

Editorial note (modified February 25,2008)

Collected here are the various reports of experiments conducted on the Barkhausen noise which could well be the cause of the up-conversion in the initial LIGO interferometers. For the person with no time, only the last reports with the most likely correct interpretation matter. At the end there are several reports that estimate the contribution of Barkhausen noise to the quiescent spectrum and show that the control currents into the ETM coils make fluctuating magnet moments of the magnets with sufficient amplitude, when they interact with the spatial magnetic field gradients of the PAM magnets, to explain a significant part of the up-conversion noise in the 50 to 100Hz band.

A report produced by David Kelley a freshman at MIT, shows that SmCo magnets of identical magnetic moment and geometry as the NdFeB magnets used in the initial interferometer, are at least 1/500 times less noisy. Another report shows that SmCo magnets, processed through the baking procedure used by the project to vacuum qualify and to cure the epoxy, are also noise free. The magnetic moments of both types of magnets in the magnetic open circuit application of the test masses are with about 5% the same. NdFeB magnets both glued and unglued are noisy.

Late in the experiments it became noticed that NdFeB magnets always detuned the bridge circuit while SmCo never did. The magnetic permeability for the NdFeB magnets is larger than 1 while that of the SmCo magnets is very close to 1. Implying that there are still magnetic domains to be turned in the NdFeB, the magnet, in the open magnetic circuit conditions of the test masses, is not saturated

Once the spacings of the PAM and control magnets were measured, it became clear that, especially for close spacings comparable to the magnet lengths, the point dipole approximation used in prior estimates for the Barkhausen noise contribution were too large, in fact, above the measured values of the up-conversion in situ. The scaling of the magnet/magnet forces with separation was done numerically and the agreement between the in-situ measurements with magnet spacing improved considerably. Best seen in the other compendium of reports on in-situ measurements.

At the end is the elog entry at Livingston showing another way of getting the magnet force/current from the magnet coil combination. The measurement drops the magnet through the coil with a known velocity. The reciprocity of volts/velocity = force/current is demonstrated and shows the adopted value of 1.6×10^{-2} newtons/amp per coil .

The other reports are demonstration of the attempts to try understand observations made by Sam Waldman and Robert Schofield as well as Brian O'Reilly of up-conversion when there is excess excitation of the interferometer at low frequencies. Some students or even a historian might enjoy reading about the meander of experimental sleuthing.

R. Weiss October 8, 2007 , Feb 25, 2008

NOTES ON BARKHAUSEN NOISE

Rainer Weiss

September 26, 2003 (modified December 2003, March 2006)

INTRODUCTION

Andri Gretarsson has become interested in the possible contribution of Barkhausen noise to the LIGO 1 noise budget and is thinking of carrying out an investigation.

This note, based in part on some earlier calculations that are in paper form in the DCC, shows that we may want to investigate the possibility of a limiting Barkhausen noise in the LIGO 1 test masses. The best evidence we have currently that we are not being seriously compromised is from the displacement spectra from the 40 meter and the TAMA data. The noise in these measurements does not quite reach the SRD. The final result of the estimate in these notes is that we should take Barkhausen noise more seriously.

At the end of the note a simple electronics test is described that sets limits on the Barkhausen noise due to the magnets on the test masses. The limits attained are at the current (2006) level of the interferometer sensitivity.

BASIC CONCEPT OF THE NOISE SOURCE

The noise is due to the discrete magnetic domains in a ferromagnetic material. The domains are between 0.1 to 10 microns in size and are characterized by having all electrons in the domain polarized with their magnetic moments in the same direction. The domains are in effect the “quanta” for the magnetization process. There is friction at the domain walls when orienting in response to an external H field. The friction causes magnetic losses in the material when driven by AC fields as well as magnetization fluctuations since the domains do not follow the external driving field in complete synchronism or smoothly.

The model I use in the calculations below is that the grains rotate with a Poisson distribution with the average rate of rotation determined by the driving field. The Barkhausen effect is a means of converting large amplitude low frequency drive currents into a broad band noise.

The magnetic material used in the test mass magnets (NEO-35) has a unit cell consisting of $\text{Nd}_2\text{Fe}_{14}\text{B}$ (28 magnetizing electrons, mass = 1068 amu). The magnetization curves of the material are shown in Figure 1. The material has a very high saturation magnetization when magnetized. At $H = 0$ the magnetization is almost complete with every electron in the material oriented along (opposite) the initial polarizing field. The operating point of the magnets on the test masses lies toward negative (depolarizing) H around -300 kA/m where the magnetization at 25C is close to $6.3 \times 10^5 \text{ J/Tesla m}^3$ and the slope of the magnetization $dM/dH = 0.8 \text{ J/A Tesla m}^2$. I do not have a good estimate of the grain size and will carry it as a parameter.

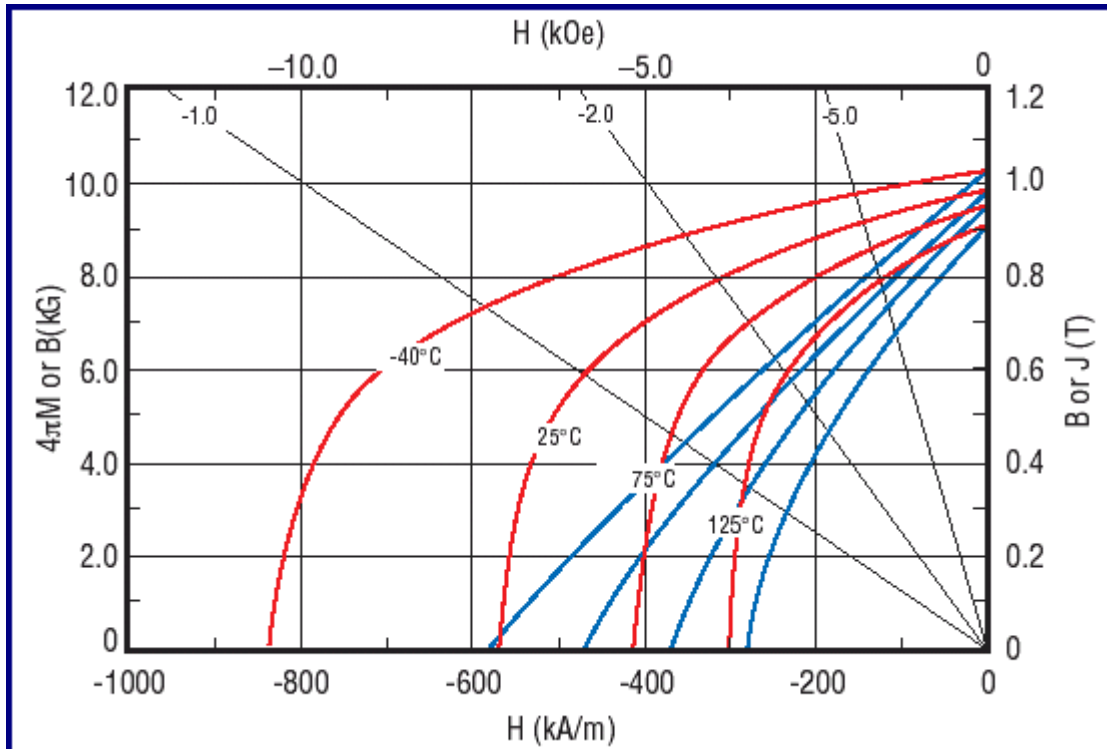


Figure 1 The hysteresis curve for the OSEM magnet material (Neodymium-Iron- Boron). The red curves show the magnetization $\mu_0 M$ in the material as a function of the depolarizing H field for several temperatures. The blue curves are the relation between B and H . The black straight lines are curves of constant μ .

THE CALCULATION

The steps in the Barkhausen noise estimate are the following:

- 1) Determine the relation between the current in the coil and H field on the magnet.
- 2) Estimate the magnetic moment associated with the domain.
- 3) Using the dM/dH at the operating point determine the number of domains that rotate with current change in the coils.
- 4) Use the measured magnetic force as a function of coil current and the estimate for the magnetic moment of the magnet and that of the domain to determine the force associated with rotation of a single domain.
- 5) Assume Poisson statistics to relate the average force due to the changing current and the spectral density of the fluctuating force due to the pulses from the rotating domains.
- 6) Estimate the displacement noise spectrum from the noise force spectrum.
- 7) Estimate the voltage noise generated in an OSEM coil due to the Barkhausen noise.

Coil and Magnet Parameters

Assume the number of turns $N = 200$ and the radius of the coil $r = 3/8'' = 9.5 \times 10^{-3}$ meters, then in the middle of the coil $B/i = 0.013$ Tesla/amp which corresponds to $H/i = 1.04 \times 10^4$ 1/m in the vacuum around the magnet.

The magnet has the dimensions length $L = 3.17$ mm and radius $a = 0.95$ mm, the volume of the magnet $V = 9.0 \times 10^{-9}$ m³. Using the bulk magnetization, the magnetic moment of the magnet $\mu = 5.7 \times 10^{-3}$ J/Tesla. The change in magnetic moment with external current is

$$\Delta\mu = V \frac{dM}{dH} \frac{H}{i} \Delta i = 7.5 \times 10^{-5} \text{ J/Tesla amp}$$

The magnetic moment of the domain, assuming that it is spherical with a radius r , varies as $\mu_d = M V = 2.6 \times 10^{-12} r(\text{microns})^3$.

In order to use the Poisson statistics the number of domains that rotate with change in the coil current is needed to estimate the average number of pulses per second

$$\frac{\Delta n}{\Delta i} = \left(\frac{\Delta\mu}{\mu_d} \right) \frac{1}{\mu_d} = \frac{2.8 \times 10^7}{r(\text{microns})^3} \text{ domain rotations/amp}$$

The force exerted on the test mass per magnet and coil combination has been measured as $F/i = 3 \times 10^{-2}$ Newtons/amp. The force is proportional to the magnetic moment of the magnet times the gradient of the coil's magnetic field at the magnet. Using the estimated value of the magnetic moment and the linearity with the magnetic moment, the change in force with change in the magnetic moment is

$$\Delta F(t) = \frac{\alpha \Delta\mu(t)}{\mu} \langle i \rangle_\tau = 5.3 \Delta\mu(t) \langle i \rangle_\tau \text{ Newtons}$$

The average current in the coil, $\langle i \rangle_\tau$, should really be interpreted as the average of the absolute value of the coil current since the polarity of the noise forces does not enter into the noise estimate. The "quantized" force associated with the rotation of a single domain becomes

$$F_{\text{domain}} = \frac{\alpha \mu_{\text{domain}}}{\mu_{\text{magnet}}} \langle i \rangle_\tau = 1.4 \times 10^{-11} r(\text{microns})^3 \langle i \rangle_\tau \text{ Newtons}$$

The noise spectral density from the Poisson distribution of domain rotations as the coil current varies is given (similarly to the shot noise for independent electron flow) as

$$F(f) = F_{\text{domain}} \sqrt{2 \dot{N}_{\text{domain}}}$$

$\dot{N}_{\text{domain}} = \frac{\Delta n}{\Delta i} \frac{\Delta i}{\Delta t}$ is the average rate of domain rotations and is driven by the change in the

current in the coil. For sinusoidal currents $\dot{N}_{\text{domain}} = 2\pi f i \frac{\Delta n}{\Delta i}$ where f is the frequency

and i is the amplitude of the oscillating current. I realize that the separation of the average current and the oscillating current is not rigorous and needs to be revisited if it becomes important to make a better estimate of the noise. Combining the various previous factors gives the result

$$F(f) = 2.6 \times 10^{-7} \langle i \rangle_\tau \sqrt{f_{\text{current}} i r(\text{microns})^3} \text{ Newtons}/\sqrt{\text{Hz}}$$

The motion of a single 10 kg test mass in response to this force density with uncorrelated domain rotations in the four magnets gives

$$x(f) = \frac{2F(f)}{m\omega^2} = \frac{1.3 \times 10^{-9}}{f^2} \langle i \rangle_{\tau} \sqrt{if_{current} r(\text{microns})^3} \text{ meters}/\sqrt{\text{Hz}}$$

The force density is not negligible for likely parameters. Using a signal frequency of 100 Hz an average current of 10 mA, domains with 0.1 micron radius and an oscillating current at 1 Hz with amplitude 10 mA gives $x(f)$ of a few 10^{-18} meters/ $\sqrt{\text{Hz}}$

Table top experiment to limit the noise source

Figure 2 shows a schematic diagram of an experiment to measure the flux change associated with the domain rotations. The sensitivity of the experiment should allow measurement to a domain size of 0.1 microns in one of the existing NdFeB magnets or even smaller if several magnets are available. The experiment is the classical method of demonstrating the Barkhausen effect.

Estimates for the experiment:

The flux in the coil due to the magnetic moment of the magnet

$$\Phi = \frac{\mu_0 N \mu_{magnet}}{r_{coil}} . \text{ Here } r \text{ is the radius the coil, } N \text{ is the number of turns in the coil and}$$

μ_{magnet} is the magnetic moment of the magnet. The rotation of a domain causes a voltage

$$\text{at the output of the coil } V_{domain} = \frac{\mu_0 N \mu_{domain}}{r_{coil} \tau_{relaxation}} . \text{ Using the same kind of Poisson reasoning}$$

the voltage spectral density across the coil will be

$$V(f) = V_{domain} \sqrt{2N} = V_{domain} \sqrt{2 \frac{\Delta n}{\Delta i} \frac{\Delta i}{\Delta t}} = \frac{1.3 \times 10^{-9}}{\tau_{relax}} \sqrt{fir(\text{microns})^3} \text{ volts}/\sqrt{\text{Hz}}$$

Using a domain rotation time (relaxation time) of 10^{-4} seconds, coil oscillation current frequency of 1 Hz with amplitude 10 mA and a domain radius of 0.1 microns one gets a voltage spectral density of $V(f) = 400 \text{ nVolts}/\sqrt{\text{Hz}}$. If more sensitivity is needed it should be possible to use a higher oscillation frequency, or a constellation of magnets and possibly upto 1 Ampere of drive current.

