1 Introduction: Some history

In his first publication on the possibility of gravitational waves in General Relativity Einstein (Einstein 1916) estimates the power radiated by a source and asserts "...in any case one can think of (it) will have a practically vanishing value ". Never mind that in the same paper he wrongly predicts monopole sources but also with his remarkable physical insight he establishes correctly the nature of the wave solutions as traveling strains in space-time and gets the physical dimensions right enough to get an idea of the energetics. I have not found notes in the Einstein papers from that epoch but it is not hard to guess how he came to this pessimistic conclusion. It would not be out of character (Pais 1982) to imagine that after designing a Hertzian experiment to generate and detect gravitational waves with the materials and techniques at hand in 1916 he was confronted with impossible numbers, not merely small factors, but many orders of magnitude (the latest in our epoch (Grishchuk and Sazhin 1974) almost made it by a factor of 10^5). A reasonable guess is he would next have turned to astrophysical sources both to determine if there was a measurable gravitational wave strain but also an effect on their motion observable through a telescope. He might well have chosen to estimate the effect of gravitational wave emission on a binary star system in our own galaxy. One close enough to observe would have had several solar masses, a period of 10 years and be located at distance of 100 light years. He would have estimated a gravitational wave strain of 10^{-24} at a 5 year period, at the time an inconceivably small number to measure. The binary system would look invarying, losing half its mechanical energy to gravitational waves in 10^{21} years. By the way, Einstein corrects his mistake in a second paper dedicated entirely to gravitational waves (Einstein 1918) in which he derives the quadrupole formula.

The troubles did not, however, end there. The formulation for the power carried away from a source, the Einstein pseudo-tensor relating the intensity of the radiation to the square of the second derivative in time of the gravitational wave metric components, the gravitational equivalent of Poynting's theorem in electricity and magnetism, gave different estimates in different coordinate systems. Worse still, perturbation calculations of the gravitational radiation by isolated binary systems showed the orbiting masses gaining energy as they radiated (Kennewick 2007). The theoretical problems led to increased skepticism of the reality of the radiation and, in particular, whether energy was actually carried by the waves themselves. A watershed occurred in 1957 at the Chapel Hill conference on Gravitation and General Relativity where Bondi, Wheeler and Weber argued strongly for the reality of the waves with compelling gedanken experiments. The most influential arguments were given by Pirani (Pirani 1956, Pirani 1957) who showed rigorously the spacing of free particles traveling along neighboring geodesics (the geodesic deviation) would be changed in a coordinate independent manner by a passing gravitational wave.

What has happened in the past 100 years to make gravitational wave astrophysics a vital new field? The answer lies both in new astronomical discoveries and the remarkable scientific and technological developments of the last century, in particular, the advances in measurement capability and data analysis. All of which were needed in gaining the strong evidence for gravitational waves found by Hulse and Taylor in a binary neutron star system (Taylor and Weisberg 1982). The developments also led Weber to initiate his pioneering searches for gravitational waves (Weber 1960) and subsequently led to the development of long baseline interferometric detectors (Weiss 1972, Drever 1983), the subject of this book.

2 The gravitational wave spectrum

Figure 1 shows the gravitational wave spectrum being explored in 2016 by a variety of techniques. Many of the sources were already described and their strengths estimated in a prescient article on gravitational wave sources by Thorne (Thorne 1987). The vertical scale is the rms gravitational wave strain, h, while the horizontal scale is the frequency of the waves. At the lowest frequencies, periods of the age of the universe, is the prediction of primordial gravitational waves generated by quantum fluctuations during the inflationary expansion of the universe (Grishchuk 1975),(Starobinsky 1979),(Guth 1981),(Linde 1982,1983),(Abbott and Wise 1984). The radiation is indirectly measurable by the density and temperature fluctuations it induces in the primeval plasma just at the time of decoupling of the electromagnetic radiation from the matter, about 300000 years after inflation, and once again at the time when the universe becomes reionized during the formation of the first stars ($10 \geq z \geq 6$). The gravitational waves streaming from all directions lead to quadrupolar temperature patterns in the plasma which can be measured through the spatial pattern of the polarization of the plasma thermal emission. (Thomson scattering and B modes) (Seljak and Zaldarriaga 1997),(Kamionkowski, Kosowsky and Stebbins 1997). The direct measurement of the gravitational radiation from the simplest model of the inflationary epoch is unlikely with any of the instrumentation available in 2016 and would require a specially designed space mission (Big-bang Observer) (Harry et al 2006).

The next higher frequency band in the spectrum, at periods of a few years, may be observable by timing the arrival of pulses from an assembly of stable milli-second pulsars distributed over the sky (Foster and Backer 1990). Gravitational waves will cause fluctuations in the arrival time of the pulses and if there are sufficient number of pulsars in the sample may even show a quadrupolar spatial pattern. The gravitational wave sources are the collisions of 10^{11} to 10^{12} solar mass blackholes residing in early galaxies (Jaffe and Backer 2003). Most galaxies are now known to house black holes near their centers but of a more modest size. The enormous masses are a result of the increased stellar production in the first generation of galaxies in the universe. The pulsar timing may detect an unresolved background noise from many such black hole colli-

