Sources of Gravitational Radiation

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GRAVITATIONAL RADIATION - THE STATUS OF THE EXPERIMENTS AND PROSPECTS FOR THE FUTURE

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The equivalent of a gravitational Hertzian experiment using a terrestrial source and receiver is not feasible at present or in the forseeable future. The search for gravitational radiation for this reason has concentrated on detecting gravitational emission from astrophysical sources. These experiments could serve the dual purpose of establishing the existence and properties of the radiation as well as to open a new window on the universe which will probe relativistic astrophysics and cosmology.

The best estimates for the gravitational radiation incident on the earth from astrophysical sources are summarized in the notes of the second discussion section of this conference and in two current review articles by Tyson and Giffard (1978) and Douglass and Braginsky (1979).

The source estimates are divided into three categories. Impulsive sources such as isolated stellar collapses and collisions, periodic sources arbitrarily defined as those producing waves with a 1000 or more cycles such as binary stellar systems and stellar oscillations, and finally the radiation by aggregates of sources which could produce a stochastic background of gravitational waves such as might be generated by a turbulent or chaotic primeval universe.

This review of the status and prospects for detection is divided in the same manner. Acoustically coupled antennas are discussed first and then followed by a discussion of "free" mass electromagnetically coupled antennas both of the Doppler ranging and interferometrically sensed types.

ACOUSTICALLY COUPLED ANTENNAS

This type of antenna, pioneered by J. Weber (1969) and analyzed by Giffard (1976) has four components: the resonator, motion transducer, amplifier and output filter as schematized in Fig. 1. The resonator is characterized by its mass, m, operating temperature, T, oscillator quality factor, Q, resonant frequency ω_0 or equivalently by its length l and speed of sound, c_s, in the resonator material. The resonator is driven by both the gravitational radiation which is the signal, and stochastic forces due to thermal excitations as well as the amplifier noise transmitted through the transducer (back reaction forces). The transducer, if linear, can be specified by the electromechanical coupling matrix, (Hunt, 1954).

$$\begin{pmatrix} F \\ \phi \end{pmatrix} = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} \begin{pmatrix} u \\ I \end{pmatrix} \tag{1}$$

F is the force applied to the resonator by the transducer and u the velocity of the resonator, ϕ the voltage at the transducer output and I the current. Z_{11} is the mechanical impedance of the transducer, Z_{22} the electrical output impedance and Z_{12} the electromechanical coupling impedance. The matrix is symmetric in magnitude of the matrix elements $|Z_{12}| = |Z_{21}|$, which follows from reciprocity. Z_{12} is responsible for converting amplifier noise to a noise force on the resonator. A useful quantity, the electromechanical coupling efficiency, β, is defined as the ratio of the electrical energy stored in the transducer to the mechanical energy stored in the resonator, Gibbons and Hawking, (1971). In terms of the transducer matrix elements β is given by

$$\beta = \frac{|Z_{21}|^2}{|Z_{22}|\ m\omega} \tag{2}$$

Another quantity often used is the displacement sensitivity of the transducer

$$\alpha = |Z_{21}|\omega \quad \text{volts/cm} \tag{3}$$

The amplifier can be characterized by a series voltage noise source with spectral density $e_n^2(f)$ volts^2/Hz, a shunt current noise generator with spectral density $i_n^2(f)$ amp^2/Hz, followed by an ideal noise free amplifier. The input impedance of the amplifier is assumed large relative to the noise matching impedance defined as

$$|Z_{match}| = \left(\frac{e_n^2(f)}{i_n^2(f)}\right)^{1/2} \tag{4}$$

If the input of the amplifier is terminated in Z_{match} the voltage and current noise contributions become equal.

With the amplifier matched, the amplifier noise can be expressed in terms of the spectral power density of an equivalent thermal source at temperature T_{amp} given by

$$\frac{hf}{e^{hf/kT_{amp}} - 1} = \left(e_n^2(f) \ i_n^2(f) \right)^{1/2} \tag{5}$$

if the power gain of the amplifier is much larger than one (Weber, 1957, Heffner, 1962). In the Rayleigh-Jeans limit ($hf/kT << 1$), this expression reduces to the familiar Nyquist formula

$$T_{amp} = \frac{\left(e_n^2(f) \ i_n^2(f) \right)^{1/2}}{k} \tag{5a}$$

While if the amplifier is limited by spontaneous emission noise at its input - the quantum limit for a linear amplifier - the equivalent temperature becomes

$$T_{amp} = \frac{hf}{k \ \ell n 2} \tag{6}$$

which is 6.9×10^{-8} °K at 1 kHz.

The equivalent temperature and matching impedance completely characterize the amplifier in signal to noise calculations. In principle the electrical impedance transformation that matches the transducer to the amplifier can be noise free.

The amplifier does not have to operate at the audio frequency of the acoustic resonator, where low noise amplifiers are not readily available. The transducer can be a variable capacitor or inductor (or microwave cavity) which is driven by a high frequency carrier signal. Side bands are introduced in the output signal of the transducer with amplitude determined by the resonator motion and offset frequencies determined by the resonator frequency. The transducer becomes the first stage of a parametric amplifier and is then followed by a low noise amplifier at the carrier frequency. The utility of this scheme lies in the translation of the frequency to a region where a good amplifier exists.

The parametric amplifier in the ideal limit is governed by the Manley-Rowe (Manley and Rowe, 1956, Rowe, 1958) relations which state that the rate of signal photons at the output and input can at best be the same. The power gain is

$$\frac{P_{out}}{P_{in}} \leq \frac{\omega_{out}}{\omega_{in}} \tag{7}$$

and the equivalent noise temperature at ω_{in} can at best be

$$T_{in} = T_{out} \frac{\omega_{in}}{\omega_{out}} \tag{8}$$

where T_{out} is the noise temperature of the amplifier following the parametric converter, providing the converter is noise free. In principle then, if an amplifier can be made to operate at the quantum limit at some frequency by parametric conversion the quantum limit can be achieved at another frequency.

The final component of the system is a filter which can be optimized if the system noise spectrum, $G_{nn}(\omega)$, and the signal power spectrum $G_{ss}(\omega)$ are known. The optimal linear filter can be determined by using the Wiener-Hopf (Wiener, 1949) theorem which states that the transfer function of the output filter should be

$$T(\omega) \cong \frac{G_{ss}(\omega)}{G_{nn}(\omega) + G_{ss}(\omega)} \tag{9}$$

The optimization of the output filter becomes important when the signal to noise is small and requires knowledge of the signal power spectrum. This is one of the major experimental motivations for theoretical pulse shape predictions.

DETECTION CRITERIA FOR ACOUSTIC ANTENNA SYSTEMS

In this section the generalized formulations of the preceding section are applied to detection criteria for the three classes of gravitational wave sources. The system noise, expressed in the frequency domain, is used to derive an rms output noise which is then expressed in terms of the minimum detectable gravitational wave signal.

a) Impulsive Sources

The following calculation assumes that the gravitational wave pulse length is short compared to the relaxation time of the resonator. The output filter is not optimized but rather a simple integrator with integration time, t_{int}.