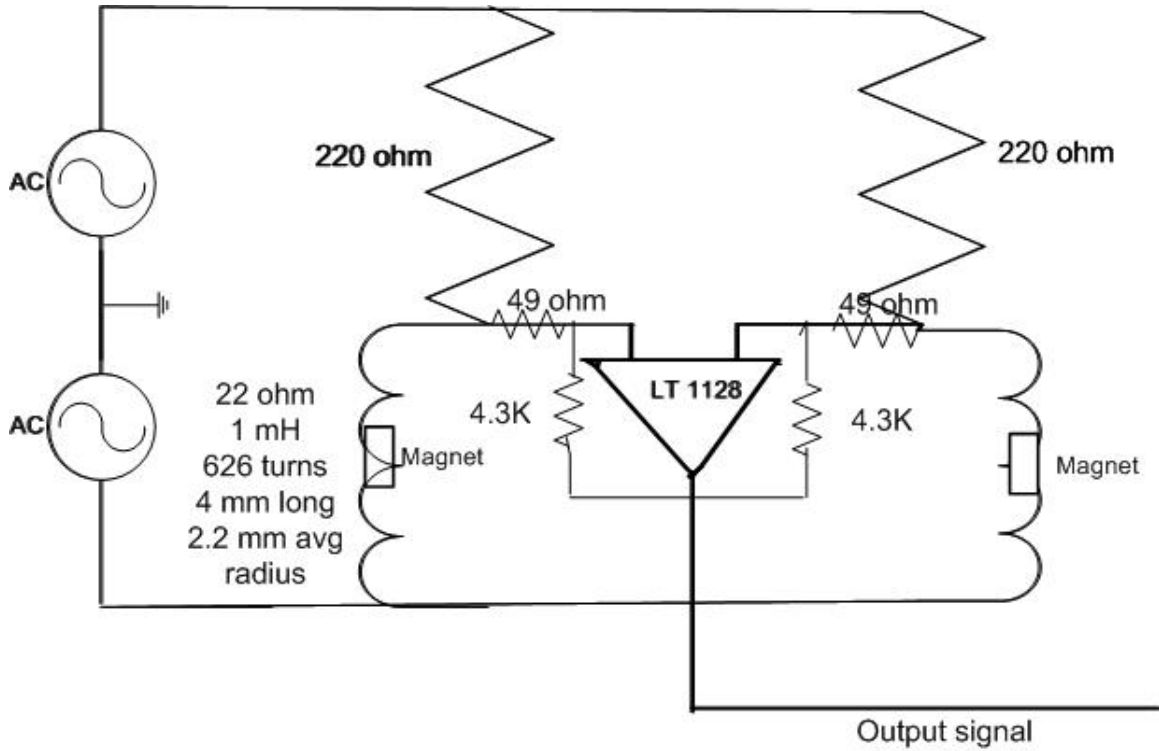


Figure 2 Simple induction experiment using a bridge circuit and a low noise op amp to read the bridge unbalance due to the posited Barkhausen flux change in the OSEM coil associated with the magnets under investigation. One of the coils has a large parallel trimming resistor and small parallel capacitor to null the in and out of phase voltage difference between the two sides of the bridge (across the input of the op amp).

The actual experiment

Without reference to the dynamics inside the magnets the voltage induced in the coil can be written from Faraday's law directly. The field from the magnet in terms of its magnetic moment is

$$B_z = \frac{\mu_0 M}{A_{mag} L_{mag}}$$

where A_{mag} is the cross section of the magnet and L_{mag} the length. M is the magnetic moment. For estimation purposes the cross-section of the magnet is also the area associated with flux in the coil. The voltage induced in the coil when the magnet moment changes is

$$V = N_s \frac{d\phi}{dt} = \frac{\mu_0 N_{coil}}{L_{coil}} \frac{dM}{dt}$$

here N_s is the number of turns of the coil in the length of the magnet. Expressed in terms of the Fourier amplitude of the voltage and magnetic moment, the magnetic moment spectral density in terms of the induced voltage spectral density becomes

$$M(\omega) = \frac{L_{coil} V(\omega)}{\mu_0 N_{coil} \omega}$$

Placing such a magnet in the OSEM coil would make a time varying force

$$F(\omega) = M(\omega) \frac{dB}{dx} = \frac{L_{coil} V(\omega)}{N_{coil} \omega} \frac{N_{osem}}{L_{osem} a} i(drive + bias)$$

a is the radius of the OSEM coil and N_{osem} and L_{osem} are the number of turns and the length of the OSEM coil. Since an upper limit is all that the experiment can give using the present data, the OSEM currents from both the drive and the bias are lumped together. The magnetic moment variation is proportional to the drive frequency and its amplitude, for this upper limit estimate these are lumped together in the value of $V(\omega)$, which sets the experimental limit.

Results of the experiment

drive frequency Hz	drive amplitude rms mA	bridge unbalance at drive frequency microVolts rms	V(f) 40 to 1000Hz nV/sqrt(Hz)
10	0	0	1.62
10	2.3	3.4	1.62
10	6.9	1.2	1.66
10	13.8	3.3	1.72
70	0	0	1.49
70	2.3	0.40	1.50
70	6.9	0.42	1.52
70	13.8	1.11	1.54
620	2.3	0.71	1.50
620	6.9	0.83	1.52
620	13.8	1.84	1.58

The results are almost the same with the magnets not in the coils. The noise is therefore best used as an upper limit. Using the value 1.5nV/sqrt(Hz) as the upper limit and taking into account that in the LIGO there are 16 magnets associated with the 4 test masses and that there is a typical bias current of 20 ma, the estimated upper limit for the displacement noise from the Barkhausen effect is less than 3×10^{-19} meters/sqrt(Hz) at 100 Hz.

The results also give an upper limit for the domain size of 0.01microns (much smaller than I had thought). The rise in the noise with drive current is due to the nonlinearity in the preamplifier. The same rise is observed when there are no magnets in the coils. Suspect that the large common mode signals are causing the non-linear response. If this measurement is to be done again it would be better to use a high quality audio transformer between the differential points of the bridge and then couple the transformer secondary to the low noise operational amplifier. Another variant of the experiment should be tried in which the magnets are glued to their standoffs and the standoff in turn are glued to some fused silica. There may well be a shape change of the magnet associated with the Barkhausen effect that causes more noise when the magnet is constrained by the glueing.

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More Sensitive Bridge Experiment to Search for Up-Conversion in the Magnets and Coils

R. Weiss September 19, 2006

Abstract

The table-top bridge experiment was improved so that it is possible to measure voltage noise smaller than $0.5 \text{ nV}/\sqrt{\text{Hz}}$ on the bridge elements. The impedance of the bridge has been reduced to allow peak drive currents of 100ma. With these changes several effects were measured: the current induced noise in film resistors, the mechanically induced noise in the coils and (possibly) the Barkhausen noise in the magnets. The up-converted noise in the 20 to 100Hz band when the bridge is driven by low frequency (7Hz) currents of 20 to 100 ma is at a level that could cause the up-conversion we are seeing in initial LIGO. More work needs to be done in seperating the magnetic Barkhausen noise from the noise due to internal coil motion. In particular, the coil noise needs to be measured with the OSEM coils rather than the specially wound coils designed for this experiment.

Introduction

The table-top bridge experiment to measure the Barkhausen noise in the magnets was improved by adding a step up transformer and using an improved low noise FET input amplifier. The resistors in the bridge were reduced to match the transformer primary impedance of 30 ohms. A schematic of the circuit is shown in **Figure 1**.

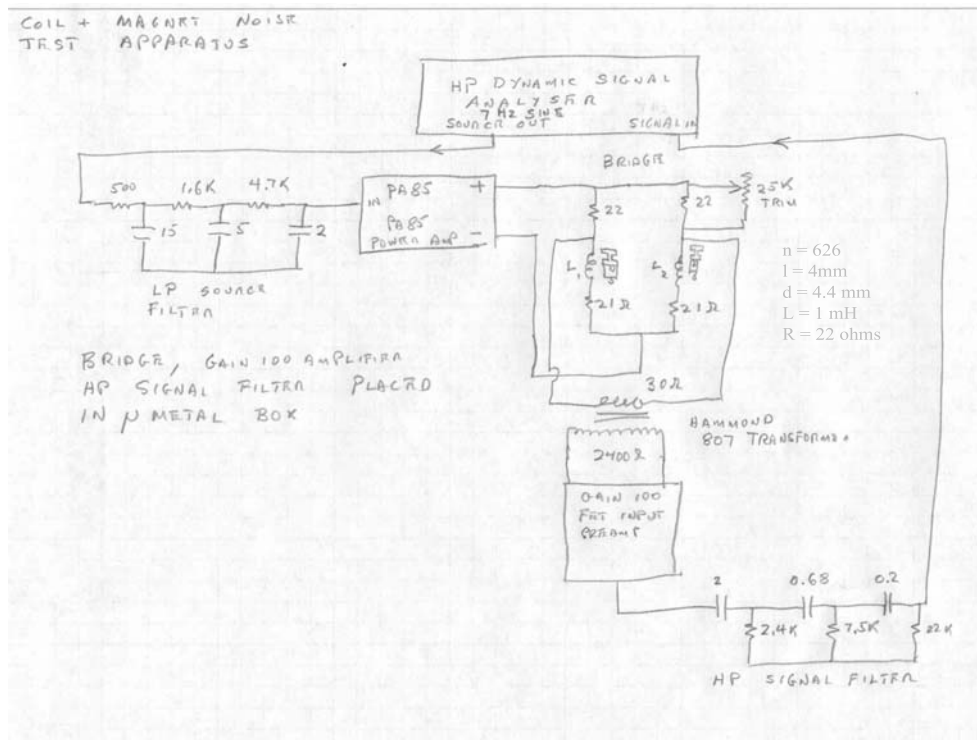


Figure 1 Schematic of the experiment

Measurements The measurements consisted of recording the spectrum under the following conditions :

no drive	22 ohm film resistors	coil without magnets	coil with magnets
1 V pk 7 Hz sine 22ma pk	“	“	“
2 V pk 7 Hz sine 44ma pk	“	“	“
3 V pk 7 Hz sine 67ma pk	“	“	“

The spectra shown in the figures have been corrected for the transfer function of the bridge transformer, the high pass filter and the FET preamplifier. The voltage noise amplitude is referred to the primary of the transformer.

The noise measured with no drive is close to that expected from the incoherent addition of the thermal noise in the 22 ohm resistors in each bridge element, $0.6\text{nV}/\sqrt{\text{Hz}}$ across the transformer primary when summed and the various impedances are taken into account. The undriven noise for all the cases is very close to identical. Note, that the bridge and transformer need to be placed in a magnetic shield otherwise gradients in the 60Hz dominate the rms signal.

As can be seen by looking at the figures excess noise arises when the bridge is driven. I thought that replacing the coils by film resistors would be sufficient to eliminate the excess noise when driven but this is not the case. It has always been known that carbon resistors show excess current noise with a $1/f$ spectrum, as much as $1\ \mu\text{Vrms}$ in a band of 100Hz per volt across the resistor. To my surprise even the thin film resistors exhibit approximately 10nVrms per volt across the resistor. In the next incarnation of this experiment will use encapsulated wirewound or Vishay metal foil resistors (the audiophile crazies guessed right on this but I am sure cannot really hear the difference between the resistors). One worry I had about this excess noise was that the digital signal generator in the HP dynamic analyser was the source, that is why the additional low pass filter was placed between the source output of the analyser and the bridge. With this low pass filter and the further reduction from the bridge balance, the largest contribution from this source ($700\text{nV}/\sqrt{\text{Hz}}$ at 100Hz with 3V pk output at 7 Hz) would have been $0.003\text{nV}/\sqrt{\text{Hz}}$ at the primary of the transformer in the 100 Hz band.

Figure 2 through **Figure 4** shows the progression of the up-conversion noise as the 7Hz sinusoidal drive voltage is changed from 1,2 and 3 Volts pk. Each figure has a reference, the red curve, with no drive voltage. The violet curve is the noise of the film resistors where the coils have been replaced by the resistors. The green curve is the noise with the empty coils in two elements of the bridge but still 2 film resistors in the other two elements. The difference between the green and violet curves is plotted in **Figure 5** for all three drive voltages. The extra noise from the coil is from physical motion of the windings. The coil noise was almost an order magnitude larger before the windings were immobilized by a thin layer of coil dope which should have penetrated the windings before setting. Rana's guess that mechanical motion of the coils could be a noise source in LIGO is not so crazy and the force noise that could arise from this motion is estimated later in the note.