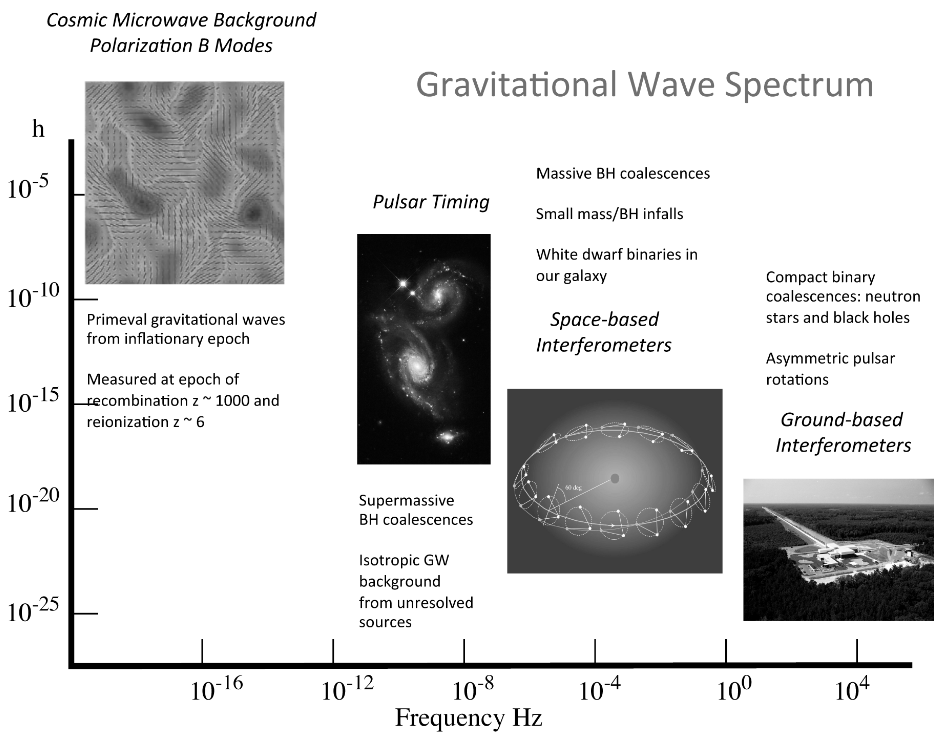


Figure 1: Spectrum of gravitational waves: sources and techniques

sions (Hellings and Downs 1983), (Jenet etal 2005) and, if nature is kind, a few individual events will stand out.

The spectral region between periods of a few hours to fractions of an hour contains the emission by black holes in the collision of more recently formed galaxies such as our Milky Way. Also, the radiation by smaller objects falling into these black holes as well as a virtual continuum of narrow lines due to white dwarf binary systems in our own galaxy. The frequency band has been chosen as the target for the Laser Interferometer Space Antenna (eLISA/NGO 2012), a mission now 40 years in planning in both the United States and Europe.

The high frequency region above a few Hz is accessible by interferometers on the ground and is the primary subject of this book. It contains the radiation by the coalescence of compact binary systems of neutron stars and black holes in the mass range from solar mass to 100's of solar masses. As well as the radiation by pulsars themselves as asymmetric rotors and impulsive sources such as supernova explosions if they are sufficiently non-spherical.

3 Ground based interferometric detectors

The worldwide network of large baseline interferometric detectors currently consists of: the two 4km LIGO detectors in the United States in Washington State and Louisiana , the 3km VIRGO detector in Cascina, Italy and the 600m GEO detector in Hannover, Germany (GEO 2014). A 3km KAGRA detector is being constructed in the Kamioka mine in Japan (KAGRA 2015) and a 4km detector identical to the LIGO detectors is being planned for a location in India. The detectors are all part of a network to localize gravitational wave sources with sufficient precision on the sky to enable finding electromagnetic counterparts and thereby to enhance the science being discovered by coupling to more conventional astronomy.

All the detectors are based on variants of the Michelson interferometer shown in Figure 3. The Michelson geometry trades on the kinematics and polarization of the gravitational wave. The interferometer is most sensitive to waves traveling perpendicular to the plane of the interferometer with its metric components along the arms, causing phase shifts in the light traversing one arm to be positive while those in the other to be negative, exactly the differential change for which the light intensity at the photodetector will vary maximally.

The phase shifts can be interpreted in several ways and these different interpretations have resulted in misunderstandings, in fact, in the early days of gravitational wave detection some thought an interferometer would not be able to detect gravitational waves at all (Saulson1987). One way to determine the interaction of a gravitational wave with the interferometer (Weber 1960) is to express the gravitational wave in terms of the contracted Reimann tensor as a tidal gravitational force in an otherwise inertial frame where the mirrors respond

THE RADIATION FIELD

Geometric Interpretation

$$d\,s^2 \;=\; g_{ij} dx^i\, dx^j$$

$$g_{ij} \;=\; \eta_{ij} \;+\; h_{ij} \qquad \text{weak field}$$

$$\eta_{ij} \;=\; \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ 0 & & -1 & \\ & & & -1 \end{pmatrix} \quad \text{Minkowski Metric of Special Relativity}$$

Gravitational Wave Propagating in the x_1 Direction

Traceless gauge

$$h_{ij} \;=\; \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & h_{22} & h_{23} \\ 0 & 0 & h_{32} & h_{33} \end{pmatrix} \quad \text{all } h_{ij} \ll 1$$

Plane Wave

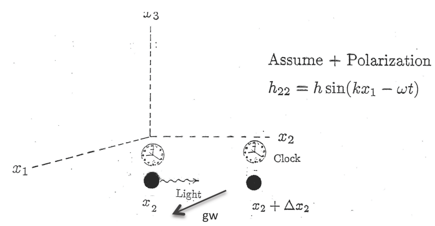
$\mathbf{h_{22} \;=\; -h_{33}}$ $\qquad$ $\mathbf{h_{23} \;=\; h_{32}}$

+ polarization $\qquad$ **×polarization**

And All Only Function of $x_1 \;-\; ct$

coordinate time of clocks remains proper time

spatial coordinates of clocks remain fixed



Minkowski

$$\Delta s^2 \;=\; 0 \;=\; c^2 \Delta\, t^2 \;-\; \Big(1 + h\sin(kx_1 - \omega t)\Big)\, \Delta\, x_2^2$$