The rms noise in the antenna expressed as an equivalent energy in the resonator is given by

$$\Delta E = \frac{t_{int}}{m} \sum_{i=1}^{n} F_i^2(f) + \frac{m\,\omega_0^2}{4\,t_{int}}\,x^2(f) \tag{10}$$

where $F_n^2(f)$ is the spectral density of the stochastic forces on the resonator in dynes^2/Hz and $x^2(f)$ is the spectral density of the equivalent displacement noise in cm^2/Hz due to the amplifier and transducer.

If the resonator is well isolated from external perturbations (ground noise, magnetic pulses, cosmic rays, etc.) the dominant stochastic forces are the thermal Nyquist forces given by

$$F_{th}^2(f) = \frac{4\,k\,T\,m\,\omega_0}{Q} \tag{11}$$

and the back reaction force due to the amplifier noise acting on the resonator through the transducer given by

$$F_{br}^2(f) = |Z_{12}|^2\; i_n^2(f) \tag{12}$$

The equivalent displacement noise spectral density is

$$x^2(f) = \frac{\left(e_n^2(f) + |Z_{22}|^2\; i_n^2(f)\right)}{|Z_{21}|^2\;\omega_0^2} \tag{13}$$

Combining these equations results in an expression for the rms noise in the system

$$\Delta E = \frac{t_{int}}{m}\left(\frac{4\,k\,T\,m\,\omega_0}{Q} + |Z_{12}|^2\; i_n^2\right) + \frac{m}{4t_{int}}\;\frac{\left(e_n^2(f)+|Z_{22}|^2 i_n^2(f)\right)}{|Z_{21}|^2} \tag{14}$$

The rms noise is minimized when t_{int} is chosen to make the two terms equal, this condition is given by

$$t_{int}^2 = \frac{m^2\left(e_n^2(f) + |Z_{22}|^2\; i_n^2(f)\right)}{\frac{16\,k\,T\,m\,\omega_0}{Q}\;|Z_{21}|^2 + 4\;|Z_{12}|^2\;|Z_{21}|^2\; i_n^2(f)} \tag{15}$$

The minimum noise becomes

$$\Delta E_{min} = \left[\left(\frac{e_n^2(f) + |Z_{22}|^2\ i_n^2(f)}{|Z_{12}|^2}\right)\left(\frac{4\,k\,T\,m\,\omega_0}{Q} + |Z_{12}|^2\ i_n^2(f)\right)\right]^{1/2} \tag{16}$$

where the reciprocity condition has been used.

Equation (16) can be simplified if re expressed in terms of β (Eq. (2)) and if the amplifier is matched to the transducer $|Z_{match}| = |Z_{22}|$, (Eq. (4)).

$$\Delta E_{min} = \left[\left(\frac{8\ kT_{amp}}{\beta}\right)\left(\frac{4\ kT}{Q} + 4\ kT_{amp}\ \beta\right)\right]^{1/2} \tag{17}$$

The minimum excitation energy in the resonator is related to the minimum detectable gravitational pulse flux by

$$F_{min}(\nu_0) = \frac{\Delta E_{min}}{\sigma_T} \tag{18}$$

where, σ_T, the total absorption cross section for a longitudinal resonator averaged over all polarization, is given by (Misner, Thorne and Wheeler, 1973)

$$\sigma_T = \frac{32}{15\pi}\frac{G}{c^3}\ m\ c_s^2 \tag{19}$$

The initial rms strain in the resonator is

$$\frac{\Delta \ell}{\ell} \doteq \left(\frac{2\pi G}{c^3}\right)^{1/2} F^{1/2}(\nu_0) = \pi\left(\frac{15}{16}\right)^{1/2}\left(\frac{\Delta E_{min}}{Mc_s^2}\right)^{1/2} \tag{20}$$

Two limiting cases are important. The first is the case when the noise is dominated by the Nyquist forces so that the back reaction noise force can be neglected. This has been the case for all acoustic antennas constructed, including those now in operation and projected for the next several years. Assuming that the transducer is optimally matched to the amplifier, the optimum integration time becomes

$$t_{int}^2 = \frac{T_{amp}\ Q}{2\ T\ \beta\ \omega_0^2} \tag{21}$$

The minimum detectable flux and strain are

$$F(\nu) = \frac{15\pi\, c^3}{32\ G\ mc_s^2}\left(\frac{32\ kT_{amp}kT}{\beta Q}\right)^{1/2} \tag{22}$$

$$\frac{\Delta \ell}{\ell} = \pi\left(\frac{15}{16}\right)^{1/2} \frac{1}{(m\ c_s^2)^{1/2}}\left(\frac{32\ kT_{amp}\ kT}{\beta\ Q}\right)^{1/4}$$

Representative experimental parameters for this case are given in Table 2.

The other limiting case occurs when the back reaction noise dominates ($T \to 0$).

For this case the optimum integration time becomes

$$t_{int} = \frac{1}{\sqrt{2}\ \beta\omega_0} \tag{23}$$

and the minimum detectable energy change becomes

$$\Delta E = 4\ \sqrt{2}\ kT_{amp} \tag{24}$$

If the amplifier is quantum noise limited (Eq. 6) the minimum flux and strain are

$$F(\nu) \geq \frac{\sqrt{2}\ 15\pi}{(\ln 2)32}\ \frac{c^3}{G}\ \frac{\hbar\omega}{mc_s^2} \tag{25}$$

$$F_{GPU}(\nu) \geq 2.1 \times 10^{-7}\left(\frac{f}{kHz}\right)^{1/2} \frac{1}{\left(m/10^6\right)} \qquad \text{Aluminium}$$

$$\frac{\Delta \ell}{\ell} > \left(\frac{\sqrt{2}\ 15\ \pi^2}{\ln 2\ 16}\right)^{1/2}\left(\frac{\hbar\omega}{mc^2}\right)^{1/2}$$

$$\frac{\Delta \ell}{\ell} \geq 1.8 \times 10^{-20}\left(\frac{f}{kHz}\right)^{1/2} \frac{1}{\left(m/10^6\right)^{1/2}} \qquad \text{Aluminium}$$

Table 1. Useful Conversion Equations (cgs units).

$$\frac{c^3}{G} = 4 \times 10^{38} \text{ ergs/cm}^2\text{Hz} \qquad 1 \text{ GPU} = 1 \times 10^5 \text{ ergs/cm}^2\text{Hz}$$

IMPULSIVE SOURCES BANDWIDTH EQUAL TO FREQUENCY

$$F(\nu) = \frac{c^3}{2\pi G}\left(\frac{\Delta\ell}{\ell}\right)^2 = 6.4 \times 10^{37}\left(\frac{\Delta\ell}{\ell}\right)^2 \text{ ergs/cm}^2\text{Hz}$$

$$\frac{\Delta\ell}{\ell} = 4 \times 10^{-17}\left(\frac{F(\nu)}{\text{GPU}}\right)^{1/2}$$

IMPULSIVE MINIMUM RMS FLUX AND STRAIN IN TERMS OF SYSTEM EQUIVALENT TEMPERATURE (ALUMINIUM RESONATOR)

$$F_{GPU}(\nu) = 2.3\,\frac{T_{eff}}{\left(\frac{m}{10^6}\right)} \qquad \frac{\Delta\ell}{\ell} = 6 \times 10^{-17}\left(\frac{T_{eff}}{\left(\frac{m}{10^6}\right)}\right)^{1/2}$$