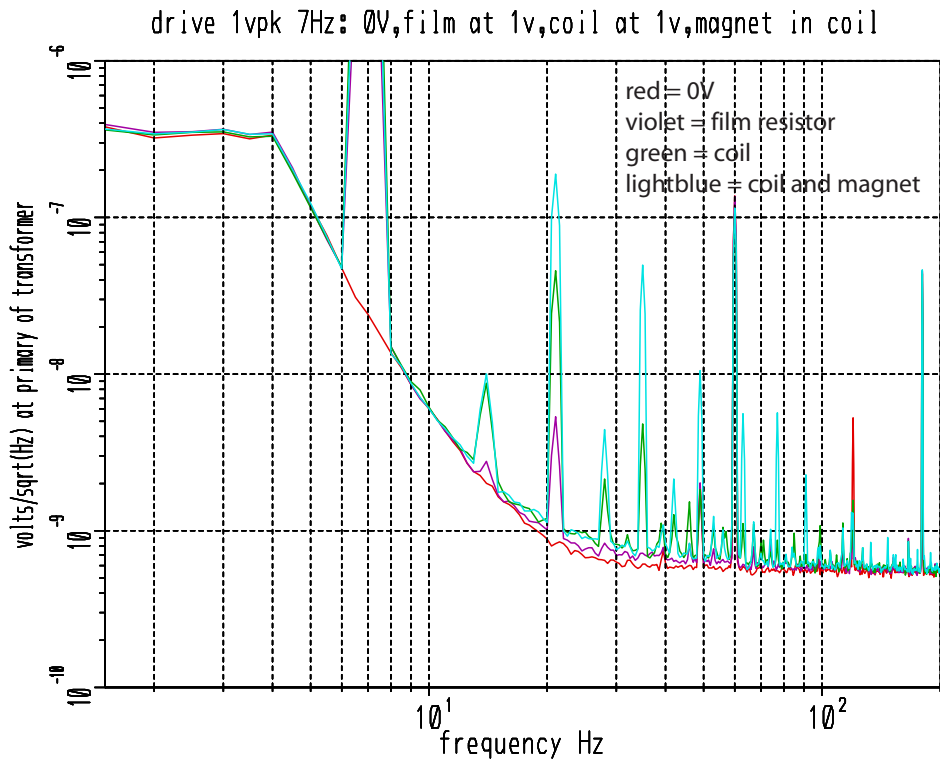


Figure 2 1 volt pk 7Hz sinusoidal excitation

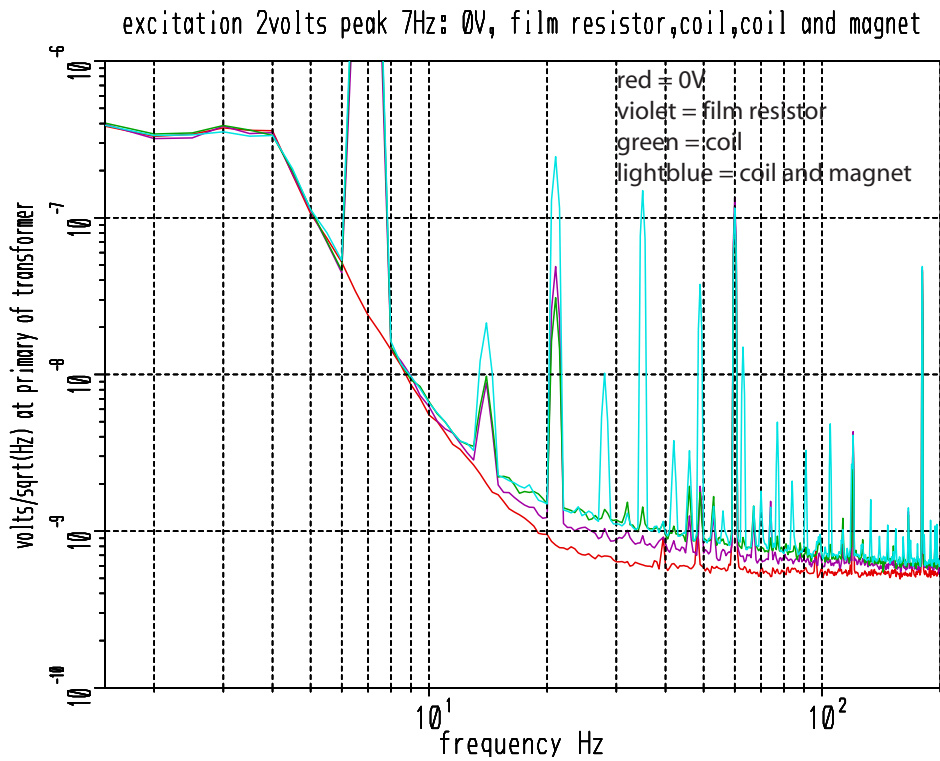


Figure 3 2 Volt pk 7Hz sinusoidal excitation

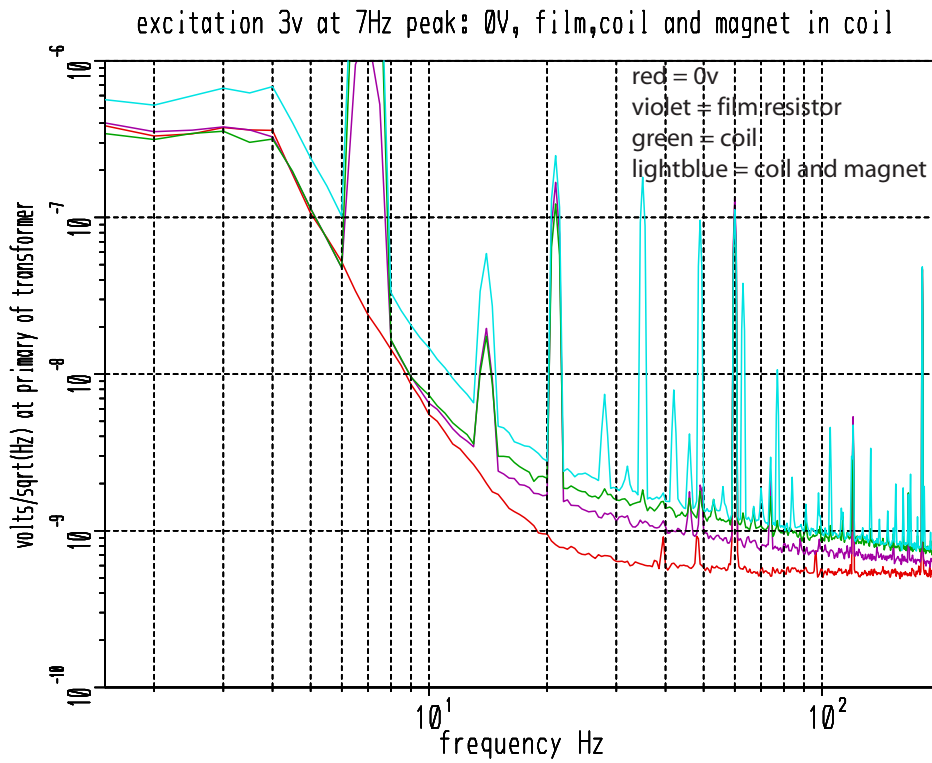


Figure 4 3 Volt pk 7Hz sinusoidal excitation

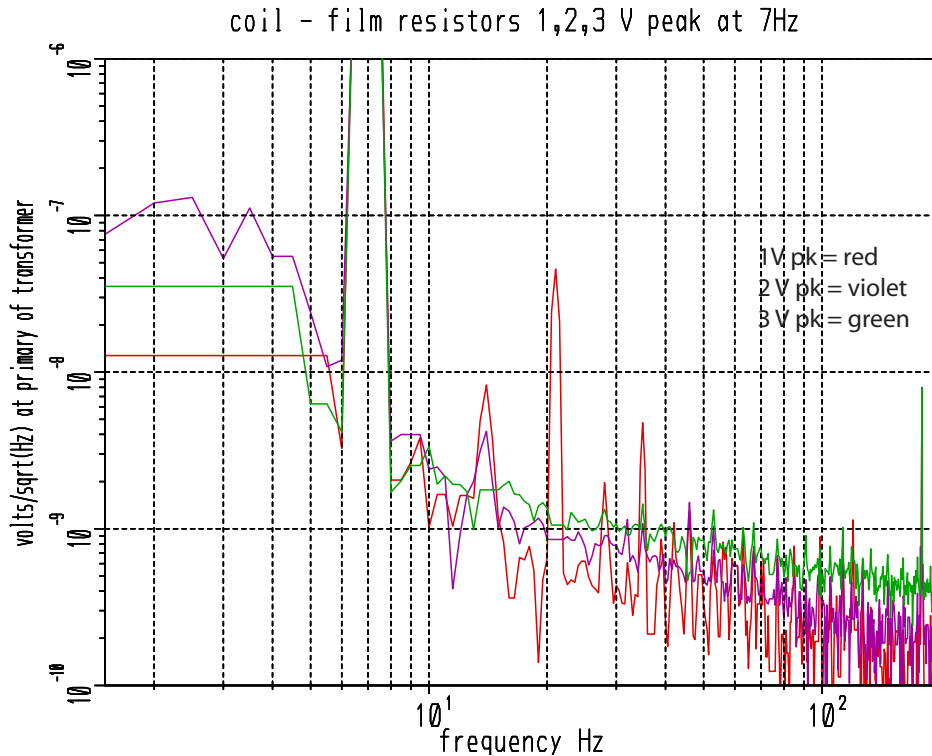


Figure 5 Difference between spectra of empty coil and film resistors for the various drive voltages. Negative values in the difference were removed by setting to the nearest positive value.

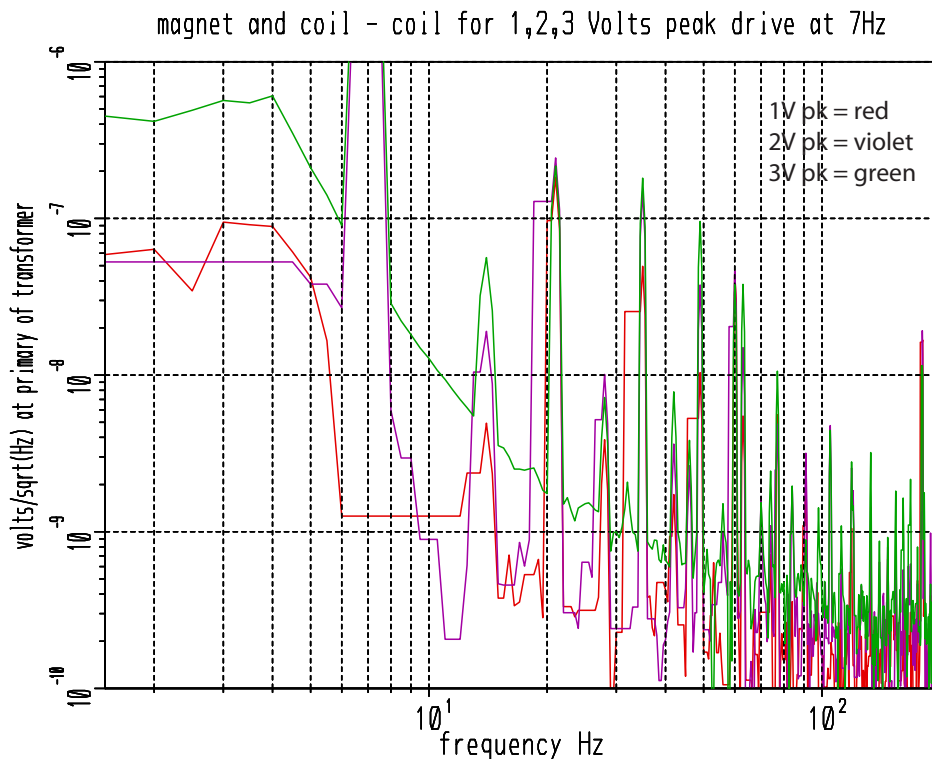


Figure 6 Difference between spectra of coil with magnet and empty coil for the various drive voltages. Negative values in the difference were removed by setting to the nearest positive value.

As can be seen in all the figures the magnet causes additional noise but also discrete harmonics of the drive frequency. My suspicion is that the additional noise and especially the drive harmonics are enlarged physical motions induced in the coil by the magnet B field interacting with the coil current and are most likely not Barkhausen noise although this is not really established by the measurements. The coil motions without the magnet should grow as the square of the current while with the magnet they would be proportional to the current. There is not enough data to make this distinction.

As will be shown in the estimates below, the noise could explain the up-conversion in LIGO and it will become important to measure the physical motions of the actual OSEM coils which could be much larger than the coils in this experiment.

Estimates

For the estimates the excess noise spectrum in voltage is assumed to have a $1/f$ character.

Assumption 1: Varying voltage $V(f)$ comes from a varying magnetic field $B(f)$ from the magnet

No model is used short of the assumption that the entire voltage fluctuation is due to a fluctuation in the magnetic field of the magnet. When the force is calculated the time varying force on the current carrying coil windings from this varying magnet field is estimated. The force on the coil must be the same as the force on the magnet by momentum conservation.

Need to establish the relation between $V(f)$ and $B(f)$ on the magnet. Do not assume that the variations have to come from the Barkhausen effect as calculated before. The coil has $N = 626$ turns and the radius of the magnet $a_m = 0.95 \times 10^{-3}$ meters. For the sake of the approximation, the magnetic field is primarily confined to the magnet. The fluctuating B field and the fluctuating voltage are related by Faraday's law as

$$B(f) = \frac{V(f)}{N \pi a_m^2 \omega}$$

The voltage fluctuations are almost given by $V(f) = \frac{1.5 \times 10^{-6}}{f} i(\text{amp pk}) \text{ Volts}/\sqrt{\text{Hz}}$ so that the

fluctuating magnetic field from the magnet is $B(f) = \frac{1.3 \times 10^{-4}}{f^2} i(\text{amp pk}) \text{ tesla}/\sqrt{\text{Hz}}$.

The conversion to a force on the OSEM coil uses the turns in this coil $N_{\text{osem}} = 200$ and the radius of the coil $a_{\text{osem}} = 9.5 \times 10^{-3}$ meters. The force spectral density is

$$F(f) = 2\pi a_{\text{osem}} B(f) N_{\text{osem}} \left(\frac{a_m}{a_{\text{osem}}} \right)^3 = \frac{1 \times 10^{-7}}{f^2} i(\text{amp pk}) \text{ newtons}/\sqrt{\text{Hz}}$$

Finally the conversion to a displacement of the mass which assumes the incoherent sum of the up-conversion in 4 coil/magnet motors.

$$x(f) = \frac{2 \times 10^{-7}}{(2\pi)^2 m f^4} i(\text{amp pk}) \text{ meters}/\sqrt{\text{Hz}}$$

with a mirror mass of 10kg and a peak current of 30 ma (assume this is the current used by Sam Waldman in his experiment to measure the up-conversion spectrum with a drive at 8Hz), the estimated spectrum becomes 6×10^{-18} at 40 Hz and 2×10^{-19} meters/ $\sqrt{\text{Hz}}$ at 100 Hz. Sam did measure a $1/f^4$ spectrum with about 4×10^{-18} meter/ $\sqrt{\text{Hz}}$ at 40Hz. So one cannot throw out the bridge experiment results. The trouble is that there is no proof that the voltage fluctuation is due to a magnetic field fluctuation in the magnet. The spectrum of voltage fluctuations does not match the model developed for a Barkhausen spectrum.

Suppose it is really due to a mechanical motion of the coil windings and would this also be able to provide a fluctuating force. The next estimate is dedicated to this but there is no reason that the bridge coils behave the same way as the OSEM coils. The bridge coils were doped down, the OSEM coils are looser and could actually wiggle more. Need to measure the OSEM coils in the bridge rig to find out.

