APPENDIX A

ACOUSTIC AND ELECTROMAGNETICALLY COUPLED ANTENNAE IN THE NAIVE QUANTUM LIMIT

Acoustic antennae are now operating at noise levels two orders of magnitude in amplitude and correspondingly four orders of magnitude in power above the naive quantum limit. The limit is set by a straightforward application of quantum mechanics to the interaction of the transducer and associated amplifier with the acoustic resonator. The opinion held by some working in the quantum theory of measurement is that in principle the naive quantum limit need not impose a firm limit to the detection of gravitational radiation by acoustic resonators (or electromagnetically coupled antennae). Transducer schemes have been proposed which could circumvent the naive quantum limit (quantum non-demolition systems), however they appear difficult to implement and the gain in sensitivity over the naive quantum limit is unfortunately a strong inverse function of the power losses in these schemes. It is in our opinion a fair assumption that even if the naive quantum limit is not a limit it will be difficult to make much progress to get below it. For the sake of the ensuing calculations, we assume that the naive quantum limit is a real limit.

Present day prototype interferometric antennae have not yet approached the present performance of acoustic antennae. Therefore it may seem silly at this stage to contemplate their quantum limited performance. However, in discussing the ultimate idealized performance of either system, the quantum limit is a hard boundary, and in principle getting to it is a matter of technical improvement.

The result of this section is that the ratio of the quantum noise limited strain sensitivity of an idealized electromagnetically coupled antenna to that of an idealized acoustically coupled antenna of the same mass is close to the ratio of the velocity of sound in the acoustic antenna to the velocity of light, a factor of the order of 10^5. The quantum limit of a 1 ton acoustic antenna is of the order of a few times 10^{-23} strain/$\mathrm{Hz}^{1/2}$, while the quantum limit of an interferometric antenna with 1 ton end masses and of optimal length is a few times 10^{-28} strain/$\mathrm{Hz}^{1/2}$. A perusal of the section of this report on the noise sources in the electromagnetic antenna indicates that attaining such a sensitivity would pose a formidable technical challenge. At 1 KHz, but not at lower frequencies, the light power required is beyond reason. However, it is worth noting that a long baseline electromagnetically coupled antenna using present day engineering practice will perform close to an order of magnitude better in amplitude sensitivity than the quantum limited acoustic antenna and furthermore there is about a factor 10^4 margin before one hits the fundamental limit.

Naive Quantum Limit of an Idealized Acoustic Antenna

The treatment follows the presentation given by Weiss in Sources of Gravitational Radiation, L. Smarr, Editor, Cambridge Univ. Press, 1979. The gravitational wave tidal force density acting on the acoustic antenna of mass m and length ℓ is given by

$$F_g^2(f) = \left(\frac{m\ell\omega^2 h(f)}{2}\right)^2 \qquad (1)$$

where h(f) is the gravitational wave strain amplitude density and ω the frequency. The motion transducer attached to the acoustic antenna is described by the matrix

$$\begin{pmatrix} F \\ \phi \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} u \\ I \end{pmatrix} \qquad \underset{\text{reciprocity}}{|Z_{12}| = |Z_{21}|} \tag{2}$$

where F is the force exerted by the transducer on the antenna, u the velocity of the antenna motions at the transducer, I the current running through and ϕ the voltage across the transducer.

A useful quantity defining the transducer is the ratio of the electromagnetic energy stored in the transducer to the mechanical energy stored in the acoustic resonator. This is given by

$$\beta = \frac{|Z_{21}|^2}{|Z_{22}| \; m\omega} \tag{3}$$

Another useful quantity is the position sensitivity of the transducer

$$\alpha = |Z_{21}|\omega \quad \text{volts/cm} \tag{4}$$

The noise in the amplifier that follows the transducer is characterized by a series voltage noise generator $e_n^2(f)$, and a shunt current noise generator, $i_n^2(f)$. When the transducer is matched to the amplifier, the noise power from these two noise generators become equal.