RELATION BETWEEN FLUX AND STRAIN AMPLITUDE IN A MONOCHROMATIC WAVE

$$\frac{\Delta\ell}{\ell} = \left(\frac{G}{2\pi c^3}\right)^{1/2}\frac{I_g^{1/2}}{f} = 2 \times 10^{-20}\,\frac{I_g^{1/2}}{f}$$

SPECTRAL DENSITY OF STRAIN

$$\left(\frac{\Delta\ell}{\ell}(f)\right)^2 = \frac{4}{\pi}\frac{G}{c^3}\frac{I_g(f)}{f^2} = 3.2 \times 10^{-39}\,\frac{I_g(f)}{f^2}\ \text{Hz}^{-1}$$

TYPE	f_o Hz	m gm $x10^6$	1 cm	Q	T °K	t_{obs} sec	$x^2(f)$ cm^2/Hz	$\lvert Z_{12}\rvert^2$ cgs	β	T_{eq} °K	F(ν) min GPU	$\frac{\Delta\ell}{\ell}$
Weber type cylindrical bar PZT transducer Tyson (1973)	710	3.6	357	$2.2x10^5$	300	0.7	$2.3x10^{-29}$	$1.6x10^{-2}$	$2x10^{-4}$	20°	15	$1.5x10^{-16}$
Split bar PZT transducer Hough et al. (1975) Drever et al. (1973)	1050 ±400	0.3	155	$6.6x10^3$	300	$6x10^{-4}$	$3x10^{-32}$	8	.18	4.5	34	$2.3x10^{-16}$
Cryogenic aluminium bar squid variable inductable transducer Boughn et al. (1977)	1315	.68	200	$6.6x10^4$	4.4	0.3				0.39	1.3	$4.5x10^{-17}$
Quadrupole Antenna thermally tuned to pulsar NP0532 DC capacitive transducer Hirakawa et al. 1978)	60.2	.078 effective	110 x 110	$4.5x10^3$	300		$1.2x10^{-24}$	$3.1x10^{-2}$	$1.4x10^{-3}$			$6x10^{-17}/Hz^{1/2}$

Table 2. Representative Characteristics of Acoustic Antennae

b) Periodic Sources

The analysis for the performance of acoustic resonators driven by a periodic gravitational wave follows a comparable prescription. The spectral density of the displacement noise is

$$x_n^2\,(f) = |T(f)|^2 \sum_{i=1}^{m} F_{n_i}^2(f) + \frac{1}{|Z_{21}|^2\omega^2}\left(e_n^2(f) + |Z_{21}|^2\, i_n^2(f)\right) \tag{26}$$

where the first term represents the sum of the stochastic forces on the resonator. $|T\ (f)|$ is the magnitude of the force to displacement transfer function of the resonator

$$|T(f)|^2 = \left|\frac{x(f)}{F(f)}\right|^2 = \left(m^2\left[(\omega_0^2 - \omega^2)^2 + \left(\frac{\omega\omega_0}{Q}\right)^2\right]\right)^{-1} \tag{27}$$

The spectral density of the gravitational gradient force due to the wave is

$$F_g^2(f) = \left(ml\ \omega^2\left(\frac{\Delta\ell}{\ell}\right)\right)^2 \delta(f - f_0) \tag{28}$$

The criterion for detection is that the signal power by greater than the noise

$$\int F_g^2(f)\ |T(f)|^2\ df > \int x_n^2(f)\ df \tag{29}$$

Considering only the case where the resonator is tuned to the frequency of the source and the transducer impedance is matched to the amplifier, the minimum detectable wave strain becomes

$$\frac{\Delta\ell}{\ell} \geq \left[\frac{1}{2t_{int}\ m\ \omega_0^3\ l^2}\left(\frac{4kT}{Q} + 4kT_{amp}\beta + \frac{8kT_{amp}}{\beta\ Q^2}\right)\right]^{1/2} \tag{30}$$

In this equation it is assumed that the Nyquist forces and back reaction forces are the only stochastic forces and that the bandwidth of the measurement is $\Delta f = 1/2\ t_{int}$ - where t_{int} is the total time of observation.

The demands on the transducer - amplifier combination are not as difficult to meet in detecting periodic sources as for impulsive ones. The interplay of the backreaction and broadband amplifier noise as seen in equation (30) is minimized when the transducer coupling is $\beta = \sqrt{2}/Q$, a lightly coupled transducer. Unlike the situation in detecting impulsive events, the strain limit continues to improve as $1/Q^{1/2}$ even when the amplifier noise dominates over the Nyquist noise.

At frequencies lower than 100 Hz the longitudinal resonators become impractically long - 50 meters at 60 Hz for aluminium, so that plate or mass loaded resonators have been used and are contemplated for future experiments. An example of present techniques is the room temperature experiment of Hirakawa et. al. (1978) (Table 2) which operates at a noise level of $\Delta\ell/\ell/\mathrm{Hz}^{1/2} \sim 6 \times 10^{-17}$ at 60.2 Hz, the Crab pulsar frequency.

One can contemplate quantum noise limited amplifiers for the detection of periodic sources. However, the optimal performance of the antenna will require that the physical temperature of the resonator be at the equivalent quantum temperature (Eq. 6); at 60.2 Hz this corresponds to 4×10^{-9} °K. Such low temperatures are not technically possible at present. Nevertheless, a single crystal resonator at 60 Hz with a Q of 10^9, an oscillating mass of 100 kg and length 1 meter could have a quantum noise limit $\Delta\ell/\ell/\mathrm{Hz}^{1/2} \sim 4 \times 10^{-25}$. A more realistic estimate for the next decade might be such an antenna operated at 0.1 °K which would have a limit of $\Delta\ell/\ell/\mathrm{Hz}^{1/2} \sim 2 \times 10^{-21}$.

c) Stochastic Gravitational Sources-Gravitational Radiometry

A detection scheme for a stochastic background of gravitational radiation requires the cross correlation of the output of at least two antennas, (Fig. 2). The premise is that the gravitational radiation noise signals are correlated in the antennas while the internal noise in each antenna due to other sources is uncorrelated and will average to zero in the cross correlation.

The cross correlation detection criterion is (Bendat, 1958)

$$\left(\frac{\Delta\ell}{\ell}(f)\right)^2_{grav} > \frac{\left(\Delta\ell/\ell\ (f)\right)^2_{internal}}{(\Delta f\ t_{int})^{1/2}} \tag{31}$$

where Δf is the bandwidth of the antenna system, t_{int} the post multiplication integration time, $(\Delta\ell/\ell(f))^2_{internal}$ is the spectral density of the internal antenna equivalent strain noise while $(\Delta\ell/\ell(f))^2_{grav}$ is the spectral density of the gravitational strain noise - the signal.

The signal to noise calculation is more complicated for stochastic sources than for the periodic and impulsive sources and therefore is carried out only approximately here.