Assumption 2 $V(f)$ comes from mechanical motions in the coil

The interesting aspect of this approach is that it is not necessary to actually calculate the motion of the coil turns to estimate the fluctuating force arising from these motions as long as the voltage spectrum is measurable. This will serve well if further measurements are attempted on the OSEM coils themselves. The basis of the estimate is to use the magnetic field from the magnet (consid-

ered unvarying for this estimate) and have it interact with the moving coils carrying the current. Assume that this current is not producing an appreciable part of the magnetic field. Again use conservation of momentum and calculate the varying force on the coil as the way to get the fluctuating force on the magnet. The fluctuating force on the coil is

$$F(f) = 2\pi r_{\text{osem}} i(\text{amp pk}) x(f) \alpha N_{\text{osem}} \left(\frac{dB_r}{dx} \right)_{\text{magnet}}$$

Here r_{osem} is the radius of the OSEM coil, $x(f)$ is the motion of a turn along the longitudinal direction, α is the fraction of the turns partaking in the motion and the derivative is the change in radial field of the magnet along the x direction. It is assumed that the magnet is centered in the coil to avoid an unpleasant integral. The field and field derivative are given by

$$B_r = B_0 \cos\theta \left(\frac{a_m}{r_{\text{osem}}} \right)^3 \quad \frac{dB_r}{dx} = \frac{B_0}{r_{\text{osem}}} \left(\frac{a_m}{r_{\text{osem}}} \right)^3$$

The voltage induced in the moving coil turn from the field of the magnet is determined by using the motional electric field generated in the wire by the translational wire motion in the field of the magnet (a coil rotation could also cause a voltage but would not cause a force unless the magnet is off axis, need to think about this possibility further because it could cause the voltage to be a too generous indicator of the fluctuating force).

$$V(f) = 2\pi f x(f) B_0 \left(\frac{a_m}{r_{\text{osem}}} \right)^3 (2\pi r_{\text{osem}} \alpha N_{\text{osem}})$$

Expressing $x(f)$ in terms of the $V(f)$ and then inserting into the equation for the fluctuating force gives, the following simple relation for the $F(f)$

$$F(f) = \frac{i(\text{amp pk}) V(f)}{2\pi f r_{\text{osem}}}$$

The mirror displacement noise for 4 coil magnet motors added incoherently would be

$$x(f) = \frac{2 i(\text{amp pk}) V(f)}{r_{\text{osem}} m (2\pi f)^3}$$

If the OSEM coils behave similarly to the doped bridge coils, $V(f)$ would vary as $1/f$ and the displacement noise would once again vary $1/f^4$. The displacement noise would vary as i^2 . The noise would be 5×10^{-19} at 100Hz and 2×10^{-17} meters/ $\sqrt{\text{Hz}}$ at 40 Hz using 20 ma of coil current, values that are almost a factor of 5 too large to explain the up conversion in initial LIGO.

More measurements of up-conversion with the bridge experiment

R. Weiss October 21, 2006

Summary

The bridge experiment was run with OSEM and the previous small coils and the film resistors were replaced by wirewound resistors. The results are:

- 1) The wire wound resistors do not show upconversion (in the parlance of the electrical engineers, do not have a voltage coefficient) and the windings are well enough potted to avoid motion from the induced magnetic fields interacting with the current loops.
- 2) The good OSEM coils do not show up-conversion under any of the conditions in the experiment, these were: empty coils, coils loaded with the small magnets that had glued dumbbells, coils loaded with large magnets but no glued component. The sensitivity of the measurements with the small magnets is about 50 times worse than with the small coils due to the poorer coupling to the magnets and the lower number of turns. The fact that the OSEM coils with the large magnets do not show up-conversion is interesting and could be due to several causes.
- 3) One of the OSEM coils had, what can only be attributed to an internal short, which was making contact tentatively. The up-conversion in this coil was alarmingly large.
- 4) Measurements with the wirewound resistors and the small coils with and without magnets are consistent with the earlier results.
- 5) Have still not been able to separate the up-conversion from the magnet alone and that due to motions of the coil, although it is clear that there is motion in the coil at the harmonics of the drive and these are made larger when the magnet is in the coil.

What to do next

Several other variants should be tried:

- 1) To understand why the large magnet in the OSEM coil did not show up-conversion, it might be worthwhile gluing some aluminium to it with the same epoxy as on the testmasses and trying the up-conversion measurement again.
- 2) Conversely, the small coil experiment should be tried with small magnets that do not have aluminium dumbbells glued to them.
- 3) To determine the motion of the OSEM coil windings, it would be useful to drive the coil harder to establish when and if the harmonics grow with the square of the current. Before this can be done more filtering is needed to reduce the drive in the harmonics from simple distortion in the drive waveform. It will become increasingly difficult to increase the drive amplitude and maintain bridge balance due to the change of the coil resistance with temperature. It may be necessary to

place the coils in oil or other thermally conducting media to reduce the thermal gradients.

4) Another variant on the magnets could be to assemble a group of the small magnets in a plastic container, all polarized in the same direction (tending to fly apart) and improve the OSEM coil coupling to the magnets. Then to look again at up-conversion in this assembly of magnets. Worth finding out if the glued dumbbell is the cause of the up-conversion in the small coil measurement first.

This is getting to be a project!

osem coil with internal short 0 to 1V pk drive at 8Hz

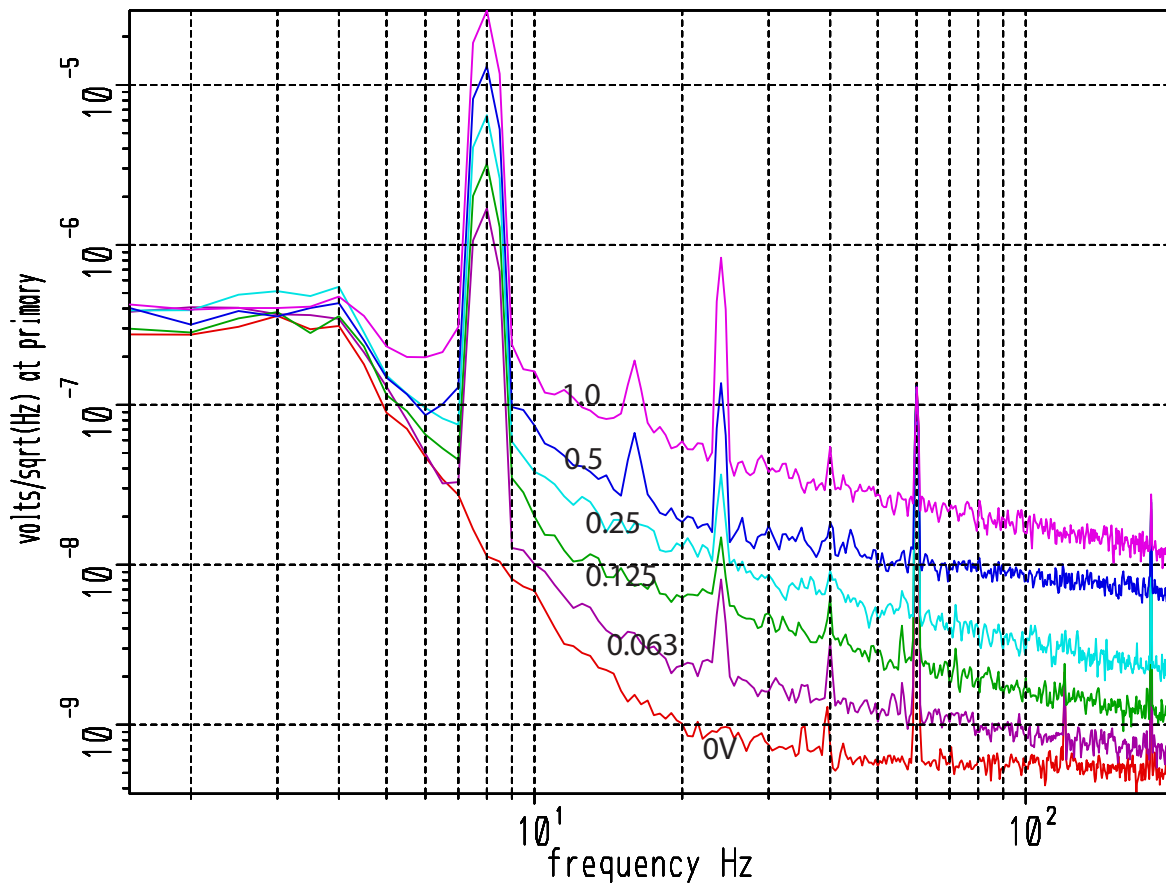


Figure 1 The up-conversion in an OSEM coil when there is a tentative short between windings

osem coil with no magnet \emptyset and 5V pk drive at 8Hz

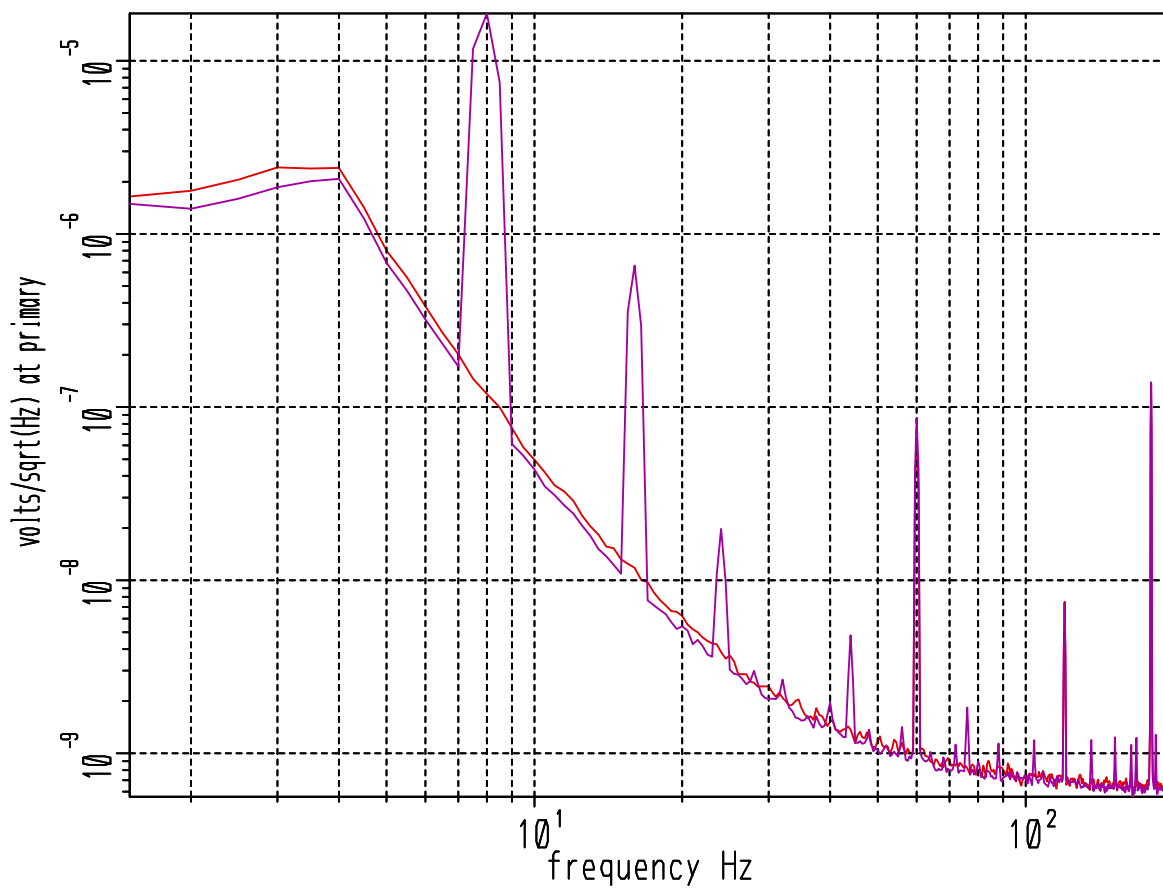


Figure 2 OSEM coil without a magnet under conditions of no drive and 130 ma peak drive at 8 Hz.

osem coil with small magnet: 0 and 5Vpk drive at 8Hz

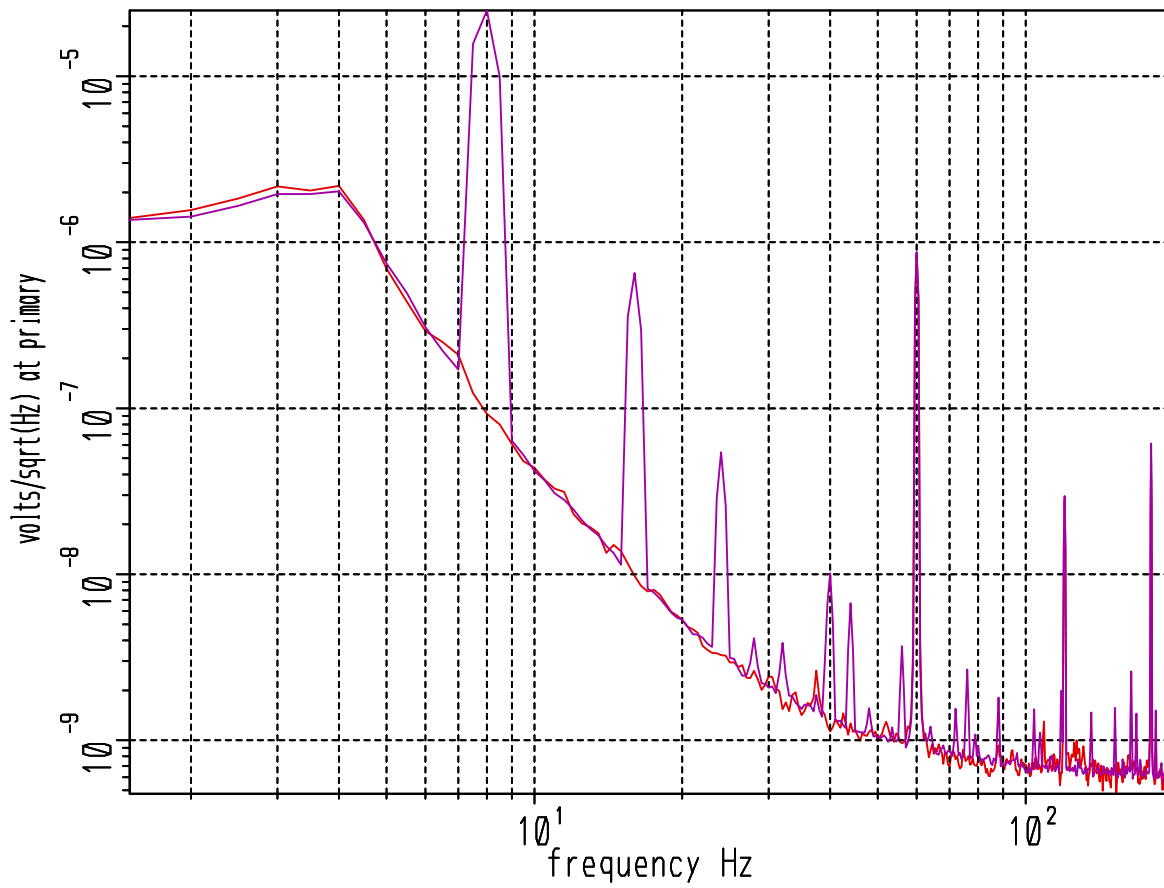


Figure 3 OSEM coil with small magnet having a glued aluminium dumbbell.