LIGHT RAY

Let $\quad \Delta t \;\ll\; \dfrac{1}{\omega} \qquad h \;\ll\; 1$

$$c\,\Delta\, t \;\cong\; \Big(1 + \frac{h}{2}\sin(kx_1 - \omega t)\Big)\Delta x_2$$

INFERRED DISTANCE BETWEEN POINTS

$$\frac{\delta(c\,\Delta t)}{\Delta\, x_2} \;=\; \frac{h}{2}\sin(kx_1 - \omega t) \qquad \text{Time Dependent } \textbf{Strain}$$

$$\frac{\Delta\, l}{l} \;=\; \frac{h}{2} \qquad \text{The Measurable Quantity}$$

Figure 2: Sensing the gravitational wave metric with light. The transverse traceless gauge for the gravitational wave leaves the clocks keeping proper time and their coordinate positions fixed. The coordinates for the emission and the reception of light is associated with a null interval as in Special Relativity.

to the tidal forces in proportion to their mass. The mirror accelerations are proportional to the second derivative of h and proportional (as all tidal forces) to the mirror separation. This is the most Newtonian like description. There is no interaction between the light and the gravitational wave and no thought of the distortion of space-time. Another description, particularly appropriate to thinking about the interferometer is shown in Figure 2. The gravitational wave is expressed in a coordinate system with a transverse traceless metric for the waves. The spatial coordinates of the mirrors maintain fixed values as the wave passes through and the coordinate clocks as well as proper clocks are unaffected by the wave. However, the time it takes the light to traverse the arms changes with the presence of the gravitational wave, taking longer in one arm and shorter in the other. The driver here is the time varying spatial components of the gravitational wave metric tensor h. In this description it is most economical to think that space itself has deformed between the mirrors held at fixed locations in the space. And still a third description (Cooperstock and Faraoni 1993) in the transverse traceless gauge is to write Maxwell's equations in the perturbed metric of the gravitational wave in the interferometer arms and to solve for the gravitational wave perturbed electromagnetic fields of the light in the arms. The result of the perturbation are phase modulated sidebands of the light at frequencies above and below the input laser light separated from the carrier by the gravitational wave frequency. All three interpretations alone give the same result, the trouble comes when one mixes the interpretations.

An elegant way to analyze the Michelson interferometer when operated as a gravitational wave detector is to note the symmetries. The beam splitter has the property that reflection on one side flips the sign of the optical electric field, the antisymmetric side (port), while reflections from the other side, the symmetric side (port), maintain the sign[1] If this were not the case, the interferometer could not conserve energy. The input laser carrier light is injected into the interferometer at the symmetric port while the output light to the photo detector emerges from the antisymmetric port. Ideally the interferometer is set up to have the light spend equal time in the two arms. If the optical transmission is the same in the two arms, the carrier light returning to the beam splitter will cancel at the antisymmetric port and if in addition the losses are small, all the carrier light at the input will emerge at the symmetric port. The phase modulation sidebands induced by the gravitational wave will have opposite sign in the two arms due to the gravitational wave polarization. They will therefore add at the antisymmetric port and cancel at symmetric one. To convert the phase modulation sidebands at the antisymmetric port into amplitude modulated sidebands and thereby make them detectable, a small amount of carrier is deliberately left at the antisymmetric port. The above describes the basic detector.

In addition there are some useful ideas to increase the carrier power in the interferometer by reflecting the carrier back into symmetric port (power

[1] Strictly true for splitters composed of dielectric layers on a glass substrate used in many precision interferometers. The phase change is determined by the Fresnel field continuity equations.

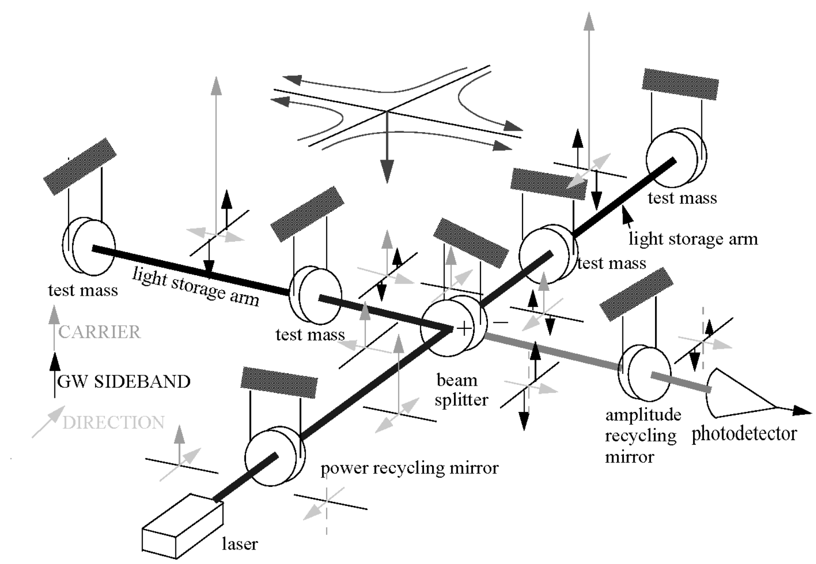


Figure 3: Optical signals in a power and signal recycled Michelson/Fabry-Perot Interferometer. The carrier and gravitational wave induced sidebands at different locations in the interferometer held at its operating point are shown. A small amount of carrier (not shown in the figure) is required at the photodetector to beat against the sidebands to produce the output signal.

recycling mirror) (Drever 1983), (Schilling 1982) and to tailor the spectral response of the interferometer by reflecting the phase modulated sidebands back into the antisymmetric port (signal recycling mirror) (Drever 1983), (Meers 1988). The power recycling mirror is chosen to have a transmission equal to the losses in the entire interferometer. The Michelson as well as the Fabry-Perot cavities in the arms are all resonant while the there is close to complete cancellation of the fields at the antisymmetric port. Under these conditions no carrier is returned to the laser and one can think of the interferometer as a resonant matched load for the laser. The resonant optical cavity consisting of the entire interferometer serves to buildup the power at the beam splitter as well as to filter the spatial and temporal fluctuations of the input light. This common mode resonance also serves to act as a final frequency reference for the input light. The role of the signal recycling mirror is more subtle. The initial idea was to reflect the gravitational wave induced sidebands back into the interferometer and thereby buildup their amplitude by resonance at the cost of bandwidth. In actual practice, by detuning the interferometer as well as adjusting the position and transmission of the mirror a large variety of spectral responses and phase sensitivities can be realized. The various arrows in Figure 3 show the gravitational wave sidebands at different locations in the interferometer and might be helpful in understanding the algebra of subsequent chapters *Chapter 2*.