Z_{22}, the transducer electrical impedence, should be $\ell_n(f)/i_n(f)$. Under these circumstances the noise in the amplifier, if it is quantum limited, (limited by spontaneous emission at the input,) is given by

$$\ell_n(f) i_n(f) = \frac{\hbar\omega}{\ell n 2} \tag{5}$$

The amplifier noise sets the displacement noise density to

$$x^2(f) = \frac{2|Z_{22}| e_n(f) i_n(f)}{\alpha^2} \quad \frac{2\hbar}{m\beta \ell n 2 \omega^2} \tag{6}$$

and the shunt current noise generator drives the acoustic resonator through the transducer with a back reaction force density given by

$$F^2_{br}(f) = \frac{\beta\hbar\omega^2 m}{\ell n 2} \tag{7}$$

The gravitational strain sensitivity limit is derived from the condition that the gravitational tidal force density should have a larger effect than the stochastic force densities and the displacement noise density

$$F^2_{grav}(f) \geq F^2_{br}(f) + \frac{x^2(f)}{|T(\omega)|^2} + F^2_{th}(f) \tag{8}$$

where $|T(\omega)|$ is the magnitude of the acoustic resonator displacement to force transfer function given by

$$|T(\omega)|^2 = \frac{1}{m^2[(\omega^2-\omega_o^2)^2 + (\frac{\omega\omega_o}{Q})^2]} \tag{9}$$

$F_{th}^2(f)$ is the thermal Nyquist force given by

$$F_{th}^2(f) = \frac{4kTm\omega_o}{Q} \qquad (10)$$

Q is the quality factor of the resonator and ω_o its resonance frequency in the observed mode.

The limiting gravitational strain amplitude densities are for three frequency regimes.

$$h(f) \quad \frac{2}{\ell\omega}\left(\frac{\hbar}{\ell n 2m}\right)^{\frac{1}{2}} \rightarrow \begin{cases} \left[\beta + \frac{2}{\beta}\left(\frac{\omega_o}{\omega}\right)^4 + \frac{4kT\ell n2}{h\omega_o Q}\right]^{\frac{1}{2}} & \omega<\omega_o \\ \left[\beta + \frac{2}{\beta Q^2} + \frac{4kT\ell n2}{h\omega_o Q}\right]^{\frac{1}{2}} & \omega=\omega_o \\ \left[\beta + \frac{2}{\beta} + \frac{4kT\ell n2}{h\omega_o Q}\right] & \omega>\omega_o \end{cases} \qquad (11)$$

Except for the case directly on resonance, it is best to make the transducer coupling as strong as possible $\beta \sim 1$. The thermal noise term is then negligible. On resonance the best value for β is approximately $\sim 1/Q$ if the thermal noise can be made small enough.

The first factor of Eq. 11 may be reformulated since the length of the resonator and its resonance frequency are related by the sound speed c_s as

$$\ell = \frac{\pi c_s}{\omega} \qquad (12)$$

The quantum limit of the general case, near resonance, is then given as

To make ... AT ω_o

$\frac{KT \; \ell n2}{h\omega_o Q} < \beta$ $\quad \omega_o = 10^4$, $\beta = 1$ $\quad \frac{1}{Q} < \beta \frac{h\omega_o}{KT \ell n2}$

$h(f) \sim \frac{2}{\pi c_s}$

$$h(f) \geq \frac{2}{\pi c_s} \left(\frac{\hbar}{m \ell n 2}\right)^{\frac{1}{2}} \qquad (13)$$

which corresponds to $h(f) \sim 3.8 \times 10^{-23}$ strain/$Hz^{\frac{1}{2}}$ for a $m = 10^6$ grams made of aluminum for which $c_s = 6.4 \times 10^5$ cm/sec. The directly on resonance case, useful for periodic sources with a very well defined frequency, could be better by a factor of $1/\sqrt{Q}$ if the thermal noise could be eliminated by cooling to a temperature $T < \frac{\omega_o}{k} \sim 6 \times 10^{-8}$ °K at 1KHz.

Naive Quantum Limit of an Electromagnetically Coupled Antenna

The reasoning is analogous to the acoustic antenna case. The strain spectral density when limited only by the photon counting statistics is given by

$$h^2(f) = \left(\frac{1}{8\pi c t_{st}}\right)^2 \left(\frac{hc\lambda}{\eta P}\right) \qquad (14)$$

The uncorrelated photon recoil fluctuation force on the masses plays the same role as the back reaction force in the acoustic antenna. The strain spectral density due to this force is

$$h^2(f) = 8\left(\frac{c t_{st}}{\ell^2 m \omega^2}\right)^2 \left(\frac{hP}{\lambda c}\right) \qquad (15)$$

where m is the mass of one antenna mass. The gravitational strain spectral density must be larger than the sum of 14 and 15

$$h_g^2(f) > \left(\frac{1}{8\pi c t_{st}}\right)^2 \left(\frac{hc\lambda}{\eta P}\right) + 8\left(\frac{c t_{st}}{\ell^2 m \omega^2}\right)^2 \left(\frac{hP}{\lambda c}\right) \qquad (16)$$