With acoustic resonators the antenna will generally be followed by a filter to increase the detection bandwidth, (to suppress the resonance). The filter transfer function is

$$\left|\frac{V_{out}}{V_{in}}(f)\right|^2 = |S(f)|^2 .$$

A straightforward application of Eq. 31 gives the following detection criterion

$$\int_{\omega_1}^{\omega_1+\Delta\omega} \omega^4 \, |T(f)|^2 \, |S(f)|^2 \left(\frac{\Delta\ell}{\ell}(f)\right)^2_{grav} d\omega \geq$$

$$\frac{1}{m^2\ell^2\left(\Delta f t_{int}\right)^{1/2}}\left[\left(\frac{4\,kT\omega_0 m}{Q} + 4\,kT_{amp}\,\omega_0\,m\,\beta\right)\int |T(f)|^2 \, |S(f)|^2 \, d\omega \right.$$

$$\left. + \frac{8\,kT_{amp}\,m}{\beta\,\omega_0^2}\int \frac{|S(f)|^2}{\omega^2}\,d\omega\right] \tag{32}$$

Some special cases are interesting. If the filter doesn't exist, the antennas measure only the spectral density in a bandwidth $\Delta f \sim \omega_0/\pi Q$ around the resonance frequency. The minimum is then

$$\left(\frac{\Delta\ell}{\ell}(f_0)\right)^2_{grav} > \frac{1}{m\ell^2\,\omega_0^3\,(\Delta f\, t_{int})^{1/2}}\left[\frac{4kT}{Q} + 4\,kT_{amp}\beta + \frac{8\,kT_{amp}}{\beta Q^2}\right] \tag{33}$$

The optimization is as for a periodic source.

With a "whitening" filter which satisfies $|T(f)|^2 \, |S(f)|^2 = 1/m^2$ and a detection bandwidth small compared to the center frequency, the minimum becomes

$$\left(\frac{\Delta\ell}{\ell}(f)\right)^2_{grav} > \frac{\omega_0}{m\ell^2 \ \langle\omega\rangle^4 \ (\Delta f t_{int})^{1/2}} \left[\frac{4kT}{Q} + 4\ kT\beta_{amp} + \frac{8\ kT_{amp}}{\beta}\left\{\begin{array}{ll}\left(\frac{\omega_0}{\langle\omega\rangle}\right)^2 & \omega_0>\omega \\ \cdots\cdots & \\ \left(\frac{\langle\omega\rangle}{(\omega_0)}\right)^2 & \omega_0<\omega\end{array}\right\}\right] \qquad (34)$$

which places strong demands on the transducer-amplifier combination much as in the detection of impulsive sources.

The only experiment that has attempted to measure the gravitational background noise in a manner similar to that described is Hough et. al. (1975) using a pair of split bar resonators (Table 2.). This experiment set a limit of $I_g(f) \sim 6 \times 10^5$ ergs/sec cm^2 Hz in a 160 Hz bandwidth near 1 kHz after an integration time of 90 hours. $((\Delta\ell/\ell(f))^2 \sim 2 \times 10^{-39}/\text{Hz})$.

A prospect for the future might be a longitudinal cryogenic resonator dominated by quantum amplifier noise. Taking a mass of 5×10^6 gms, $\beta \sim 1$, bandwidth $\sim 1/10\ f_0$, $f_0 \sim 1$ kHz and integration time of 10^6 seconds Eq. (34) and (6) would give

$$\left(\frac{\Delta\ell}{\ell}(f)\right)^2_{grav} \sim 10^{-49}/\text{Hz}$$

SUMMARY ON ACOUSTIC ANTENNAS

In the next few years it is expected that cryogenic single crystal and massive aluminium bars will yield impulsive source sensitivities in the range 10^{-2} to 10^{-3} GPU (rms) corresponding to impulsive strain sensitivities $\sim 10^{-18}$ (rms). Further advances will require the development of higher β transducers which have small losses, this is particularly important for the high Q single crystals. The state of amplifiers is better than that of transducers, there are several regions in the frequency spectrum where amplifiers approach the quantum limit, in particular maser amplifiers at 10 GHz and Josephson junction devices at 30 GHz. It is primarily for this reason that modulated microwave cavity transducer appear attractive.

An improvement of 10^4 in energy sensitivity over the performance of detectors available in the next few years is required before approaching the quantum limit using linear amplifiers. Nevertheless, it is a crucial question whether such a quantum limit exists (see K. Thorne et.al. article at this conference) as the prospective source intensities for reasonable event rates (1/month rather than 1/30 years) and confidence in the detection will require sensitivities smaller than the quantum limit.

In the search for periodic signals acoustic resonators, specifically cryogenic high Q single crystal, can approach interesting sensitivities, as the demands on the transducer amplifier combination are not as stringent as for impulsive sources. The obvious candidate is the Crab pulsar.

ELECTROMAGNETICALLY COUPLED ANTENNAS

The fundamental idea of electromagnetically coupled antennas is to measure the gravitational wave strain between a group of free masses using electromagnetic waves to determine their separations. The major attribute of such antennas is that the baselines can be made large, comparable to the gravitational wavelength. The large baselines increase the gravitational wave signal relative to many noise sources which do not scale with the size of the system, most importantly the uncorrelated stochastic forces on the antenna masses. Free mass antennas are broad band and can extend the observation of the gravitational radiation to low frequencies.

The discussion that follows is limited to antennas in which the electromagnetic wave travel time between antenna masses is less than the period of the gravitational waves. The inverse case is discussed in a companion article by the JPL group on Doppler ranging to spacecraft (this volume).

A schematic diagram of an element of an electromagnetically coupled antenna is shown in Fig. 3. The two arm element is essential for cancelling the effect of some noise sources in particular the phase fluctuations of the electromagnetic oscillator and also enhances the interaction with the polarization of tensor gravitational waves which produce a differential mode signal in this configuration.

For a monochromatic wave at frequency f, the measurable strain is given by

$$\frac{\Delta \ell}{\ell}(f)_{meas} = \frac{\Delta \ell}{\ell}(f)_{wave} \frac{\sin(2\pi\, t_{stor}/\tau_g)}{(2\pi\, t_{stor}/\tau_g)} \tag{35}$$

where $\Delta\ell/\ell(f)_{wave}$ is assumed much less than 1. ℓ is the physical antenna baseline while t_{stor} is the storage time of the electromagnetic waves in the antenna and τ_g is the period of the gravitational wave. It is assumed that the gravitational waves propagate at c so that for a single pass antenna $\tau_{stor}/\tau_g = \ell/\lambda$.

The criterion for detection of a gravitational wave strain spectral density is given by:

$$\left(\frac{\Delta \ell}{\ell}(f)\right)^2_{min} = \frac{1}{4\ell^2}\left[x_T^2(f) + 2\sum_{i=1}^{n}\frac{F_i^2(f)}{m^2\omega^4} + N_\lambda^2(f)\, c^2\, t^2_{stor}\right] \frac{(2\pi\, t_{stor}/\tau_g)^2}{\sin^2(2\pi\, t_{stor}/\tau_g)} \tag{36}$$

$x_T^2(f)$ is the spectral density of the differential displacement detector noise, $F_i^2(f)$ the spectral density of one of the stochastic noise forces on an antenna mass m, $N_\lambda^2(f)$ is the spectral density of uncorrelated index of refraction fluctuations in the antenna arms averaged over the length of the arms at the wavelength of the electromagnetic oscillator.