4 Noise sources

The noise sources that compromise the gravitational wave strain sensitivity are characterized in terms of perturbations that affect the ability to measure the displacement (phase noise) and those that apply random forces to the mirrors (force noise). These in turn are further characterized as being due to fundamental fluctuations based on physical laws such as thermodynamics and quantum physics or on technical problems amenable to better experimental practice.

4.1 Quantum noise

Figure 4 shows how the fundamental quantum noise limits the interferometer sensitivity *Chapter 4*. The mechanism is due to an insight by Caves (Caves 1981) who recognized that the zero point fluctuations of the electromagnetic field entering the antisymmetric port are responsible for both the phase fluctuations that limit the ability to determine the differential position of the end test masses (shot noise phase fluctuations) as well as the momentum fluctuations that differentially drive the end test masses. The quantum noise has both characters: phase and random force noise. The virtual quanta entering antisymmetric port become "real" when they interact with the carrier light in the same mode. The symmetric port of the interferometer also admits zero

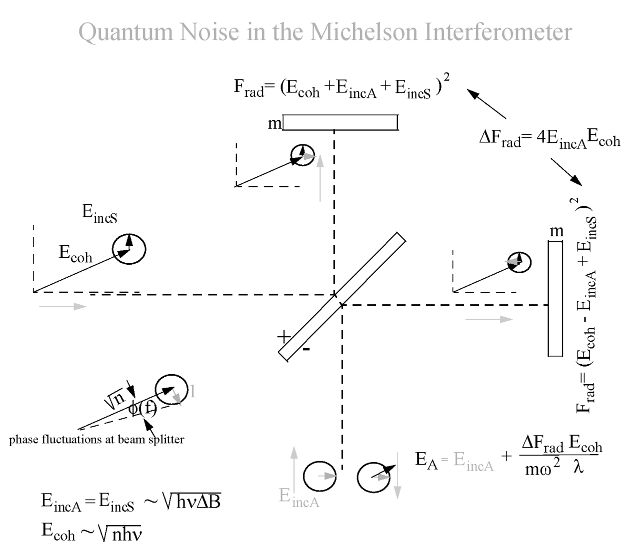


Figure 4: Quantum noise in a Michelson interferometer. Vacuum field fluctuations, the arrows in the small circles, enter the interferometer at both symmetric and antisymmetric ports. With the interferometer held at its operating point the fluctuations entering the symmetric port are cancelled at the antisymmetric port. The radiation pressure fluctuations on the two arm mirrors from the symmetric port cause common mode motions of the mirrors and are not detected as optical phase shifts at the antisymmetric port. The vacuum field fluctuations entering the antisymmetric port are responsible for both the quantum phase and momentum noise of the interferometer (assuming 100% quantum efficiency of the photodetectoror.

point fluctuations but in this mode the momenttum noise induced in the test masses as well as the intensity fluctuations are common mode and do not effect the phase at the antisymmetric port. The ratio of the phase fluctuations to the interferometer signal induced by the gravitational wave varies $1/\sqrt{P_{bs}}$ while the displacement noise due to the momentum fluctuations grow as $\sqrt{P_{tm}}$. As a consequence, if quantum noise dominates the noise budget, there is an optimum power for each frequency which yields the "naive " quantum limit for the interferometer. It is worth noting that additional detector quantum noise arises due to the statistics of the quantum detection at the photo detector, this contribution with high quantum efficiency photodetectors becomes negligible at the power usually employed in an interferometric gravitational wave detector.

The possibility to reduce the quantum noise in an interferometer below the

”naive” limit was recognized by Yuen and Caves (Yuen 1976 and Caves and Schumaker 1985). The idea was to generate a complementary entangled photon to the virtual photon in a non-linear optical device and let the two photons together enter the antisymmetric port. The two photons have the same frequency as the carrier but have sidebands arranged to cancel the phase fluctuations (phase squeezing) or their amplitude fluctuations (momentum squeezing). The two photon states are fragile and sensitive to losses which destroy their correlation. Nevertheless, squeezed light generators placed at the antisymmetric port are an important ingredient in future improved detectors especially if high power carriers cannot be handled in the interferometers due to mirror heating.