The minimum occurs when the two terms of Eq. 16 are equal. Setting a condition on the light power, P_{opt}, given by,

$$P_{opt} = \frac{\lambda c m \ell^2 \omega^2}{\sqrt{2}\, 16\pi \eta^{1/2} (c t_{st})^2} \tag{17}$$

For this power the amplitude spectral density is given by

$$h(f) > \frac{1}{\ell\omega} \left(\frac{\hbar}{\sqrt{2}\, \eta^{1/2} m} \right)^{1/2} \tag{18}$$

If the gravity antenna length is optimized,

$\ell = \pi c/\omega$

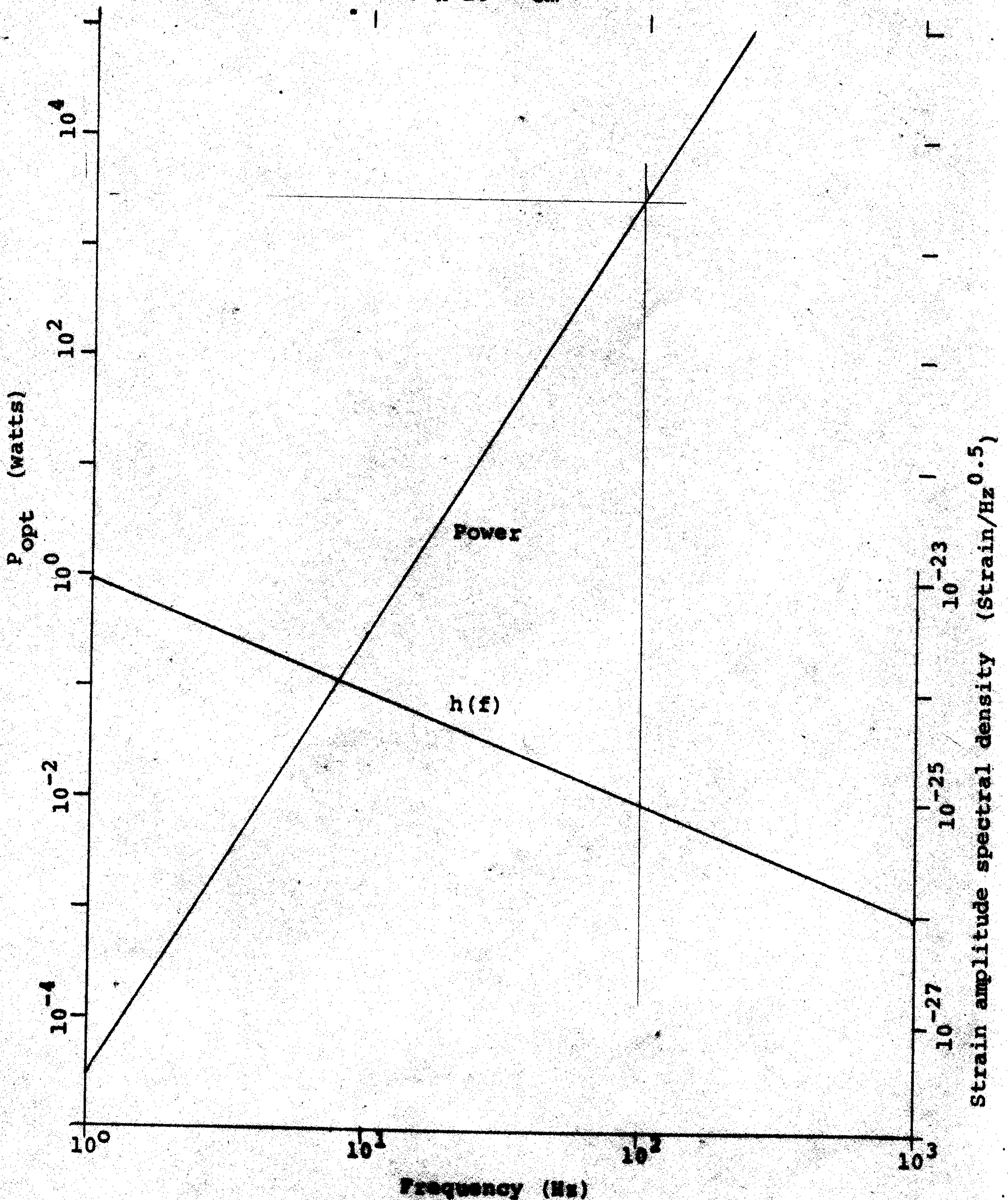
h(f) becomes

$$h(f) > \frac{1}{\pi c} \left(\frac{\hbar}{2\, \eta^{1/2} m} \right)^{1/2} \tag{19}$$

The parameters for an antenna optimized at 1KHz are not realistic. The optimum antenna would have a length of 150 km and with an end mass of 10^6 gm, the limiting $h(f) \sim 3 \times 10^{-28}$ strain/$Hz^{1/2}$. The circulating optical power required in the antenna, if $\lambda \sim 5 \times 10^{-5}$ cm, is 8×10^{10} watts!

The figure shows the optimum optical power required and the strain amplitude spectral density at this power for a quantum limited 5km long antenna in which the storage time is maintained at 1/f. The antenna end masses are 10^6 gms. At mid frequencies $\sim$100 Hz, the quantum limit could be reached in second generation improvements of such a system.

Quantum limited performance of a 5 km antenna
with $t_{stor} = 1/f$
$m = 10^6$ grams
$\lambda = 5 \times 10^{-5}$ cm



APPENDIX B

Why a Fiber Optic Antenna Will Not Suffice

When one compares the cost of the large evacuated enclosure that is required for an optical interferometer gravity wave antenna with the cost of the fiber optic links that now are used routinely for communication tasks, it seems possible that the fabrication costs, and, to some extent, the operational cost also, might be reduced significantly if single mode optical fibers could be used as the interferometer arms of the antenna. We have studied this superficially attractive alternative at some length.

We conclude that there are several compelling arguments why a fiber optic system will not function adequately as a gravity wave antenna. The arguments will be evaluated in this section.

In summary, there are three major intrinsic problems:

A. The power handling capability of single mode optical fibers is limited by intensity damage. Presently available fibers would limit the power to a level that is some 100 times lower than that required for successful operation of the antenna. In principle, this restriction could be ameliorated by increasing the diameter of the core, or by using multiple fibers.

