If we assume v_n to be zero and v_s is negligibly small before entering the plug, (2) may be integrated along its length to obtain the steady-state solution

$$\frac{1}{2}\frac{\rho_s}{\rho}v_s^2 = S\Delta T - \frac{\Delta P}{\rho} \tag{3}$$

using the fact that $\rho_n + \rho_s = \rho$ and $\partial v_s/\partial t = 0$.

The rate of heat withdrawal from the plug is $\dot{Q} \simeq L\dot{m}$. (The exact expression includes the term $TS\dot{m}$, but this is small compared to $L\dot{m}$.) If the cross-sectional area of the solid portion of the plug, A', is several orders of magnitude larger than the channel cross-sectional area, A (see description of plug below), we may assume that the heat flow is dominated by the thermal conductance of the metal, namely, kA'/l. Under these conditions, $\dot{Q} = C\Delta T$ and hence,

$$\Delta T = \frac{L\dot{m}}{C} \tag{4}$$

Also, v_s may be expressed in terms of $\dot{m}$ as

$$\rho_s v_s A = \dot{m} \tag{5}$$

Substituting (4) and (5) into (3) gives the relation

$$\frac{\dot{m}^2}{2A^2\rho_s} = \frac{\rho S L\dot{m}}{C} - \Delta P \tag{6}$$

This is a quadratic equation for $\dot{m}$ in terms of known parameters whose solution is

$$\dot{m} = \frac{\rho SL/C \pm \sqrt{(\rho SL)^2/C^2 - 2\Delta P/A^2\rho_s}}{1/A^2\rho_s} \tag{7}$$

Insertion of known values for S, L, C, P, and A, for an aluminum plug of the dimensions described below and in the range of 1.6 to 2°K shows that the second term under the radical is much smaller than the first. Expanding to a first order provides us with the following two expressions:

$$\dot{m} \simeq \frac{\Delta PC}{\rho SL} \tag{8a}$$

and

$$\dot{m} \simeq \frac{2\rho_s\rho SLA^2}{C} - \frac{\Delta PC}{\rho SL} \tag{8b}$$

Assuming that the pressure drop in the pumping line is negligible, ΔP is simply the vapor pressure inside the dewar. Since (8b) yields flow rates greatly exceeding v_{crit} in almost any physically realizable situation, it is rejected as violating the assumptions on which the derivation is based.

By substituting approximate temperature dependences of C, S, and ΔP into (8a), we obtain the temperature dependence of $\dot{m}$ as

$$\dot{m} \propto e^{-8.9/T}T^{-3.6} \tag{9}$$

This expression is monotonically increasing in the temperature range from 0°K to