4.2 Thermal noise

The other fundamental noise source with significant contributions in the current interferometers is thermal noise *Chapter 5*. Thermal noise is a fluctuating force due to the excitation of modes of motion each with an equipartition energy of kT (Saulson 1990). A profound overriding idea in statistical mechanics is the fluctuation dissipation theorem which takes on real meaning in the sensitive precision mechanical instruments such as the gravitational wave detectors. Simply stated the theorem asserts that whatever mechanism damps the system is also responsible for the fluctuations of the system around equilibrium. A simple example to demonstrate the theorem is a pendulum in an imperfect vacuum . The residual gas molecules moving at thermal speeds as they make collisions with the pendulum remove the kinetic energy from the moving pendulum, the molecules colliding with the surface moving toward them and bouncing off the surface extract more energy from the pendulum, than the energy it gains from those moving toward the pendulum and bouncing off the surface that is receding from them. The difference in momentum transfer is proportional to the velocity of the pendulum and leads to velocity dependent (viscous) damping of the pendulum motion - this is the dissipation mechanism. While this damping is going on the pendulum is also being excited into motion by the molecular collisions. In any time interval there are not exactly equal number of molecules hitting the two faces of the pendulum, there is a Poisson distribution of hits with a variance equal to the sqrt of the number hitting the surface. The fluctuations in the number cause the pendulum to move - this is the fluctuation part of the theorem, the mechanism responsible are the molecular collisions with the surface. Pursuing the example further gives the full impact of the theorem. If one further improves the vacuum to the point where the collisions are so infrequent as to cause negligible gas damping, a real pendulum will still experience damping. The damping mechanism turns out to be mechanical loss in the elasticity of the support fibers which is far more subtle to analyze but the theorem nevertheless allows one to estimate the thermally driven fluctuations. The pendulum motion along the light beam is an example of a single mechanical mode system. The thermal noise not only occurs at the resonance frequency of the system but on both sides of the resonance as determined by the transfer function of the system to a random distribution of sharp impacts (white or colored noise depending on the details of the damping mechanism).

The pendulum is a particularly useful device for making a horizontal suspension as regards thermal noise. The dissipation is in the elastic members, the bending of the support fibers at the suspension point and at the connection to the test mass. However, the energy of the horizontal oscillation is stored primarily in the gravitational potential of the test mass as it is lifted in the gravitational field of the Earth. The energy stored in bending the fiber is factors of 10^{-5} smaller. As a consequence, even though the losses in the fiber bending may account for 10^{-4} of the energy stored in the fiber it may only be a factor of 10^{-9} of the energy stored in the pendulum motion. This dilution factor does not apply to the vertical oscillations of the pendulum where all the energy is stored in elasticity in the fiber. The thermal noise is larger in the vertical. This comes back into the horizontal motion due to the curvature of the Earth which gives a vertical to horizontal noise conversion of 10^{-4} in 4km and will be larger by the square of the arm length as one contemplates longer detectors.

A similar analysis applies to estimate the fluctuations of the surface of the mirror providing one takes into account all the normal modes that contribute to the surface motion (Callen and Welton 1951). Levin (Levin 1998) has developed a formulation of the theorem that applies to a local patch of surface or volume avoiding the summation over the modes.

One strategy to reduce the influence of thermal noise in the interferometer is to assure that the resonant mechanical modes of the test masses and suspensions are either below or above the frequency band of interest to detect gravitational waves. For some modes such as the violin resonances of the support wires that fall in to the gravitational wave band, the method is to reduce the elastic losses by using low dissipation materials and then to damp the modes with negative feedback using low noise sensors and controllers, thereby effectively refrigerating the mode and "beating" the fluctuation dissipation theorem. The technique was first applied in my experience by Dicke (Roll etal 1962) in his Eötvos experiment and now finds application in almost all precision mechanical experiments - noise free damping. A more brute force approach being considered in KAGRA and possibly for other future detectors is to use cryogenics. The thermal noise amplitude should reduce with the sqrt of the absolute temperature and possibly faster if the damping mechanism in the material is also a function of temperature.

A still unsolved problem due to thermal noise are the fluctuations in both index and thickness of the multi-layer optical coatings on the mirrors. As seen in Figure 5 these dominate the noise budget in the critical mid frequency band around the best sensitivity. The solution to make optical coatings with both low optical loss as well as mechanical loss is a central problem in current research.

4.3 Seismic noise

The vibrations of the Earth are a dominant noise source in the interferometric detectors but they are not fundamental or irreducible in the same manner as the quantum noise *Chapter 6*. Seismic noise yields to ingenuity and clever engineering. The seismic noise spectrum on the Earth has components at periods of 12 and 24 hours due to the solid Earth tides, the Earth normal modes extending from periods of several hours to 20 minutes, earthquakes with spectral components between 10 minutes to 0.1 seconds, ocean waves hitting shore lines with periods primarily at 6 and 3 seconds (microseismic peaks), meteorological sources such as wind scattered by buildings and trees with periods in the minute to 0.1 second band and a host of anthropogenic sources extending from periods of seconds to 10's of milliseconds. The rms motions in the 0.1 to 10Hz band in a typical city are 20s of microns while at a rural site around 1 micron, motions so many orders of magnitude larger than anything expected from a gravitational wave in a 4km detector, necessitate that the first order of business is to make good vibration isolators.

The part of seismic noise that consists of local translational and rotational accelerations of the ground can be measured directly if one can have a reference to the local inertial frame. This is the principle of a seismometer or tilt meter which contain a mass or mass with a large moment of inertia well isolated from the local accelerations by a soft suspension (low frequency resonance) and are well damped by noise free damping systems. The techniques employed in the various interferometric detector projects combine both the isolation provided by simply suspending a platform from the ground by a low frequency period single or multiple suspension, gaining an isolation with $(f_0/f)^2$ with each isolation stage (as long as the resonances are far below the gravitational wave frequency being measured) with active feedback systems that use a seismometer mounted on the platform as inertial sensor and controllers to null the motion of the platform in inertial space. The sensitive test mass mirrors are then further suspended from these inertially stabilized platforms. The strategy that has been most successful to reduce both the influence of seismic noise and the thermal noise from the mirror suspensions is to separate the isolation functions. The platforms motions are reduced actively at low frequencies where the large motions occur and passively at high frequencies where the sensor noise dominates. The final reduction of both low and high frequency noise, once the large motions have been removed at the platform, is done with control and noise free damping of the suspensions. These exert small forces on the suspension elements which further filter the in-band noise generated in the seismic isolation system. The final suspension stage at the test mass uses low dissipation suspension components to reduce the thermal noise and extremely quiet low force controllers mounted on suspended elements to reduce coupling to the ground.