B. The power handling capability is limited even more severely by nonlinear effects caused by stimulated Raman and Brillouin scattering processes. The power level that can be handled in a long, low loss, single mode, fiber is at least 10^5 times lower than the required power level. It is not clear that there is an available solution to this intrinsic problem.

C. The thermal noise occasioned by length and index of refraction fluctuations in a single mode optical fiber at room temperature typically is $\sim 10^8$ larger in amplitude than our total allowable system noise, and cooling the fiber to liquid helium temperature only would reduce this thermal noise by a factor of 10. Again, it is not clear that there is an available solution to this intrinsic problem.

In addition to the preceding intrinsic problems, optical fibers are susceptible to external influences such as mechanical strain, temperature gradients, and magnetic fields. The effect of these external driving forces would be significant at the noise levels of interest in a gravity wave antenna. In principle, the extrinsic effects probably could be attenuated to acceptable levels by careful engineering, but the cost of supporting optical fibers without mechanical strain in a temperature stabilized and magnetically shielded environment would largely negate the cost saving originally postulated for the fiber optic antenna, and thus would defeat the purpose.

Limitation due to damage threshold of optical fibers.

The radius of the core of a conventional single mode optical fiber is about equal to the wavelength of the light that is to be transmitted and, because of this small size, a modest propagating power can create an internal power density that is sufficient to cause physical damage to the fiber.

A typical power density damage threshold for available fibers is

$I_{max} \sim 10^8 \, w/cm^2$

Thus, for a fiber with a core radius about equal to the wavelength of the green light that is preferred in a laser interferometer, that is about 5000A°, physical damage will be significant for an injected light power P of about 1 watt.

As shown elsewhere in this report, the laser power needed to achieve a satisfactory signal/noise ratio in a gravity wave antenna is about 100 watts. So the damage threshold problem, alone, immediately precludes the use of conventional optical fibers as the interferometer arms of the antenna.