The transducer noise comes from amplitude and phase noise of the electromagnetic oscillator and amplitude noise in the electromagnetic receiver and amplifier.

$$x_T^2(f) = \frac{\lambda^2}{b^2}\left[\underbrace{\frac{4h\nu}{\pi^2\,\eta P_d}}_{\text{shot noise}} + \underbrace{\frac{4\,kT_{eff}}{\pi^2\,\eta P_d}}_{\text{receiver noise}} + \underbrace{8\,\tau^2\,\delta}_{\text{phase noise}}\right] \tag{37}$$

In this expression, P_d is the power at the detector, η, the quantum efficiency of the detector; T_{eff}, the noise temperature of the receiver; b, the number of beams in the antenna arms; τ, the difference in time for the light to travel from the source to the detector in the two arms; and δ, the frequency width of the oscillator. The same expression applies whether the antenna is an interferometer or incorporated into a heterodyning scheme as in

Doppler ranging.

For fixed oscillator power, the minimum transducer noise occurs for an optimum number of beams given by

$$b = \frac{2}{1 - R}$$

where R is the reflectivity of the mirrors. P_d becomes $1/e^2$ of the oscillator power. It is worth noting that increasing b (folding the arms) decreases $x_T^2(f)$; however, it also increases the storage time. The optimum storage time is 1/2 the gravitational wave period. At those frequencies where the stochastic forces contribute more than the transducer noise, the folding is of no value and the only way to reduce the minimum detectable signal is to increase the antenna length. The folding can be thought of as means of matching the transducer noise to the stochastic forces.

The detectability criteria for the three classes of sources follow from Eq. (36). In detecting transient events the proper filter would be determined from Eq. (9) if the pulse spectrum is known. In principle this kind of antenna could observe the pulse shape providing there is enough signal power. In the calculations that follow the filter assumed will be nothing fancier than an integrator with a time constant equal to the length of the pulse

$$\frac{\Delta\ell}{\ell}_{\text{impulsive}} \geq \left[\left(\frac{\Delta\ell}{\ell}(f)\right)^2_{\min} \frac{1}{2t_{\text{pulse}}}\right]^{1/2} \qquad (38)$$

If the antenna length or storage time is optimized, the minimum detectable impulsive strain scales as $1/t_{\text{pulse}}^{3/2}$, up to the point where the stochastic forces dominate.

The detection criterion for periodic sources both known and unknown, as would be uncovered in a Fourier transformation of the antenna output, is given by

$$\frac{\Delta\ell}{\ell}_{\text{periodic}} \geq \left[\left(\frac{\Delta\ell}{\ell}(f)\right)^2_{\min} \frac{1}{2t_{\text{obs}}}\right]^{1/2} \qquad (39)$$

where t_{obs} is the length of an unapodized record.

Finally, the minimum detectable gravitational noise spectral density in a multi antenna cross correlation experiment similar to the one discussed under acoustic antennas is given by

$$\left(\frac{\Delta\ell}{\ell}(f)\right)^2_{grav} = \frac{\left(\frac{\Delta\ell}{\ell}(f)\right)^2_{min}}{(\Delta f\ t_{int})^{1/2}} \qquad (40)$$

The detectability conditions are best discussed through examples of various antenna systems

Ground Based Antennas

Preliminary work on ground based interferometric antennas is described by Weiss (1972), Moss et al. (1971), and Forward (1978). At present research groups at the Max-Planck Institute in Munich, Glasgow University and at M.I.T. are engaged in constructing equal arm optical Michelson interferometers that have the mirrors suspended in high vacuum tanks. The interferometer arms are multi pass cavities of 1 to 10 meters in length. The interferometers are illuminated by argon ion lasers. The transducer noise (Eq. 37) for these antennas is entirely due to photon shot noise, $x^2(f) \sim 10^{-32}$ cm^2/Hz using $P_{osc} \sim 1$ watt, $\lambda \sim 5000$ °A, R = 99.5%, $\eta \sim 1/2$. The arm lengths have to be maintained equal to 10^{-3} cm in order that the laser phase instability does not contribute.

The dominant stochastic forces are seismic noise and Nyquist forces from the suspension (Weiss, 1972). The unattenuated seismic noise at reasonable locations on the earth has a displacement spectrum of

$$x^2_{seismic}(f) \sim \frac{3\times10^{-14}}{f^4}\ \text{cm}^2/\text{Hz} \qquad f > 10\text{Hz}$$

A one axis suspension could in principle provide an isolation above its resonance frequency, f_o, of

$$\frac{x^2_{ant}(f)}{x^2_{seismic}(f)} \sim \left(\frac{f_o}{f}\right)^4$$

if f_o is 1/2 Hz the attenuated ground noise spectrum becomes

$$x^2_{seismic}(f) \sim \frac{2\times10^{-15}}{f^8}\ \text{cm}^2/\text{Hz}$$

which crosses the transducer noise at 145 Hz.

The Nyquist noise of the suspension above the resonance frequency at 300 °K with $Q \sim 10^5$ and antenna masses of $m \sim 20$ kg is

$$x^2_{\text{thermal}}(f) \sim \left(\frac{1}{m^2\,\omega^4}\right)\frac{4\,kTm\,\omega_0}{Q} \sim \frac{10^{-25}}{f^4} \quad cm^2/Hz$$

which crosses the transducer noise at about 60 Hz.

From this example it is clear that ground noise is the dominant problem in these antennas and cunning is needed in suspension design to provide seismic isolation without compromising the suspension Q. The near term prospects for these antennas operating above 300 Hz is a spectral strain sensitivity of $(\Delta\ell/\ell\,(f))^2 \sim 10^{-38}\ Hz^{-1}$.

A larger ground based version of such an antenna, although expensive, is a distinct possibility. To maintain the same detected power, the optics must scale in linear dimension as $\sqrt{\lambda\ell}$. The crossing point of the uncorrelated ground noise and Nyquist noise with the transducer noise is independent of the baseline. For example, a 10 km antenna with 15 passes per arm or a 1 km antenna with 150 passes would have an optimal storage time for 1 m sec pulses. The minimum detectable spectral density becomes $(\Delta\ell/\ell\ (f))^2 = 10^{-42}\ Hz^{-1}$ which corresponds to an rms impulsive strain sensitivity of $\Delta\ell/\ell \sim 2 \times 10^{-20}$ for millisecond pulses using a 1 watt laser. This limit could be reduced by another factor of 10 when 100 watt argon ion lasers become available in the near future.

If the ground noise isolation can be improved substantially with active isolation systems, such an antenna could measure the Crab pulsar with a strain sensitivity of $\Delta\ell/\ell \sim 2 \times 10^{-25}/\tau^{1/2}$ (days).

Space Antennas

Electromagnetically coupled antennas operated in space offer very large baselines and freedom from seismic noise; extending measurements of the gravitational radiation spectrum to frequencies lower than 10 Hz.

