mirror for $p \;\leq\; 20$ are limits due to the finite mirror size.

The most difficult surface specification to meet is the large scale error on the spherical back arm cavity mirror where the Gaussian spot size is largest in the proposed LIGO cavity geometry.

THE SURFACE SPECIFICATIONS

Front arm cavity mirror - flat

Zernike sums over a 10 cm radius aperture

$$\sum_{n=8}^{\infty} a_{n,0}^2 \;\leq\; \frac{(1\;-\;C)}{(578)} \;\leq\; 5\times 10^{-6}$$

$$\sum_{n=18}^{\infty} a_{n,2}^2 \;\leq\; \frac{(1\;-\;C)}{(288)} \;\leq\; 1\times 10^{-5}$$

$$\sum_{n=25}^{\infty} a_{n,4}^2 \;\leq\; \frac{(1\;-\;C)}{(162)} \;\leq\; 2\times 10^{-5}$$

$$\sum_{n=25}^{\infty} a_{n,6}^2 \;\leq\; \frac{(1\;-\;C)}{(72)} \;\leq\; 4\times 10^{-5}$$

Zernike sums over a 5 cm radius aperture

$$\sum_{n=4}^{\infty} a_{n,0}^2 \;\leq\; \frac{(1\;-\;C)}{(300)} \;\leq\; 1\times 10^{-5}$$

$$\sum_{n=8}^{\infty} a_{n,2}^2 \;\leq\; \frac{(1\;-\;C)}{(140)} \;\leq\; 2\times 10^{-5}$$

$$\sum_{n=10}^{\infty} a_{n,4}^2 \;\leq\; \frac{(1\;-\;C)}{(80)} \;\leq\; 3\times 10^{-5}$$

$$\sum_{n=18}^{\infty} a_{n,6}^2 \;\leq\; \frac{(1\;-\;C)}{(60)} \;\leq\; 5\times 10^{-5}$$

Weighted Laguerre - Gauss sums

$$\sum_{p=2}^{\infty}\sum_{m=0}^{p} b_{p,m,0,0}^2 \;\leq\; \frac{(1\;-\;C)}{4970} \;\leq\; 6\times 10^{-7} \quad p,\; m \text{ even}$$

$$\sum_{p=1}^{\infty}\sum_{m=1}^{p} b_{p,m,0,0}^2 \;\leq\; \frac{(1\;-\;C)}{1264} \;\leq\; 2\times 10^{-6} \quad p,\; m \text{ odd}$$

Rear arm cavity mirror - spherical

Zernike sums over a 10 cm radius aperture

$$\sum_{n=8}^{\infty} a_{n,0}^2 \;\leq\; \frac{(1\;-\;C)}{(960)} \;\leq\; 3\times 10^{-6}$$

$$\sum_{n=18}^{\infty} a_{n,2}^2 \;\leq\; \frac{(1\;-\;C)}{(478)} \;\leq\; 6\times 10^{-6}$$

$$\sum_{n=25}^{\infty} a_{n,4}^2 \;\leq\; \frac{(1\;-\;C)}{(270)} \;\leq\; 1\times 10^{-5}$$

$$\sum_{n=25}^{\infty} a_{n,6}^2 \;\leq\; \frac{(1\;-\;C)}{(120)} \;\leq\; 2\times 10^{-5}$$

Zernike sums over a 5 cm radius aperture

$$\sum_{n=4}^{\infty} a_{n,0}^2 \;\leq\; \frac{(1\;-\;C)}{(490)} \;\leq\; 6\times 10^{-6}$$

$$\sum_{n=8}^{\infty} a_{n,2}^2 \;\leq\; \frac{(1\;-\;C)}{(230)} \;\leq\; 1\times 10^{-5}$$

$$\sum_{n=10}^{\infty} a_{n,4}^2 \;\leq\; \frac{(1\;-\;C)}{(130)} \;\leq\; 2\times 10^{-5}$$

$$\sum_{n=18}^{\infty} a_{n,6}^2 \;\leq\; \frac{(1\;-\;C)}{(100)} \;\leq\; 3\times 10^{-5}$$

Weighted Laguerre - Gauss sums

$$\sum_{p=2}^{\infty}\sum_{m=0}^{p} b_{p,m,0,0}^2 \;\leq\; \frac{(1\;-\;C)}{4970} \;\leq\; 6\times 10^{-7} \quad p,\; m \text{ even}$$

Sagitta match of spherical mirrors

$$\frac{\Delta h}{\lambda} \;\leq\; 2.5\sqrt{(1\;-\;C)} \;\leq\; 0.1$$

Power Spectrum Parameters for Both Mirrors

Spatial frequency range: 3 - 125 cm^{-1}

Power law exponent: $c \ = \ 1$

Correlation length: $l_{\rm cor} \ \geq \ 1 \times 10^{-1}$ cm

Power spectrum amplitude coefficient: $A \ \leq \ 2 \times 10^{-8}$ waves(5145A)^2 cm

Surface variance: $\frac{\sigma^2}{\lambda^2} \ = \ \int_1^{125} S_1(f_x) df_x \ \leq \ 1.4 \times 10^{-7}$

Surface roughness rms (in band): $\sigma \ \leq \ 2$ Angstroms

Spatial frequencies $f_x \ \geq 125$ **cm^{-1}**

Surface variance: $\frac{\sigma^2}{\lambda^2} \ = \ \int_{125}^{\infty} S_1(f_x) df_x \ \leq \ 1 \times 10^{-6}$