Limitation due to nonlinear scattering processes

Stimulated Raman and Brillouin scattering processes limit the power handling capability of optical fibers even more severely than does the physical damage threshold.

Raman scattering is a parametric photon conversion process that converts some of the energy supplied by the laser to a lower frequency.

At large power densities in a long, low loss, fiber the pump wave will leave a large enough time dependent polarization along the length of the fiber that the process can have parametric gain, resulting in both forward and backward propagation of a new wave at a frequency lower than that of the pump.

Using a criterion that the amplitude of the converted wave should everywhere be less than the amplitude of the pump, Smith shows that a good approximation to the maximum allowable input power is given by

$$P_{max} \sim 16\ A\ \alpha/\gamma_R .$$

where A is the fiber cross sectional area (cm^2)

α is the fiber attenuation constant (cm^{-1})

γ_R is the Raman conversion gain coefficient

$\sim 5 x 10^{-11}$ cm/w for amorphous glasses.

So, using $A = 7 x 10^{-9}\ cm^2$, and assuming an attenuation constant of 10^{-5} (4.3 db/km), the maximum allowable input power that avoids nonlinear effects caused by stimulated Raman scattering is 20 mw.

Stimulated Brillouin scattering is an elastooptic process. The electric fields associated with the forward travelling injected wave cause a spatial variation in the refractive index of the fiber, with an acoustic wavelength along the fiber equal to the wavelength of the injected light in the medium. The result is to form a diffraction grating which moves along the fiber at the velocity of sound in the medium. Interaction of the injected wave with this moving diffraction grating gives rise to a reflected backward travelling wave. This conversion process, also, can have parametric gain.

Using the criterion that the backward wave power should not exceed the input pump power at the injection face of the fiber, Smith shows that the maximum allowable input power is given approximately by

$$P_{max} \sim 21\ A\ \alpha\ /\ \gamma_B$$

where γ_B is the Brillouin gain coefficient

$\sim 3\ 10^{-9}$ cm/w

So, for the same fiber considered earlier, the maximum allowable input power that avoids nonlinear effects caused by stimulated Brillouin scattering is about 1 mw.

Comparing this limitation with the 100 watts that is needed to achieve an adequate signal to noise ratio in the laser interferometer, it is clear that fiber optics would be a candidate only if there is a major advance in the technology. Such an advance cannot be predicted at this time.

Intrinsic thermal noise in an optical fiber.

Thermal noise directly causes fluctuations in both the length and the density of the fiber, and, due to elastooptic coupling in the material, indirectly causes fluctuations in the refraction indices.

To evaluate the effect of thermal noise, we assume a one dimensional system, estimate the number of modes per frequency interval and obtain the thermal energy per unit length. We then calculate the length change per mode and show that the mode spacing is so close that it is reasonable to treat the problem as a continuum calculation to obtain an equivalent spectral density of displacement noise given by

$$X^2(f) = \frac{kT}{2\rho c_s A f^2} \ \mathrm{cm}^2/\mathrm{Hz}$$

where k is Boltzmann's constant = 1.38×10^{-16} ergs/°K

T is temperature (deg K)

ρ is the density of the fiber material $\sim 2\ g/cm^3$

c_s is the velocity of sound in the medium $\sim 5 \times 10^5$ cm/s

A is the cross sectional area of the fiber $\sim 10^{-8}\ cm^2$

f is the signal frequency (Hz)

For a typical single mode fiber at visible wavelengths and room temperature, this yields

$$x(f) \sim \frac{10^{-6} cm/Hz^{\frac{1}{2}}}{f}$$

At a 1 kHz signal frequency and a 10 km antenna length, the strain noise is

$$h\ (f) = \frac{X(f)}{\ell} \sim 10^{-15} strain/Hz^{\frac{1}{2}}$$

BRILLOUIN GAIN

$$G = \frac{2\pi n^7 P_{12}^2 K}{c \lambda^2 \rho_0 V_a \Delta\nu_B} \frac{P_L}{A} \Big/ cm$$

V_a ACOUSTIC VELOCITY

n = INDEX OF REFRACTION

K = 1 POLARIZATION PRESERVING

$\frac{1}{2}$ RANDOM

$\Delta\nu_B$ BRILLOUIN WIDTH

REFERENCES

R.G. Smith, Optical Power Handling Capacity of Low Loss Optical Fibers as Determined by Stimulated Raman and Brillouin Scattering, App. Optics, 11, 2489 (1972)