The microwave Doppler ranging experiments using the NASA deep space network described in the accompanying article by the JPL group, are a first step in space gravitational astronomy. In the near future the limiting noise in these experiments is due to the fluctuations in electromagnetic propagation through the solar plasma at microwave frequencies. The noise power of the propagation fluctuations varies as $1/\omega^4$ where ω is the electromagnetic frequency. The strong frequency dependence of the propagation noise as well as the smaller beam sizes possible with shorter wavelengths for fixed antenna dimensions argues for the eventual use of optical frequencies in space antenna systems even though the space qualified hardware is not now available.

Several optical space antennas have been suggested (Weiss et al., 1976). One design consists of masses loosely suspended to a large cubical frame. The masses mounted at the corners of the cube are the mirror mounts for 12 interferometers, a pair on each face. Peter Bender suggested the use of a frame to solve the problem of launching and maintaining the spacing of the antenna masses. The suspension of the masses relative to the frame is critical, they must isolate the masses from the thermal noise of the frame itself. At present cubical frames as large as 10 km on a side are being contemplated by the large space structures engineering group at NASA. For frames of this size the estimated stochastic forces of the solar radiation pressure, solar wind, interplanetary dust and cosmic ray background (Fig. 4) dominate over the thermal noise of the frame at $f < 1/100$ Hz.

Although the stochastic forces of the solar radiation pressure, solar wind and interplanetary dust are orders of magnitude larger than the cosmic ray background, they can in principle be shielded, while the penetrating cosmic ray proton flux cannot.

Assuming a 100 pass interferometer using a 1 watt laser and 10^6 gm antenna masses driven by the stochastic forces of the cosmic ray protons ($F^2(f) \sim 5 \times 10^{-25}$ dynes^2/Hz); the minimum detectable gravitational strain spectral density is

$$\left(\frac{\Delta\ell}{\ell}(f)\right)^2 \begin{cases} 10^{-44}\ \mathrm{Hz}^{-1} & 10^{-2} < f < 10^{2}\ \mathrm{Hz} \\ \dfrac{10^{-52}}{f^4}\ \mathrm{Hz}^{-1} & f < 10^{-2}\ \mathrm{Hz} \end{cases}$$

In this particular example a possible fundamental noise not considered in the general treatment of electromagnetically coupled antennas may become important, namely the recoil noise of the masses in the laser field. If the light beams in the two interferometer arms can be thought of as being uncorrelated then there is an additional stochastic force on the masses given by

$$F^2(f) = \frac{4\ b^2\ h\ P}{\lambda c} \qquad (41)$$

which in this example is several 100 times larger than the proton noise.

There is a lively but unpublished controversy whether this noise term actually exists in a balanced interferometer system.

A second design uses free flying masses in drag free shielded satellites. The control of the position of the interferometer arms against the gravitational gradient forces becomes a major concern.

The near equality of the interferometer arm path lengths must be maintained so that the phase noise of the laser remains smaller than shot noise. Using 1 watt as a typical laser power, the path length difference must be maintained to 10^{-2} cm if the laser linewidth is 100 kHz, typical of frequency unstabilized lasers. In any engineering planning of such an antenna the trade off between short term frequency control of the laser and precision station keeping of the masses will become a major factor in the design.

Baselines as large as 1000 km for a single pass interferometer may still be practical. Again assuming 10^6 gm masses and only the proton flux, the minimum detectable gravitational strain spectral density is

$$\left(\frac{\Delta\ell}{\ell}(f)\right)^2 \begin{cases} 10^{-44}\ \mathrm{Hz}^{-1} & 10^{-3} < f < 10^{2}\ \mathrm{Hz} \\ \dfrac{10^{-56}}{f^4}\ \mathrm{Hz}^{-1} & f < 10^{-3}\ \mathrm{Hz} \end{cases}$$

To further reduce the effect of the stochastic noise forces at low frequencies even larger baselines are necessary. Optical Doppler ranging (optical heterodyning) becomes an attractive option. Such a scheme could involve 3 drag free spacecraft placed at the earth-moon or earth-sun Lagrange points (Bender et al., 1979). The central spacecraft has a laser which transmits simultaneously to the other two spacecrafts equipped with laser transponders. The returned signals are beat against each other. The dominant noise is due to the finite power returned. Applying equation (37), the transducer noise is

$$\left(\frac{\Delta\ell}{\ell}(f)\right)^2 = \frac{\lambda^4}{d_1^2\, d_2^2}\left[\frac{h\nu}{\eta P_2} + \frac{h\nu}{\eta P_1}\right] \qquad (42)$$

where λ is the laser wavelength, d_1 and d_2 are the diameters of the telescope mirrors on the central spacecraft and the transponding spacecraft, P_1 and P_2 are the transmitted powers of the laser on the central spacecraft and the transponder. Taking:

$\lambda = 5 \times 10^{-5}$ cm, $d_1 = 2$ meters $d_2 = 1$ meter, P_1 and $P_2 \sim 1$ watt, $\eta = 1/2$, the transducer noise becomes

$$\left(\frac{\Delta \ell}{\ell}(f)\right)^2 \sim 10^{-44}\ \mathrm{Hz}^{-1}$$

Assuming the laser frequency width, δ, is 10^5 Hz the path lengths to the transponding spacecraft must be equal to 1/100 km so that the laser phase noise remains less than the amplitude noise. The estimated proton noise is of no consequence at frequencies larger than 1/day for 10^6 gm masses placed at separations as large as the earth-sun Lagrange points.

SUMMARY OF ELECTROMAGNETIC ANTENNAS

In the next few years several short baseline interferometers will come into operation. The estimated sensitivities of these antennas for detecting impulsive sources will be comparable with the acoustic resonators being developed concurrently, but the intereferometric antennas will have broader bandwidths. These antennas will be able to set new upper limits on the spectral density of gravitational radiation from continuous and periodic sources but not at a level which is expected to be astrophysically interesting (there may always be surprises). The development of these antennas is a means of testing the noise models and is an essential step for the planning of larger baseline systems both on the ground and in space.

A thorough analysis of large baseline systems on the ground and in space has not been carried out. Current thinking is that large baseline antennas on the ground using high power lasers hold the promise of astrophysically interesting sensitivities in the frequency range above 100 Hz with the application of straightforward engineering. The most uncertain factor is the low frequency performance, f smaller than 100 Hz, which depends on the success of ground noise isolation systems that can be more elaborate than in the example sited; for example multiple suspension systems and active isolation systems. The geophysical noise from atmospheric density fluctuations and both man made and naturally occurring gravitational gradients has not been studied sufficiently.

Electromagnetically coupled antennas in space are at present the best strategy for measuring the gravitational wave spectrum at periods longer than a hundred seconds. In the short term, multi frequency microwave Doppler ranging experiments are the only possible candidates with present day space technology. A detailed analysis of Doppler ranging systems applied to the detection of gravitational wave bursts is needed.

Optical systems in space are more promising, however, the necessary space technology is far from being developed. The noise calculations need considerably more work than is presented in this article, in particular the engineering questions associated with the shielding and the effect of gravitational gradients due to the planets as well as smaller space objects have to be studied.