Osem coil with large magnet, 8Hz drive 0,2,3,4,5 volts pk

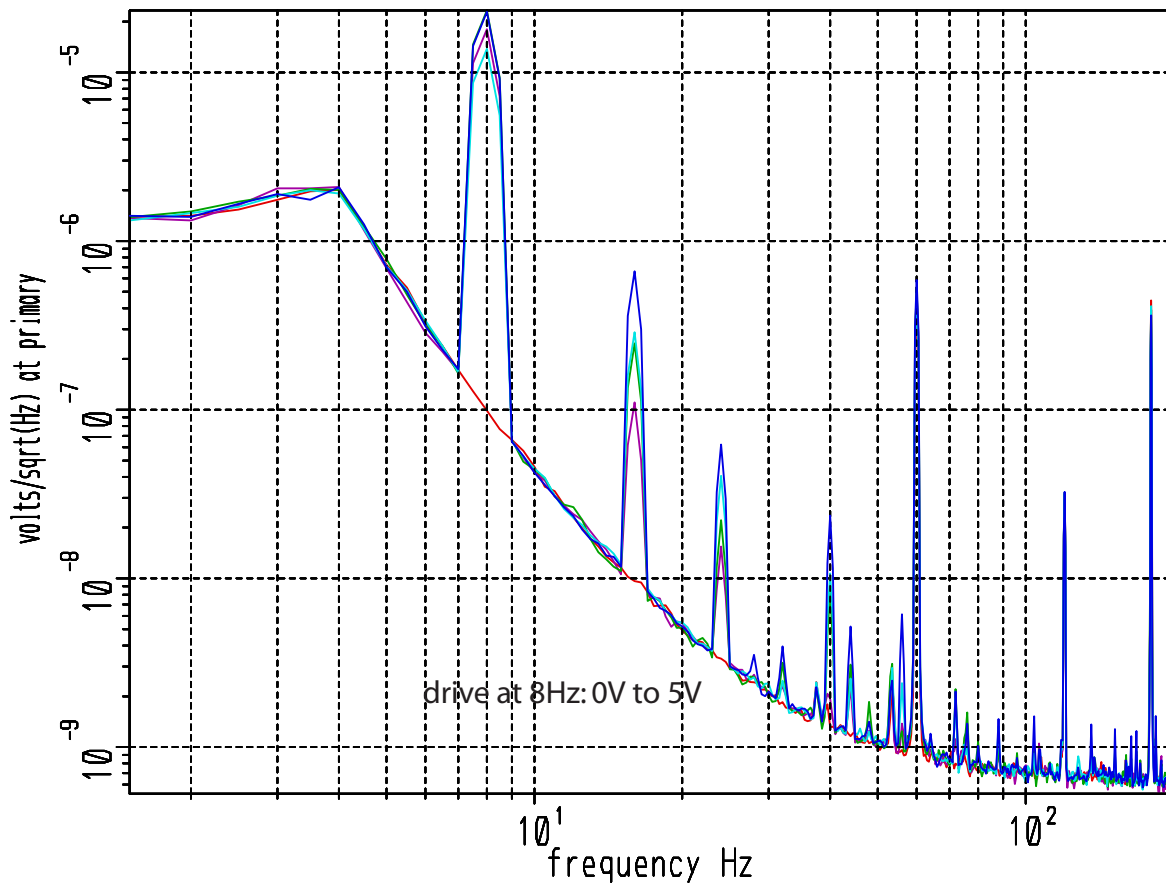


Figure 4 OSEM coil with large magnet but no glued component.

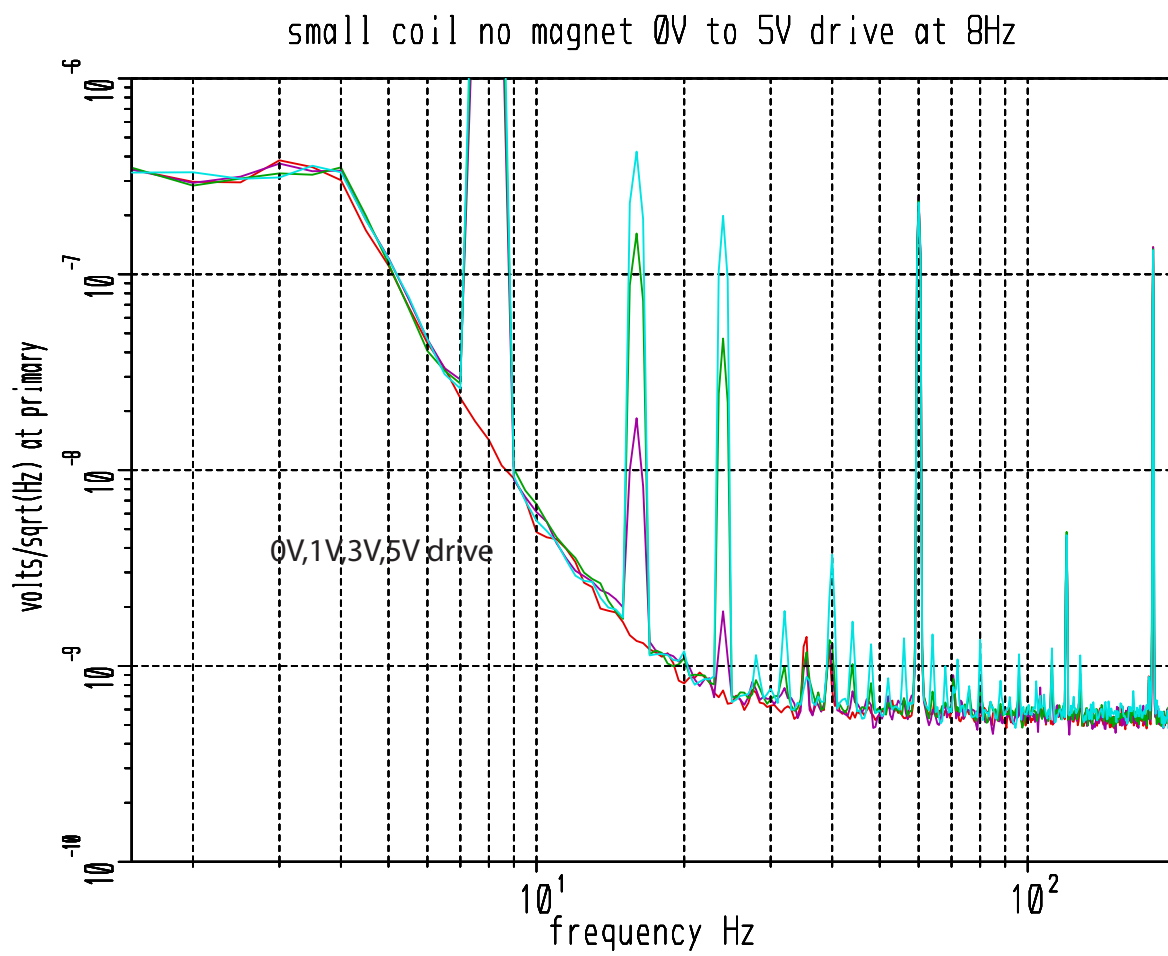


Figure 5 Small coil with no magnet but now with the quiet wirewound resistors.

small coil with magnet 0V to 5V drive at 8Hz

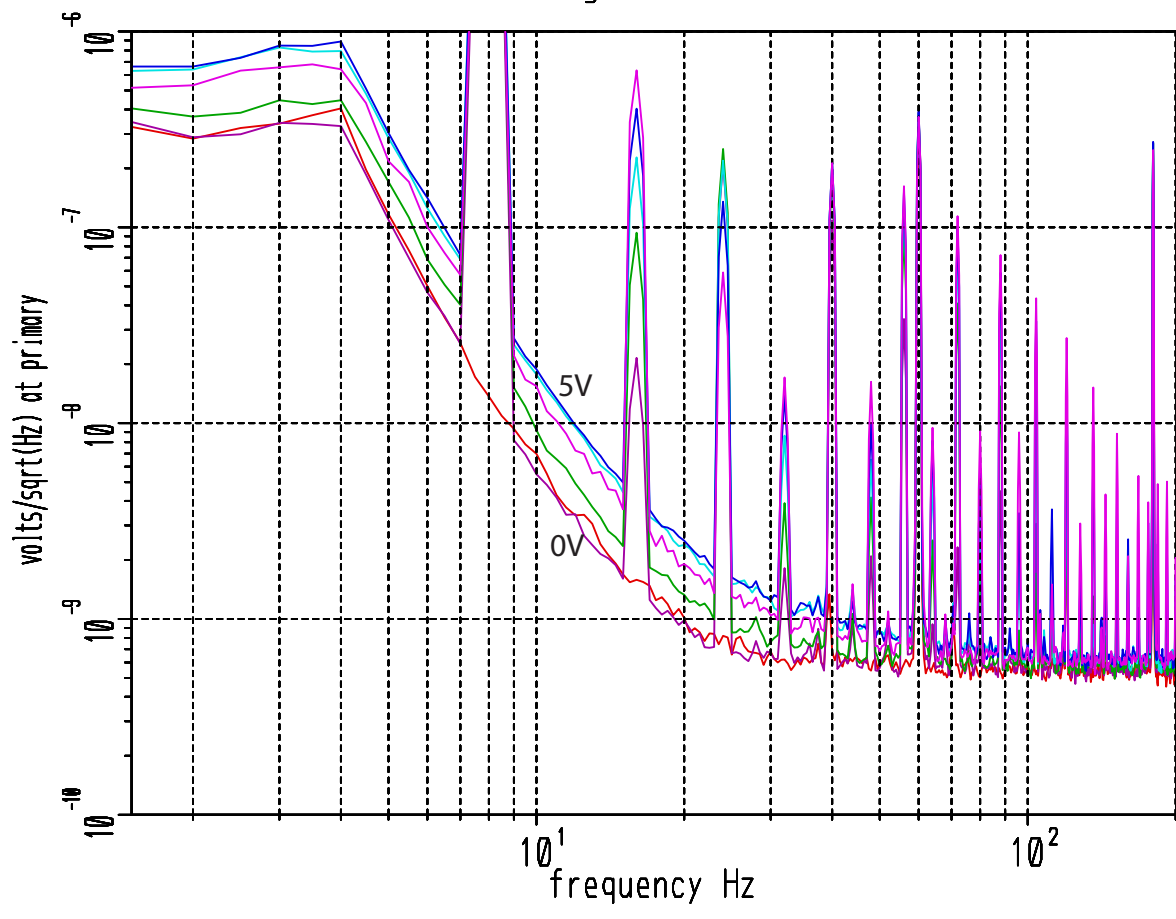


Figure 6 Small coil with magnet and glued aluminium dumbbell. The drive current was applied in equal increments with the largest value 113ma peak.

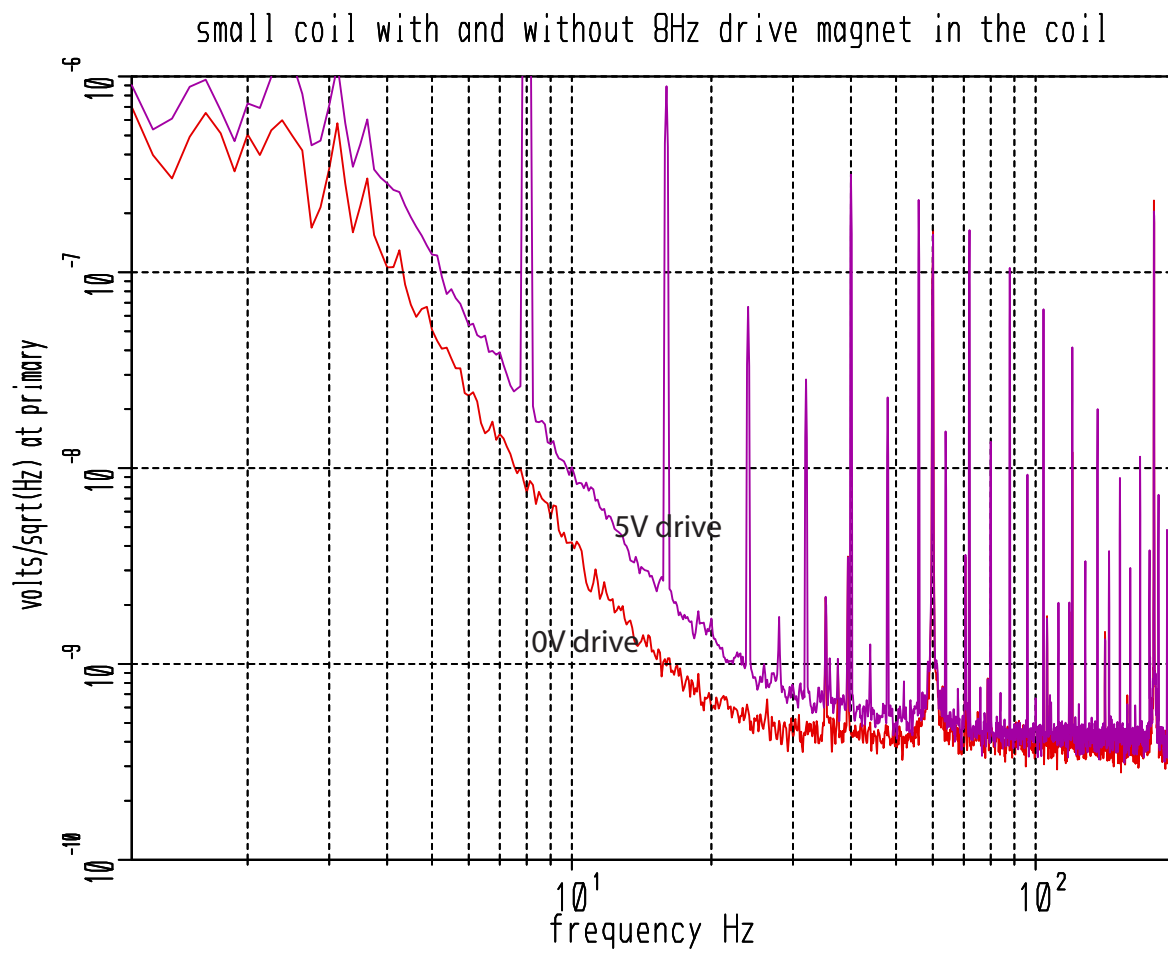


Figure 7 The same as figure 6 but with 4 times higher frequency resolution and only one drive current of 113 ma pk