A useful strategy to reduce the amount of control required by the feedback systems is to feed forward from knowledge of the perturbations or from auxiliary signals derived from external instruments. An example is the control of the arm length against the strain from the Earth tide. The Earth tide has amplitudes of a decent fraction of a mm in 4 km and angle changes of 10^{-7} radians at periods of 12 and 24 hours all of which can be programmed as a known signal to correct the isolation system position. Another example is the measurement of tilts at low frequencies (large wavelengths) by an external instrument with application through the proper proportions to the controllers in the seismic system.

A significant concern in both the seismic isolation and suspension systems is up-conversion of the large displacements at low frequencies into the gravitational wave band. Up-conversion occurs due to non-linearities in either the mechanical or electronic systems or by discontinuous behaviour in mechanical systems under stress that creep. An example observed in the initial LIGO detectors was Barkhausen noise in control magnets and other ferritic material where the low frequency large magnetic fields driven to hold the interferometer at a fixed point on a fringe caused small steplike pulses in the magnetic force as magnetic domains in the material did not immediately follow the drive fields. A corresponding possibility may occur in todays detectors since the steel springs that support the large loads in the isolation and suspension systems are known to creep or the bonding in the silicon fibers to the test mass may creep.

An area of active research is the need to be able to separate tilt from horizontal displacement in the seismic excitation of the system. The control one would exert on the seismic platform to reduce the coupling to external tilt or displacement is significantly different (a rotation vs a translation). At the moment the low frequency performance of the isolation system is compromised by not separately controlling tilt and horizontal displacement.

4.4 Newtonian gravitational gradient forces

The test masses are buffeted by random gravitational forces arising from density fluctuations in both the ground and the atmosphere (Saulson 1984, Hughes and Thorne 1998) *Chapter 6.* As the seismic isolation has improved, these forces are now the dominant noise below 10 Hz and a major reason to consider operating interferometers in space for low frequency gravitational wave detection. There is some hope to inferring the driving terms with seismometer arrays within 100's of meters of the test masses if the properties of the ground (density and compressibility) are sufficiently uniform, or can be solved for by regression (Driggers etal 2012). The signals fed forward to the test masses to correct for the gravitational gradient displacements of the test mass. Similar technique using infrasound (low frequency) pressure sensors in arrays around the test mass would be needed to correct for the displacement of the test mass due density fluctuation in the local atmosphere. The ability to gain low frequency sensitivity is strongly site dependent. There is some indication that burying the interferometer below the propagation of the bulk of the Rayleigh seismic waves would provide advantages.

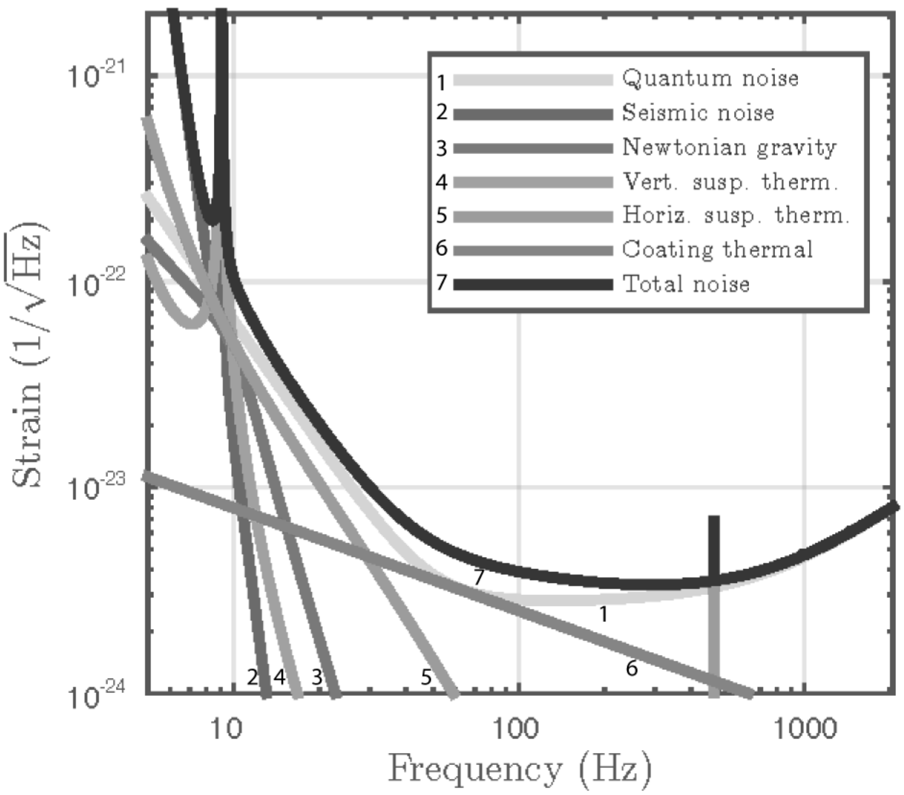


Figure 5: The advanced LIGO design noise budget (LSC 2015)

5 Techniques and "technical" noise sources

Most of the effort in bringing the detectors to design sensitivity is dealing with the reduction of noise sources that are not fundamental or irreducible. Usually better experimental practice and design are needed to overcome them. *Chapter 7* deals with some examples. Here it is worth touching on a small group that are absolutely critical.