Figures 5, 6 and 7 are a summary of the status and prospects of the performance of various antenna systems as applied to the search for impulsive, periodic and stochastic gravitational wave sources. The projections in the figures should be viewed in the spirit of educated guesses. The quantity h, the dimensionless gravitational wave amplitude, is twice the strain. All the figures are in terms of equivalent rms noise using the filtering techniques presented in the text. The projections of antenna performance for the detection of periodic and stochastic sources can be directly compared with the source predictions of the second round table of this conference. In estimating the detection probability of an impulsive source, using the results of Fig. 5, the effect of the pulse duty cycle must be included. If the noise in the antenna has a Gaussian distribution, the minimum impulsive h detectable becomes $h(\text{rms}) \times \ell n\,(Rt_p)^{-1}$, where R is the event rate and t_p the pulse length.

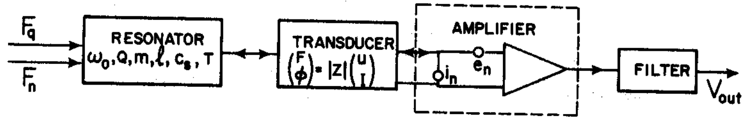

Figure 1. Basic components of an acoustically coupled antenna.

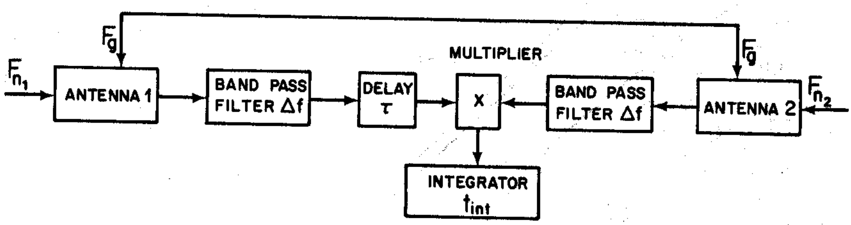

Figure 2. Schematic of a cross correlation experiment.

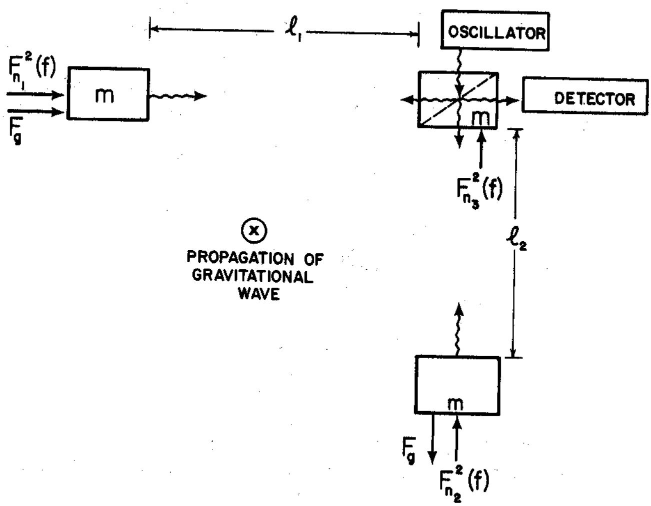

Figure 3. Schematic diagram of an element of an electromagnetically coupled antenna.

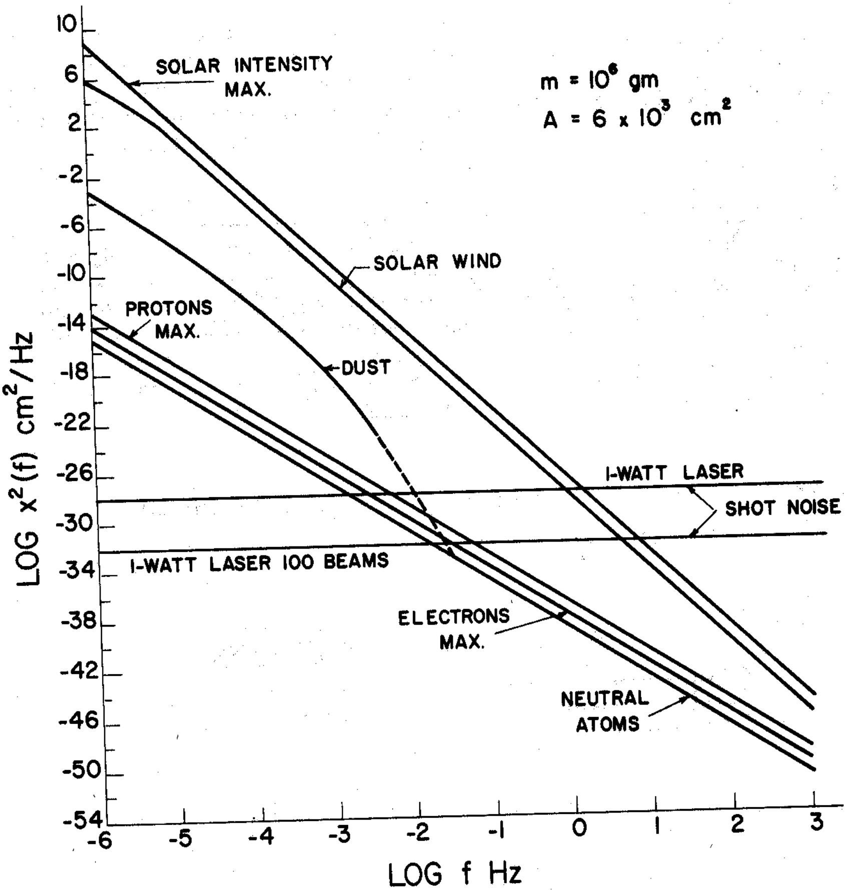

Figure 4. Estimates of displacement spectral density due to various stochastic forces in a space environment.