small coil 5V - 0V with magnet

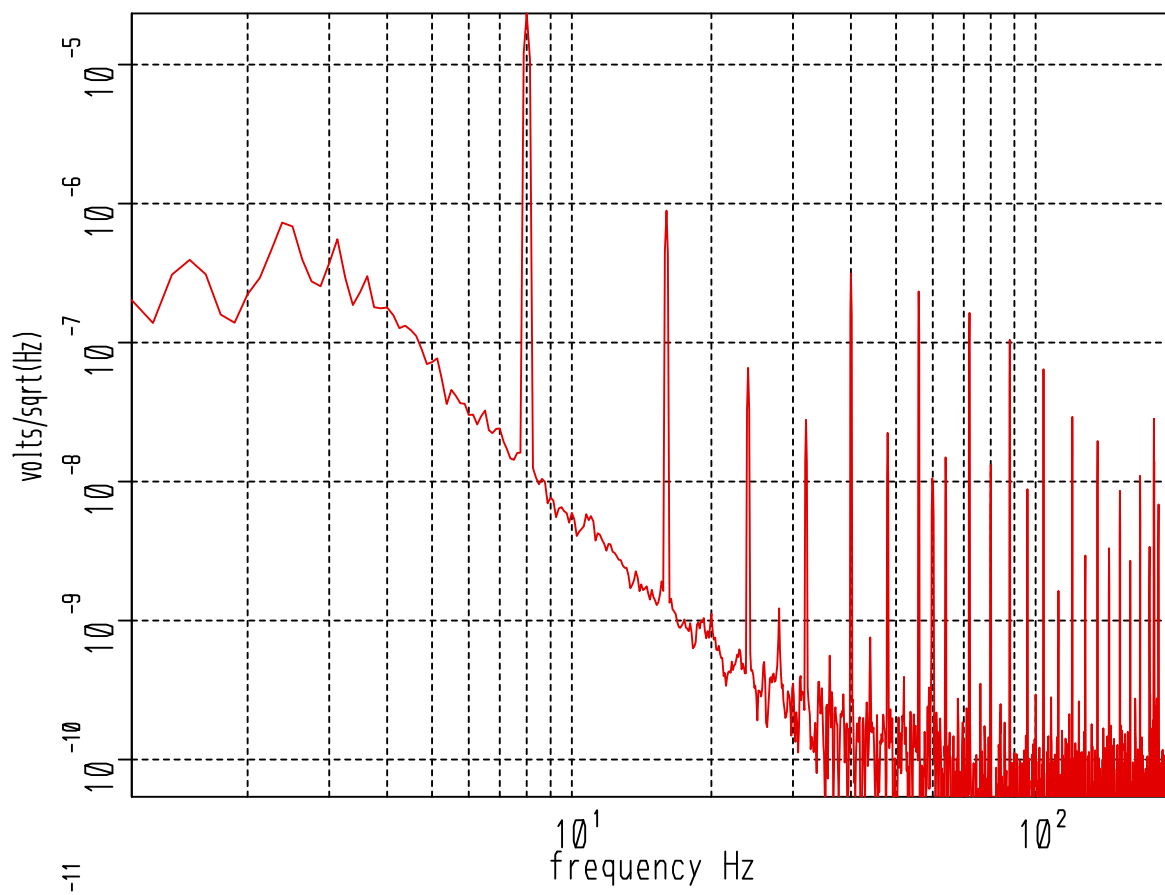


Figure 8 The difference between the driven and undriven spectra in figure 7.

Further Studies of the Magnet/Coil controller noise using the bridge excitation experiment

R. Weiss Jan 15, 2007

Introduction and summary:

This last visit to Livingston finally showed that the up-conversion noise seen in the interferometer in the band between 50 to 100Hz comes from noise induced in the magnets. The coils contribute harmonics due to forces on the windings which can be reduced considerably by immobilizing the windings (doping them down) but the broadband noise comes from the magnets directly. The mechanism is still mysterious as it does not satisfy the model for Barkhausen noise, nevertheless, the noise is certainly due to rotation of domains. Now having modelled the field around and inside the small bar magnets one finds that the depolarizing H field is not uniform and that the magnetization is not uniform nor does the field in the magnet point in the same direction throughout the magnet. The non-uniformity is particularly large in magnets with the specific width to length ratio of those we are now using and is aggravated by the fact that NdFeB magnets, though they have a large remnant magnetization in the 2nd quadrant of the hysteresis curves, do not saturate the magnetization as cleanly as the older type Samarium Cobalt magnets. The Samarium Cobalt magnets given the B,H,M load lines would only have a force 20% smaller than the current magnets and could have much less of the domain rotation noise.

The new and, now convincing, data has come about because a strongly held idea I had about the way the noise would occur turns out to have been dead wrong. In all the bridge experiments I have been careful to balance the bridge by making sure that the magnets are included in both sides of the bridge and that they are polarized in the same direction in each coil. This time to measure the change of magnetization with field in the reverse direction using the bridge, I placed only one magnet in the bridge and found an easy to measure up-conversion noise. Thinking that maybe there was something special about the magnet (for example, a crack in the material), I tried several magnets all with similar results. Next inserted two magnets but with opposite polarity in the two coils and the up-conversion noise grew over that of the single magnet. So here it is, the noise from the domain motions is correlated in the two samples at the low frequencies where we have been looking. On some reflection, it now seems it has to be that way. The domains will rotate in the direction driven by the excitation field although not necessarily without a time delay. The envelope of the noise must therefore be determined by the excitation field. This explains several confusing experiences. First in earlier experiments with the bridge, it explains why sometimes there was a small broadband noise signal, but it was never the same run to run or reproducible. It was the effect of the fluctuations in subtraction of two large numbers. The correlation of the noise envelopes also explains why the specific tests to excite the Barkhausen noise in the test masses by the so called Pringle mode (excitation of opposite forces in the different magnets of the test masses) gave no results. Note that inverting the magnet and inverting the OSEM coil so that the forces are the same in the two magnets does not eliminate the correlation. It also makes the noise a more serious problem since all four magnets when driven by the displacement excitation contribute coherently to the up-conversion noise. *It would be worthwhile seeing if the displacement damping loops on all the test masses, ITM and ETM, are blameless in making excess noise. Furthermore, it would also be useful to estimate the common mode excitation of the test masses in estimating the up-conversion noise.*

As the data below will show, the broadband noise measured with the bridge has a $\frac{A}{(f - f_0)}$ shape.

f_0 is close to the sinusoidal drive frequency. The spectrum measured in the bridge experiment is proportional to the time derivative of the flux change in the coil. The spectrum of the varying magnetization and force is proportional to $\frac{B}{f^2}$ producing a displacement spectrum that varies as

$\frac{1}{f^4}$. This is indeed the up-conversion spectrum measured by Sam Waldman and Robert Schofield.

The amplitudes of the up-conversion is reasonably the same as that predicted from these bridge measurements using the fact that all four magnets are correlated. The well defined spectrum and the correlation leads one to think that there may still be some hope in removing the up-conversion noise from the interferometer output. A possibly interesting project for an enterprising student

There are several still unexplained factors. The magnet up-conversion noise varies between the square to the 3/2 power of the drive current. For the original model of the Barkhausen Poisson noise, the dependence would have been closer to 1/2 power of the current. More measurements are need to be taken to really pin this down. I don't understand this current dependence. Another mysterious aspect is that the magnet up-conversion is almost independent of the drive frequency for a fixed drive amplitude. There is a slight rise in the noise with lower frequencies. Again quite different than the Barkhausen Poisson model which would have given a noise proportional to the derivative of the current, so growing as the frequency. The frequency dependence needs also to be investigated further. My suspicion is that the noise is related to the complex magnetization field in the magnet and that there are not only domain flips (assumed by the Poisson model) but also continuous rotations of the domains in the non-uniform field. A hint that the non-uniform field in the small magnets is part of the cause comes from attempts made to measure the up-conversion noise in magnets with much larger diameter but about the same length. Here using the OSEM coils in the bridge with magnets inserted with opposite polarity, nothing is seen even though the large magnets used with the OSEM coils should have comparable sensitivity to the small magnets in the specially wound small coils that surround the magnet.

The next experiments should :

- 1) Try Samarium Cobalt magnets of the same dimensions as the current NdFeB magnets
- 2) Try placing a small soft iron cap on one end of the NdFeB magnets to establish if the redistribution of the field, by making more of the field lines point forward, reduces the up-conversion noise.
- 3) Try the idea of a rapidly varying bias field (say, 30kHz) to see if this reduces the low frequency up-conversion noise by effectively "lubricating" the domains. A similar technique was used in early wire recorders to reduce the Barkhausen noise.
- 4) Are there differences between the magnets themselves?

The data:

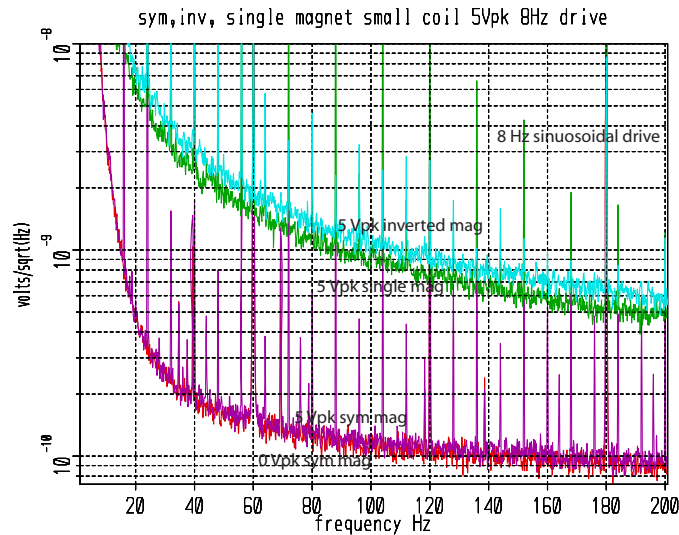


Figure 1 Mostly pedagogic picture to show the coherence of the noise envelopes in the magnets. The red curve at the bottom is the noise in the experiment with no drive on the bridge. The red-violet curve, almost coincident with the red curve at the bottom, is the case with balanced bridge driven with a 5 Vpk 8Hz excitation. The two magnets inserted in the two bridge coils are polarized in the same direction. This is the state in which the experiment was run originally. The 8Hz harmonics are excitations of some turns in the coil even after the coil was doped. The green curve at the top is the spectrum with only one magnet in one coil and the light blue is the case with oppositely polarized magnets in the two bridge coils.

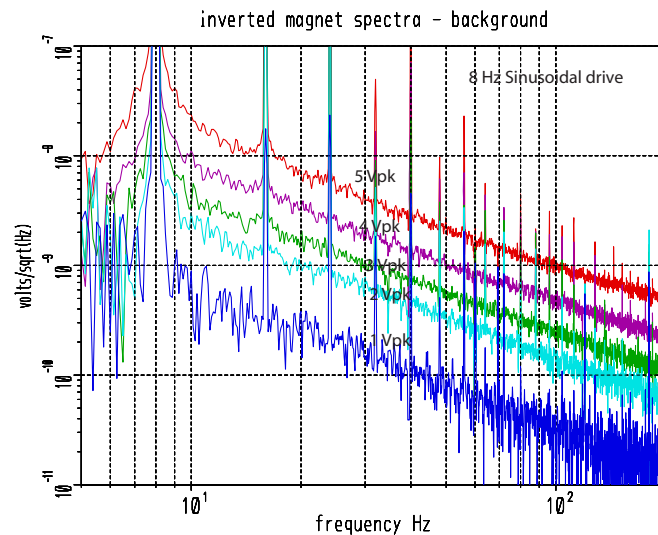


Figure 2 Up_conversion spectra of the case with two coils and two magnets oppositely polarized. The spectrum displayed is the difference between the 8Hz drive on with amplitude indicated in the figure and the background of the undriven case. The resistance of the coil and series resistor is 43 ohms (23ma for 1volt). The spectra above 40Hz have been fit by $\frac{A}{(f-f_0)}$, see **table 1**.

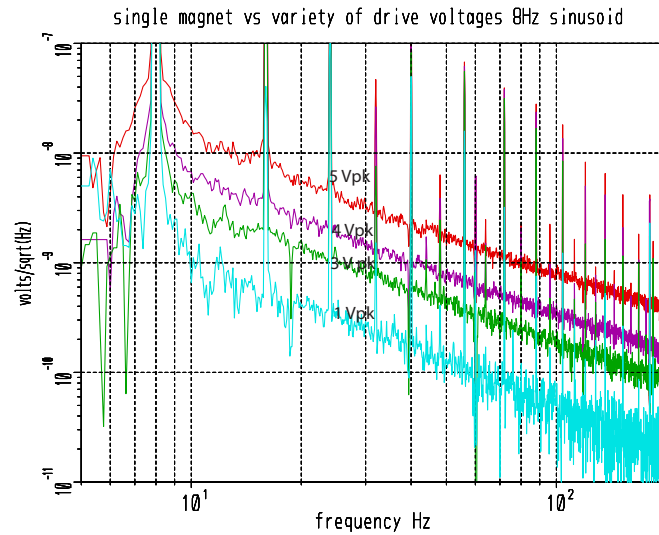


Figure 3: Similar to **Figure 2** but with only one magnet in one coil.