5.1 Scattered light: sensing noise

Scattered light derived from the high power carrier beam is the most pernicious and time consuming problem. The sources of scattering are inhomogenieties and surface errors in the mirror coatings and substrates, unintended diffraction from edges and the reflections from incomplete anti-reflection coatings and forward scattering by molecules moving through the optical beams. If the scattered light can be captured or blocked before recombining with the main beam, it will cause little harm. The slight loss in intensity out of the main beam is usually not critical. More serious is the process where a scattered light path takes a different propagation time than the main beam before recombination with the main beam. Then the phase of the main beam becomes additionally sensitive to the frequency fluctuations of the laser and more significantly the scattered path can be phase modulated by the motions of the various surfaces the scattered light encounters before being recombined. The phase change the scattered light causes in the main beam is proportional to the ratio of the amplitudes of the scattered light to the main beam light. Scattering amplitude ratios of 10^{-10} (intensity ratios of 10^{-20}) are known to have caused excess noise in the interferometers.

The other major source of scattering noise is forward scattering by molecules in the long interferometer arms *Chapter 13* . The light scattered forward by a molecule is 90 degrees out of phase with the main driving carrier with an amplitude proportional to the molecular volume. Since the molecules traverse the main beam with a variety of times, the recombination of the scattered light with the main beam causes phase fluctuations of the beam proportional to the sqrt of the number of molecules resident in the beam with a time dependence determined by the residence time of a molecule in the beam. Large molecules such as a hydrocarbon, which also move slowly, are much greater phase noise sources than hydrogen, the dominant gas in the long arms. The solution adopted by all the interferometers is to operate the long arms at ultra high vacuum. *Chapter 13*

5.2 Laser intensity and phase noise: sensing noise

Lasers for precision interferometry have improved enormously since the beginning of gravitational wave interferometry in the early 1970's. The ion lasers with powers of a few watts and wall plug efficiencies of 10^{-4} have been replaced by solid state systems with output powers of 100s watts and efficiency close to 10^{-1}. The lasers have become the most reliable sub-system in the interferometers. *Chapter 14.* Even so commercial lasers still need to be adapted for use in the interferometers. *Chapter 18 and 19*

The gravitational wave sensitivity of the interferometer is effected by the intensity noise and frequency instability of the lasers. Intensity fluctuations of the carrier do show up at the antisymmetric port of the interferometer for several reasons. The gravitational wave phase modulation sidebands need to be made into amplitude modulation to be detected by the photodetector, this requires a small amount of carrier light at the antisymmetric port. Furthermore, imperfections in the interferometer optics, in particular, an unbalance in the reflection amplitude of the two long arm cavities (for example, due to differences in the mirror reflectivites or losses) or mismatch of the optical modes coming from the two arms cause carrier light to appear at the antisymmetric port. This unwanted carrier light is filtered by an output mode cleaner to remove components not in the same optical mode, leaving the gravitational wave modulated phase sidebands and the carrier remaining from the cavity unbalance. The remaining carrier is fluctuating with the intensity noise of the laser filtered by the long arm cavities. In addition, a small difference in propagation time of the carrier from the two arms (Schnupp asymmetry) is deliberately inserted in the Michelson arms to allow RF phase modulation sidebands to appear at the antisymmetric port for other purposes than the main gravitational signal recovery. This unbalance as well as the unbalance in arm cavity storage times, cause frequency excursions of the laser to appear at the antisymmetric port and propagate through the output mode cleaner. Both the intensity and the frequency noise of the laser needs to be reduced. This is done with control loops comparing the detected light coming from the symmetric port with a DC potential as a reference for the intensity and the common mode phase of the two long arm cavities as a frequency reference.

5.3 Coupling of angular fluctuations to displacement : stochastic force noise, heating and radiation pressure instabilities

One of the more complex control systems in the interferometers is the automatic alignment system which maintains the orientation of the mirrors relative to a fiducial mirror (one of the input test masses, *this may no longer be true, the alignment system maybe an improvised combination of wavefront sensors, quadrant diodes and damping via optical levers*). The noise in the alignment system and the poorly isolated low frequency seismic noise cause the mirror alignment to fluctuate about an average angle. If the interferometer is in alignment and the beam spots on the mirrors are all centered on the rotational mode axes, the phase of the detected light at the antisymmetric port varies only in second order. If the system is misaligned or the beam spots are not centered, a first order phase shift occurs at the antisymmetric port and correspondingly produces noise in the gravitational wave channel. The slow misalignments of the interferometer ,as well as that of the injected laser beam and the orientation of the input mode cleaner need to be almost continuously corrected to maintain the interferometer at low noise performance. This will be much improved with time as more rigorous and larger dynamic range angular control systems are developed. Especially, as long term stability and extending interferometer duty cycle become paramount. *Chapter 20*

The high optical power being planned later in the evolution of advanced LIGO and VIRGO, with close to a megawatt traveling between the mirrors in the long arm cavities, leads to new considerations in the control of the interferometer. The deformation of the mirrors due to non-uniform heating by absorption in the mirrors causes the excitation of higher order transverse optical modes in the cavities. The deformations are reduced with a thermal compensation system consisting of ring heaters around the test masses and the illumination of the high reflectivity surfaces of the test mass by strongly absorbed 10 micron radiation from CO_2 lasers *Chapter 22*. Another consequence of the high power are the forces and torques applied to the test masses by radiation pressure. These are no longer negligible, causing instability of the angular controls along with cross coupling to beam centering (Sidles and Sigg 2006) and optical springs in the longitudinal directions along the optical axis of the cavities (Corbitt et al 2005) *Chapter 21*.