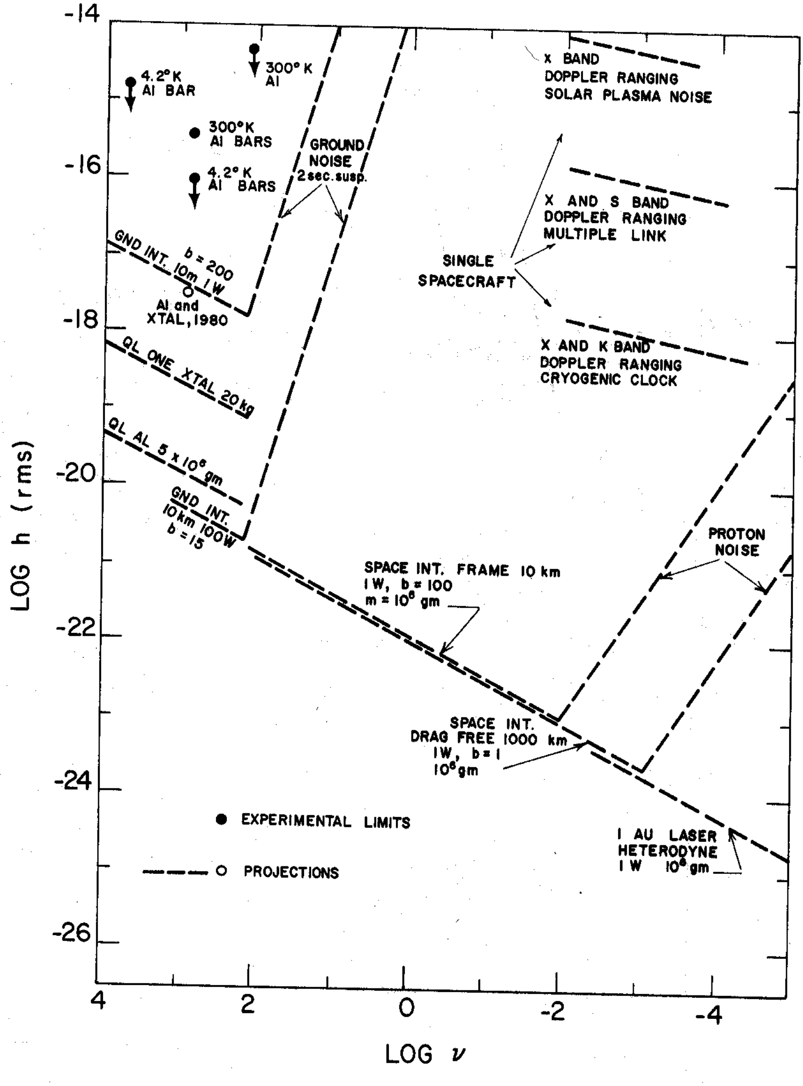

Figure 5. Experimental limits and projections for antenna performance in detecting impulsive sources. The figure presents rms values. To convert to detection probability rms values must be multiplied by the logarithmic duty cycle factor described in the text.

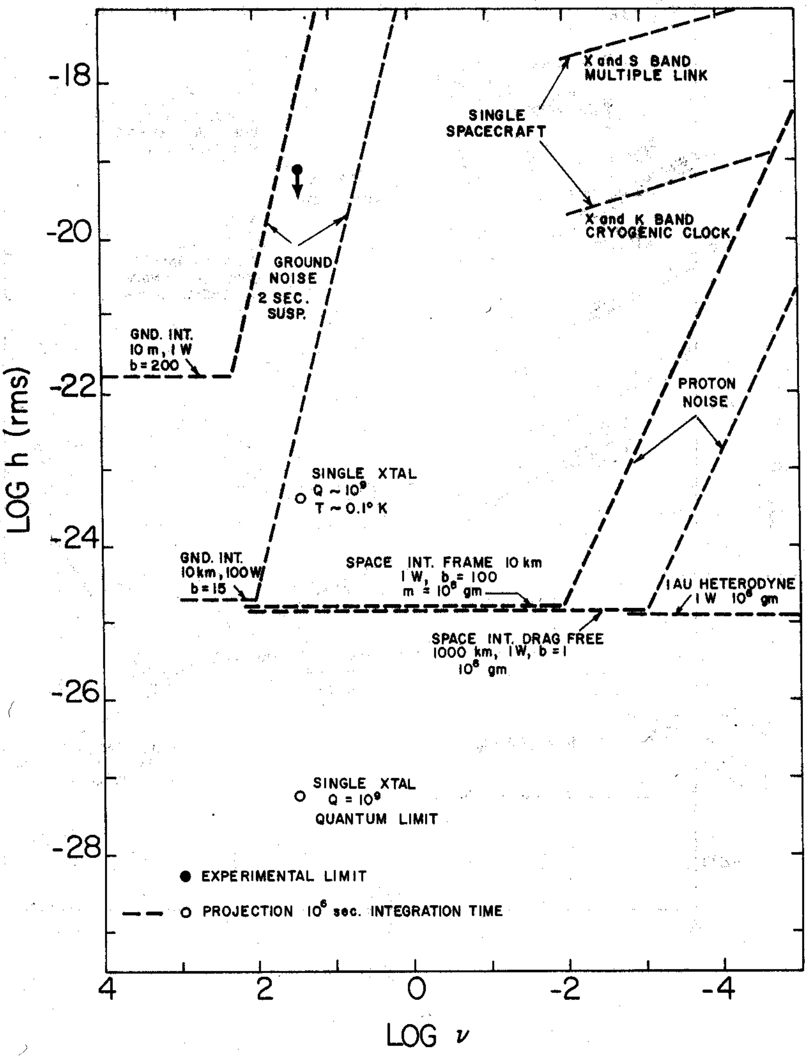

Figure 6. Experimental limits and projections for antenna performance in detecting periodic sources with an integration time of 10^6 seconds.

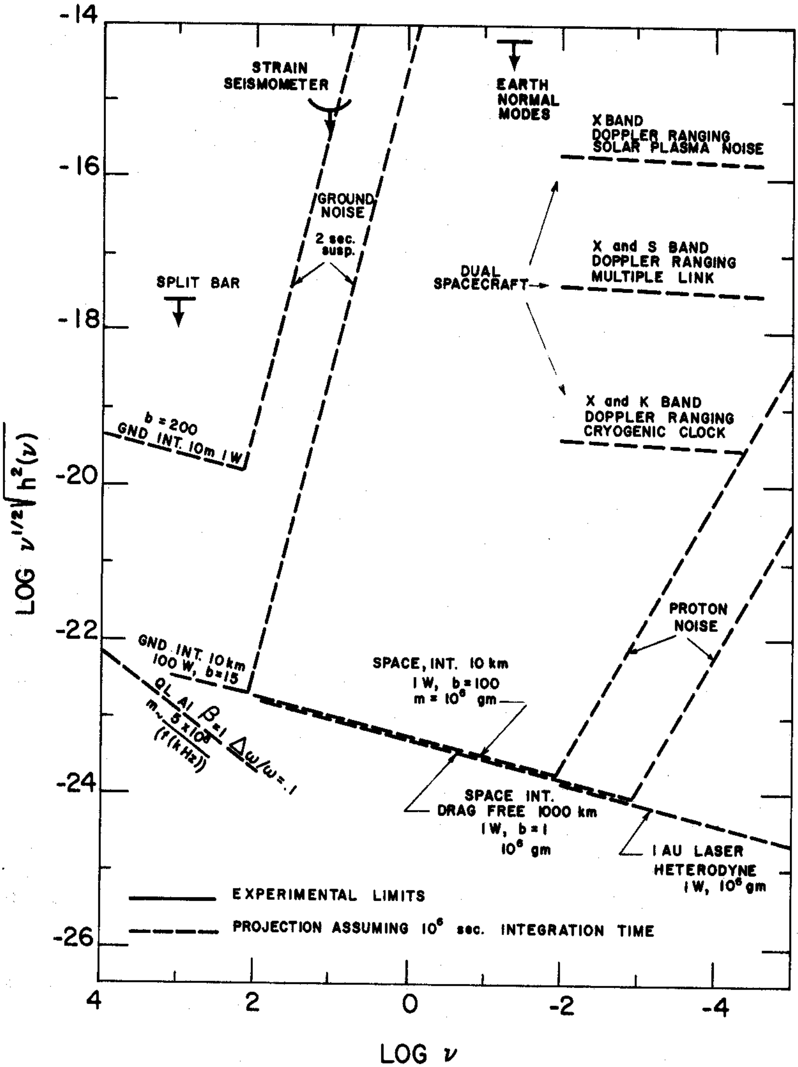

Figure 7. Experimental limits and projections for antenna performance in detecting a stochastic background using a bandwidth equal to the frequency and an integration time of 10^6 seconds.

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