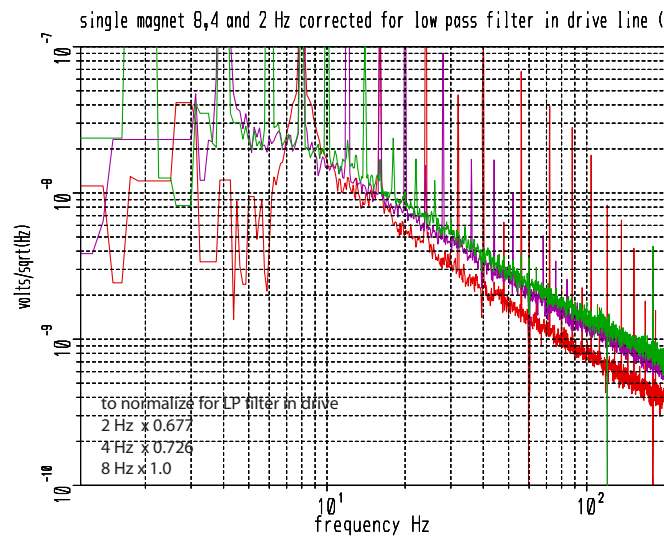


Figure 4: The figure shows the drive frequency dependence. A single magnet in one coil driven by 5 Vpk sinusoids at 8, 4 and 2 Hz . There is a low pass filter in the drive circuit to reduce harmonics. At the time the data was taken the assumption I made was a linear dependence of the noise on drive amplitude. The corrections applied to the spectra to normalize the drives and compensate for the low pass filter are given in the bottom left corner of the figure. The fact that the spectra grow with the 3/2 power of the amplitude make the normalizations somewhat invalid but nevetheless the initial assumption of a linear dependence of the up-conversion spectrum on the frequency is clearly wrong . The spectral model, results shown in **Table 1**, seems to be better than it deserves

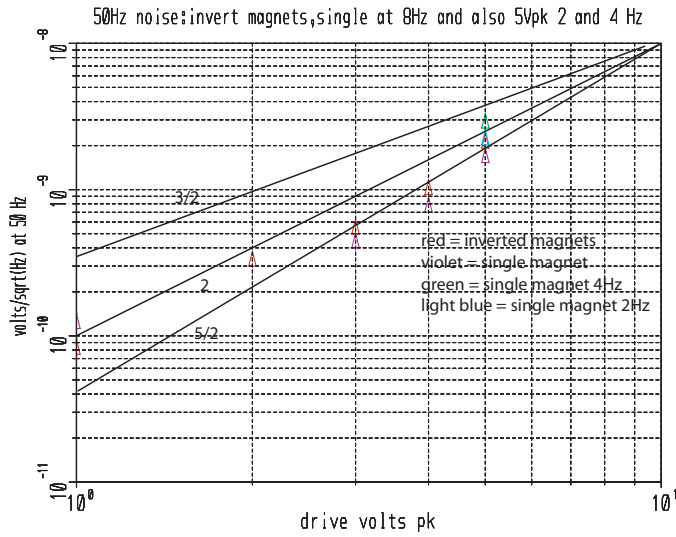


Figure 5: An attempt to show the drive amplitude dependence of the up-conversion spectrum at 50Hz. The closest dependence is the current to the 3/2 power. The data at low drive power is corrupted by the subtraction of the background. The measurement needs to be done more carefully.

Table 1: Up-conversion spectrum fit parameters, 8 Hz drive $V(f) = A/(f-f_0)$, fit between 40 to 200 Hz

Drive Vpk @ 8Hz	two magnets, one inverted $A \ V \sqrt{\text{Hz}}$	two magnets, one inverted $f_0 \text{ Hz}$	one magnet $A \ V \sqrt{\text{Hz}}$	one magnet $f_0 \text{ Hz}$
5	9.0×10^{-8}	8.4	7.3×10^{-8}	7.6
4	4.3×10^{-8}	8.2	3.1×10^{-8}	10.4
3	2.2×10^{-8}	11.0	1.6×10^{-8}	14.5
2	1.2×10^{-8}	14.0		
1	3.2×10^{-9}	11.0	3.9×10^{-9}	18.0

Note: that the normalized χ^2 runs between 1 to 10, the smallest values occurring at the largest drive.

Analysis: Table 2 lists the current parameters being used in the estimates and analysis. The end result is that it is not hard to explain the magnitude of the up-conversion with the parameters determined by the bridge experiment.

Table 2: Estimation parameters

OSEM coil N	200	magnet material	NEO-35 NdFeB
OSEM coil radius	0.953 cm	magnet radius	0.095 cm
OSEM coil length	0.476 cm	magnet length	0.317 cm
OSEM B/i	530 gauss/amp	coil/magnet force	1.6×10^{-2} newton/amp
OSEM max field grad.	1.19 i Tesla/m	magnet magnetic moment	0.014 Joules/Tesla
Small Coil N	626		
Small coil radius	0.23 cm	Small coil coupling efficiency g	$0.17 > g > 0.07$
Small coil length	0.4 cm	rms OSEM current and B	20 mA, 10 gauss (AC)
Small coil resistance	22 ohms	number of correlated magnets	$4 < Nm < 8$
Small coil B/i	1900 gauss/amp	up-conversion V(f) scaling	$i^{3/2}$
		V(f) for 10 gauss in small coil	$\frac{9.4 \times 10^{-9}}{(f - f_0)} \text{ volts}/\sqrt{\text{Hz}}$

To relate the observations made with the bridge experiment to those in the interferometer need to determine the fluctuations of the magnetic moment associated with the up-conversion voltage spectrum induced in the small coil. With the fluctuations in the magnetic moment it is easy to estimate the fluctuating force if one assumes knowledge of the magnetic field gradient from the coil at the magnet. The steps are the following (using the hateful MKS units). The fluctuations in the magnetic moment and the magnetic field are related $B(f) = \mu_0(H + M(f))$ where M is the magnetization per unit volume in the material. The voltage spectrum at the output of the small coil is then determined by Faraday's law expressed in terms of the magnetization $V(f) = \omega \Phi(f) = \omega B(f) g A_{\text{mag}} N_{\text{small coil}}$. Remembering that the magnetic moment of the magnet is the magnet volume times the magnetization, the fluctuation in the magnetic

moment is then $m(f) = \frac{V(f) L_{\text{mag}}}{\mu_0 \omega g N_{\text{small coil}}}$. The magnetic field gradient at the maximum force point out-

side of the OSEM coil at $\frac{a_{\text{osem}}}{2}$ is $\frac{dB}{dz} = \frac{0.43 \mu_0 N_{\text{osem}} i}{a_{\text{osem}}^2}$. At last, the fluctuating force is

$F(f) = m(f) \frac{dB}{dz}$. Now some choices are made. I assume that the drive current scaling for the up-con-

version is indeed the 3/2 power and that a typical rms current in the OSEM coils when there is up-conversion is 20 ma, this current would provide about 10 gauss AC field to drive the domain rotations as well as to

push on the magnet in the OSEM coil. Furthermore, I assume that the coupling efficiency of the magnet field to the small coil is $g = 0.1$. This comes from the amount of the B field returned between the poles of the magnet around the outside of the magnet within the small coil. (The coupling efficiency of the magnet to the OSEM coil almost vanishes as the coil diameter is so large, this is why using the OSEM coils in situ cannot be used to carry out these fluctuation measurements). Assume also that all four magnets on one mass act coherently in displacement forces so that the up-conversion noise spectra add in amplitude (not in power). The final result with these assumptions is a displacement noise for the two end masses added incoherently (suspect that for darm signals they should be added coherently but this is only a square root of 2) at 50Hz of 5×10^{-18} and at 100Hz 2×10^{-19} m/ $\sqrt{\text{Hz}}$.

(Hopefully) final study of the magnet behaviour responsible for the up-conversion in initial LIGO

R. Weiss May 16, 2007

Summary Report on data taken with the latest version of the bridge experiment to measure Barkhausen noise and other discontinuities in the magnetization of the NdFeB magnets used on the test masses. The results of the measurements are:

- SmCo magnets of the same size and almost equal strength to the magnets being used in initial LIGO do not show any measurable discontinuities, neither those that cause harmonic generation nor those that cause broadband Barkhausen noise.
- SmCo magnets of the same size are only a few percent weaker than the NdFeB magnets under the same “open circuit” conditions being used on the test masses.
- Calculations made in prior notes and repeated here show that the Barkhausen broadband noise generated in the magnetization of the NdFeB magnets *is responsible* for the $1/f^4$ upconversion spectrum measured in the interferometer.
- A mystery is why the harmonic generation from the discontinuity in the magnetization has not been observed in the interferometer.

Several ideas to reduce the magnet discontinuities and Barkhausen noise were tried. The results are:

- Placing soft iron collars on the magnet ends. This increased the Barkhausen noise and symmetrized it to give contributions for both field polarities.
- Chamfering and polishing the cylindrical magnet edges at the magnet ends. Here the concept was to reduce the internal tensile stresses in the magnet which are known to increase the Barkhausen noise. The result may have been a small reduction in the Barkhausen component but no reduction in the discontinuity that causes the harmonic generation.
- Using an “RF” bias magnetic field intended to facilitate the rotation of domains and thereby reduce both the discontinuities and Barkhausen noise. The bias from 5 to 50kHz with fields upto 50 Gauss were applied to the magnets while simultaneously applying the 8 Hz sinusoidal drive field. The bias fields only made matters worse and are an effective way of displaying the magnet’s non-linearity.
- The only remaining means of eliminating the up-conversion is to replace the magnets.

To prepare for the possible magnet change

Establish a minimum perturbation method to replace the current magnets for the enhanced LIGO run. It seems that a jig could be made to break the epoxy joint between the aluminium standoff and the mirror with little risk of chipping the mirror surface (shear rather than twist or bending). It then remains to establish a method of reattaching a magnet with fused silica stand offs. Aside from the established method of removing the test mass and attaching the magnets by the methods used in the initial installation one could now consider: UV cured epoxy between the standoff and the test mass, optical contacting and or silicate bonding between the standoff and the test mass. We currently have about 10 SmCo magnets from the order placed earlier this year. It would be prudent to place an order so that we have 30 more magnets (approximately \$1k) in the event that we go ahead to replace all the test mass magnets on the back faces of the 4k ETM and ITM. Fur-

ther tests of the up-conversion done after the S5 run will help in deciding whether only the ETM magnets need to be replaced. The fused silica standoffs should be ordered and polished and the magnets glued to the standoffs and baked.

Description of the experiment

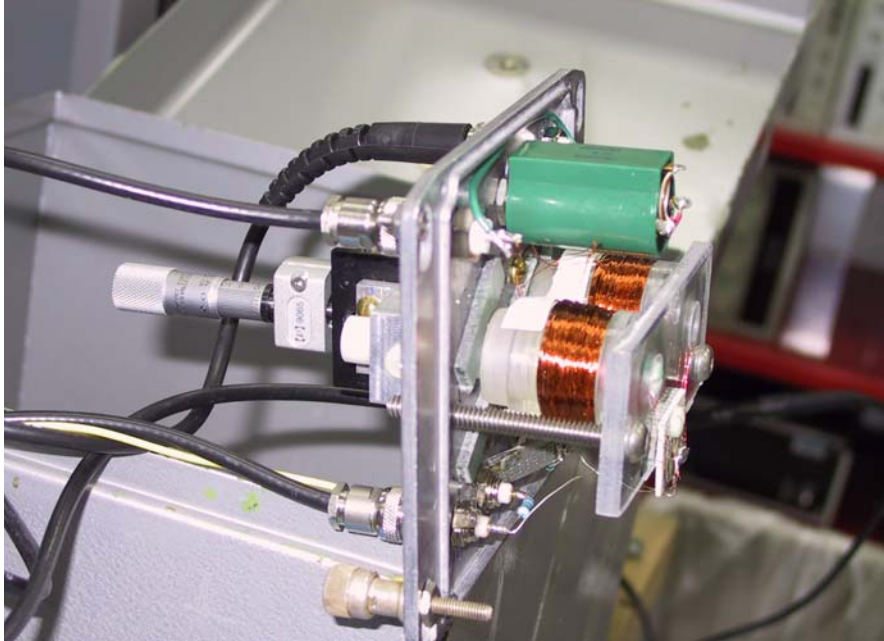


Figure 1 The latest version of the “bridge” apparatus. The drive coils, the easily seen ones, surround two smaller coils which contain the magnet(s). The smaller coils can be positioned along the longitudinal axis of the drive coils to change the mutual coupling. One of the small coils is positioned by a micrometer stage to perform the final balance (cancellation) of the drive induced currents in the two smaller coils connected in series. The “bridge” is balanced mechanically. Separating the drive and the sensing functions by using two coils eliminates the problem of thermally driven unbalance which plagued the original bridge apparatus.

Figure2 shows a schematic diagram of the entire experiment. As time wore on the rapid progress measurements were done by directly observing the response traces on the oscilloscope rather than making power spectra of the harmonics and the broadband noise. The association of the discontinuity in the oscilloscope trace with harmonic production and the fuzz in the trace with the generation of the broadband Barkhausen noise made it unnecessary to make the time consuming spectra for go/nogo decisions.