5.4 Diagnostic and noise minimization techniques *Chapter 7,15,23*

The interferometers and their sub-systems are monitored and controlled digitally. The advanced LIGO detector has tens of thousands of individual channels which allow monitoring and control access to most of the critical interferometer components and test points. Without such a system one cannot operate and successfully diagnose the complex instrument. The system has software to enable taking time series of the monitoring and test point channels and perform Fourier transformation to obtain spectra and cross spectra between channels. It is possible in real time to make measurements of the coherence between channels. The system allows one to excite the various subsystems as well. A powerful diagnostic technique to determine the contribution of the noise at the antisymmetric port output due to the noise in a particular subsystem is to stimulate the subsystem and measure the coherence (and cross spectra) between the excitation and the signal at the antisymmetric port (Shoemaker etal 1988). By establishing the transfer function, it is then possible to determine the contribution of the ambient noise at the subsystem to the noise at the antisymmetric output. This technique is used in estimating the noise budget of the instrument. With the knowledge of the coupling one can make decisions on whether to fix the noise or to regress the noise from the output signal using the monitoring signal at the subsystem. *Chapter 15*

6 Future developments

Advanced LIGO (LSC 2015) and VIRGO (VIRGO 2015) sensitivities are designed to intersect the astrophysical estimates for the source strengths at the Earth. Once detections have been made the field changes, improvements in the detectors will in part be driven by the scientific questions raised by the measured gravitational wave signals. If the initial detections are neutron star binaries there will be a strong impetus to improve the sensitivity in the high frequency region to investigate the dynamics as the stars get close together and ultimately crash into each other making a new black hole. The need will be to improve the phase sensitivity with higher power or squeezing or both as well as to develop interferometer configurations that enhance the sensitivity at high frequencies.

Many of the workers in this field are most interested in determining the properties of gravitation in the strong field which has not been tested experimentally, the observation of the birth of black holes or the coalescence of black hole binaries into a larger black hole would be a stunning signal to analyze. The first detections may well be black holes but there are no good estimates for the rate of such detections. The probability of finding black hole signals is higher at low frequencies and if none are observed with the advanced detectors, the emphasis will be to improve the detector at low frequencies.

To gain statistics for either types of events general broadband improvements in sensitivity will be urged to look deeper into the universe. The types of improvements that can be made will depend on the technology that has been developed in the current research programs. Improvements may come from operating the test masses at cryogenic temperatures, the hope is to reduce the thermal noise in the coatings which now dominate the noise budget at the most sensitive spectral region. It my also come from simply using larger beams with larger test masses or from the development of new coating techniques. Improvements at low frequencies need to reduce the Newtonian gravitational gradient noise. Some of the improvement could come from burying the detectors as is being contemplated by the Einstein gravitational wave detector (Punturo etal 2010) and by KAGRA (KAGRA 2015) with its installation in a mine. Improvements could also come from feed forward from sensor arrays.

Another direction is to build detectors with longer arms. The gravitational wave displacement grows with the length as long as the arm length remains smaller than the gravitational wave wavelength (the storage time less than the gravitational wave period). Most stochastic forces do not grow with the length short of the conversion of vertical thermal noise in the suspension into horizontal

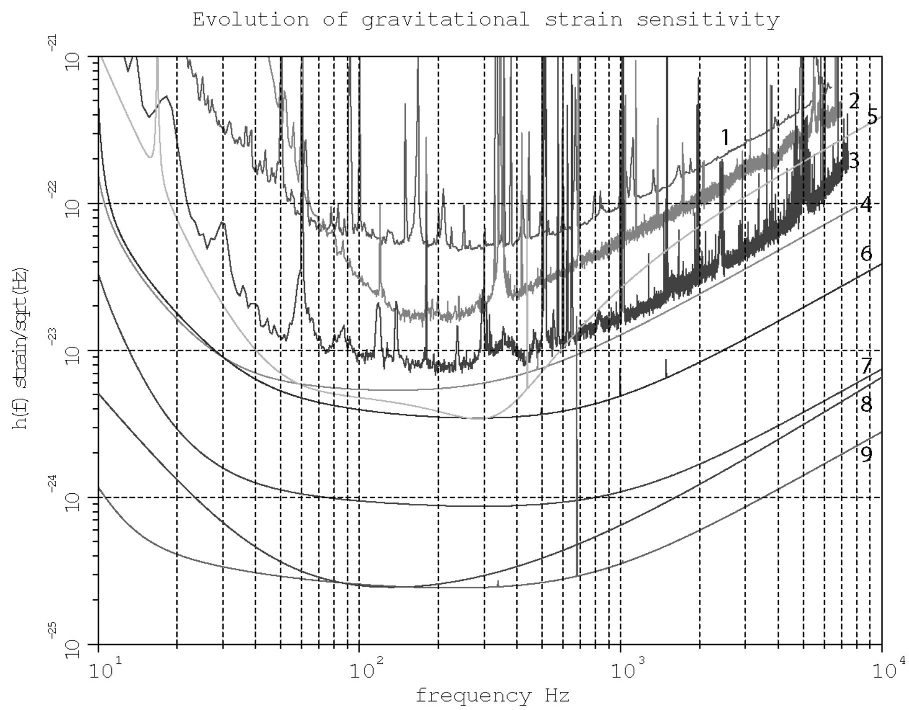


Figure 6: Evolution of the interferometers. Legend: **1** VIRGO 2009, **2** Enhanced LIGO 2009, **3** Advanced LIGO 65Mpc NS/NS 2015, **4** Advanced LIGO 150Mpc NS/NS low power, **5** Advanced VIRGO, **6** Advanced LIGO 190Mpc NS/NS high power, **7** 4km "Voyager" example 600Mpc NS/NS, **8** Einstein telescope B, **9** 40km "Cosmic Explorer" example

noise which depends on the difference in angle of the vertical at the ends of the detector. The sensing noise is also independent of the length although scattering noise could become more serious as smaller angles are involved.

Figure 6 shows some examples of the possible improvements. Several studies have been carried out within the LIGO project to determine the nature of the improvements one could make within the existing 4 km facilities, (Adhikari 2014) possibly a factor of 3 to 4. More significant improvements could be made with new facilities such as the proposed Einstein detector (Punturo et al 2015) and the 40km detector (Dwyer et al 2015). These developments could make the sensitivity improvement to bring gravitational wave observations into cosmology.

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