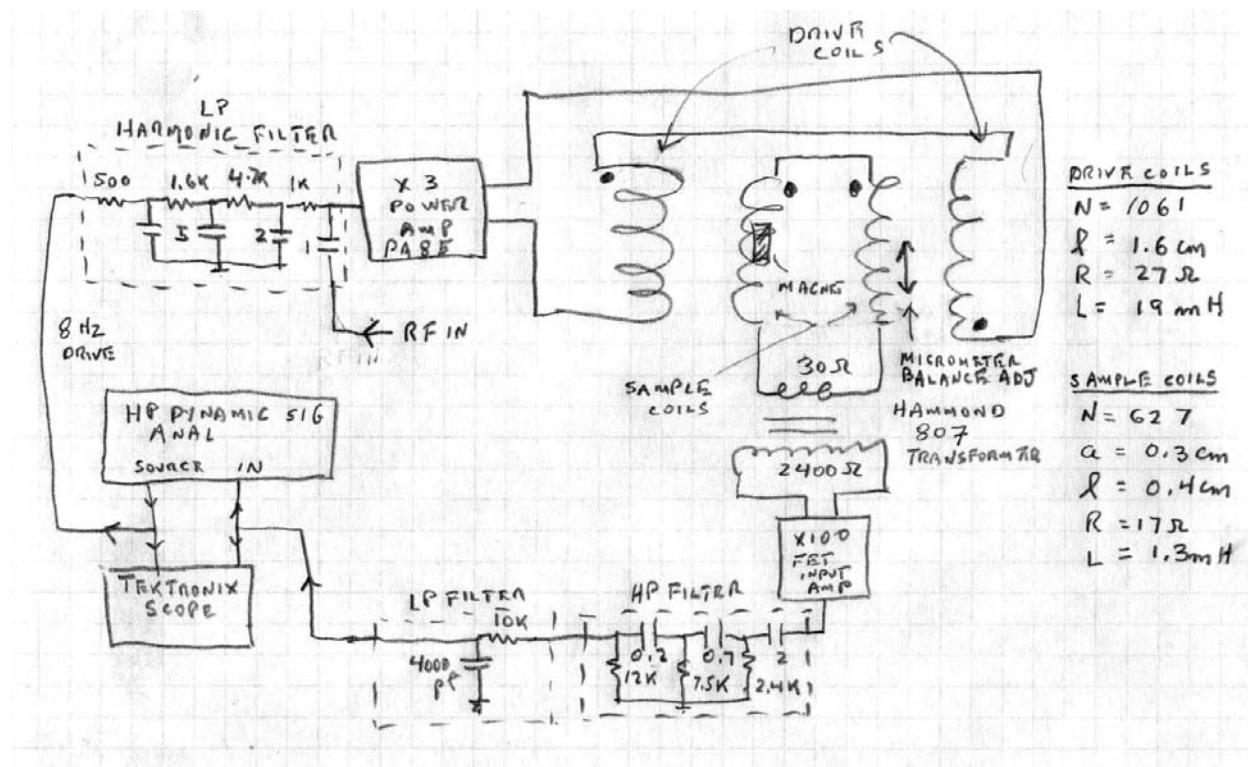


Figure 2 Schematic of the latest (May 2007) version of the “bridge” apparatus. The major change has been to make the position of the sample coils adjustable along the length of the drive coils. This to determine if the magnet non-linearity was dependent on the field or gradient of the field at the magnet. It turned out that the non-linearity was only a function of the field and that what was thought to be a gradient sensitivity had to do with motion of the magnet in the sampling coils due to inadequate clamping in some earlier measurements. The trimming to null the fundamental drive frequency current in the primary of the transformer is done by moving one of the sample coils longitudinally in its drive coil with a micrometer.

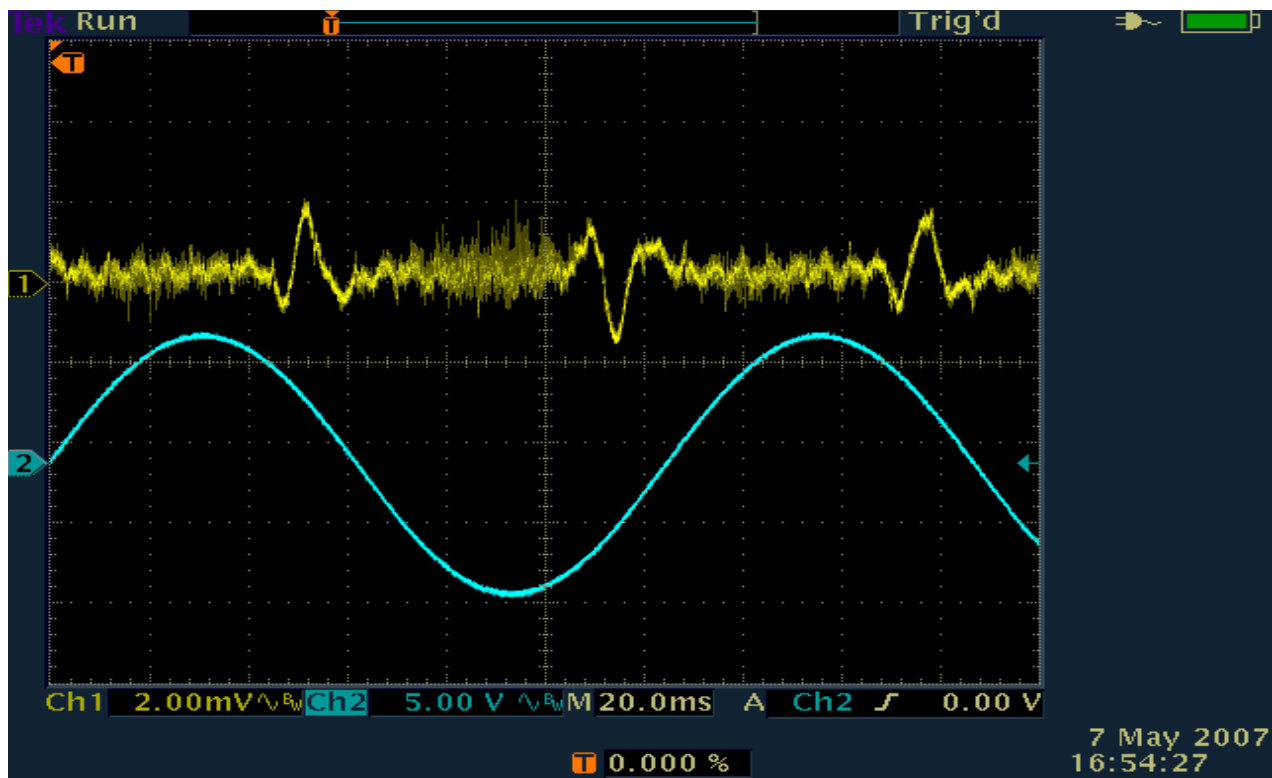


Figure 3 NdFeB magnet F with aluminium dumbbell standoff attached. The upper yellow curve is the high pass filtered output of the transformer preamplifier combination with the coils balanced to produce little 8Hz feedthrough. The lower blue curve is proportional to the drive of 5Vpk at 8Hz. The discontinuities near the zeroes of the drive cause the harmonics in the magnet spectrum while the fuzz seen mostly on the negative half of the drive cycle is due to the Barkhausen effect and causes the broadband noise. The asymmetric Barkhausen effect is due to the polarization of the magnet relative to the drive. Inverting the magnet in the coil will cause the noise to occur primarily in the positive cycle.

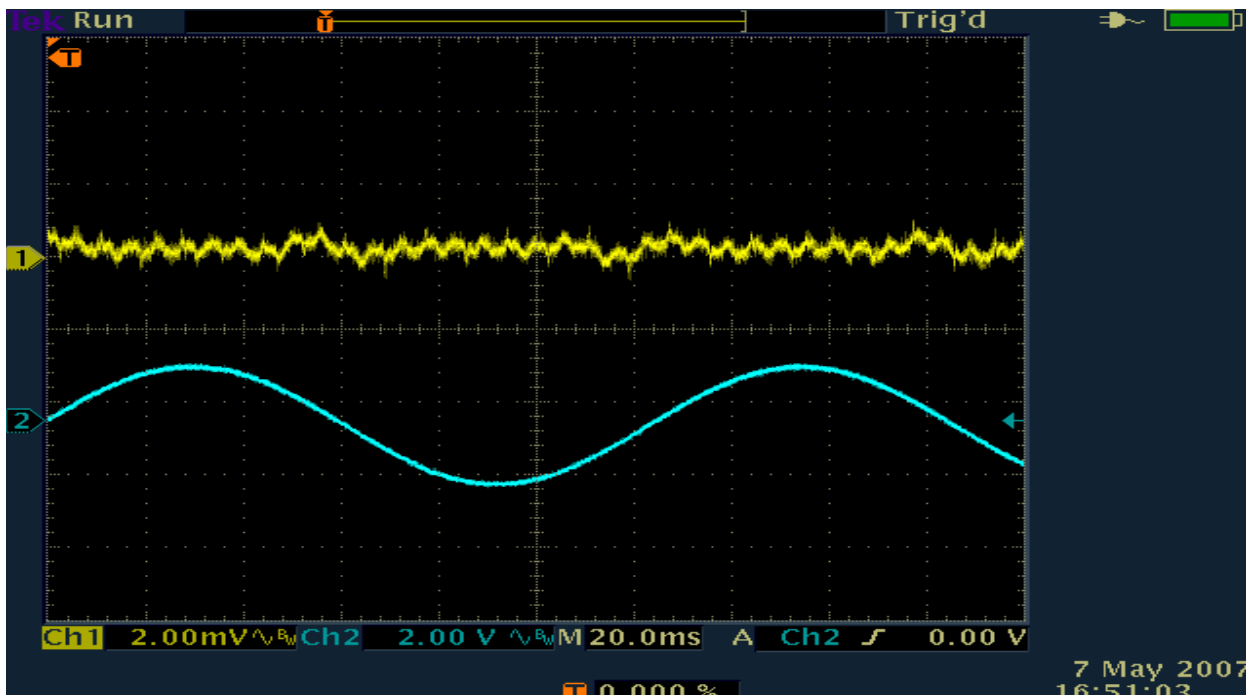


Figure 4 Same sample as **Figure 3** but with 1Vpk 8 Hz drive. The ripple in the yellow trace is due to the 60Hz line and harmonics and present without the drive.

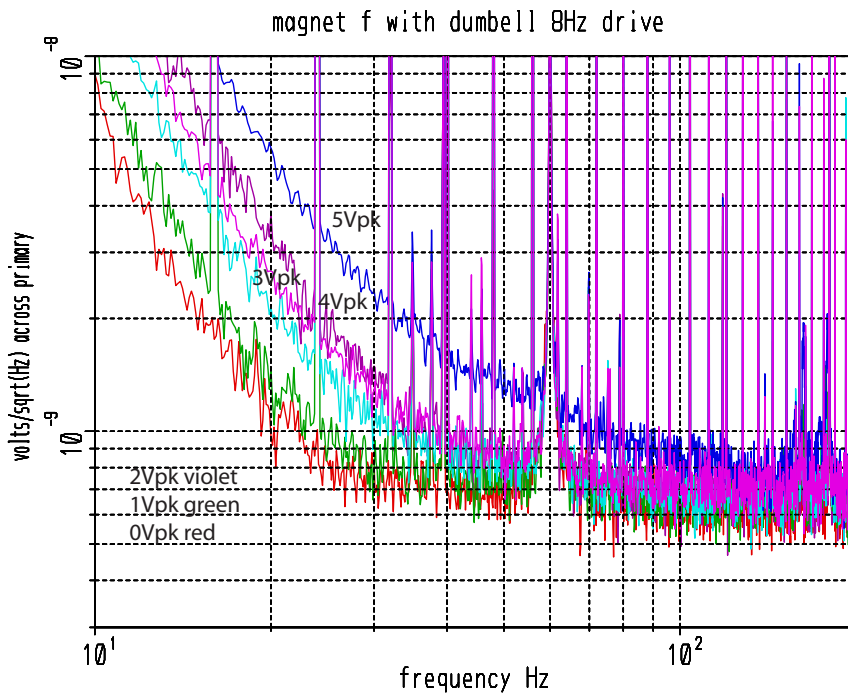


Figure 5 Frequency spectra of the Barkhausen noise and the harmonics from the discontinuity.

The spectrum has been much discussed in prior notes. The Barkhausen part leads to $1/f^4$ up-conversion spectrum observed in the interferometer. The harmonics have not been directly observed in the interferometer, which frankly I do not understand.

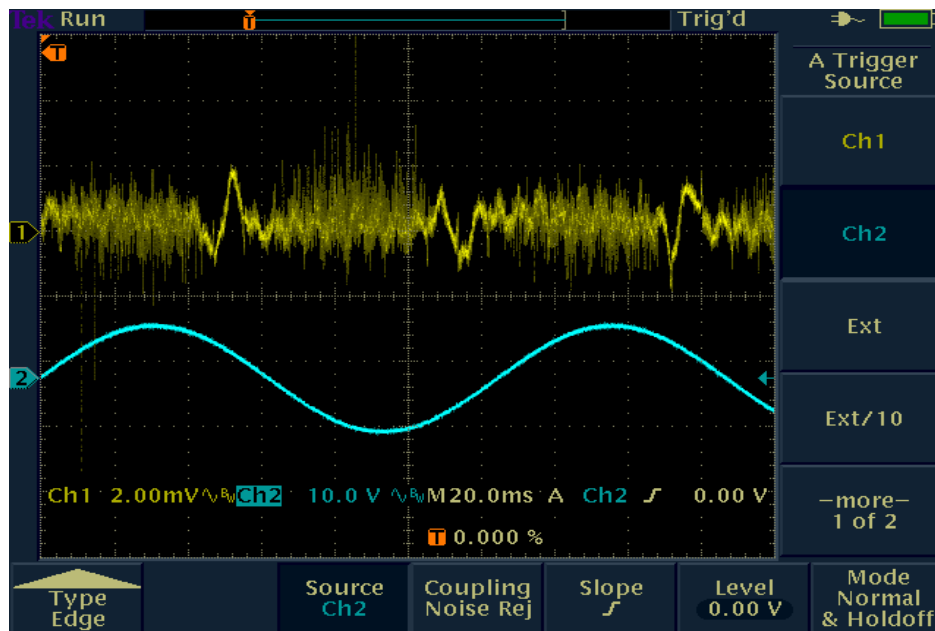


Figure 6 (Mostly for fun) The time series of the output and drive for the magnet E with two 0.002" thick collars 0.5mm wide of pure iron at each end. The Barkhausen noise is worse but symmetric with field polarity. 5Vpk drive at 8 Hz.

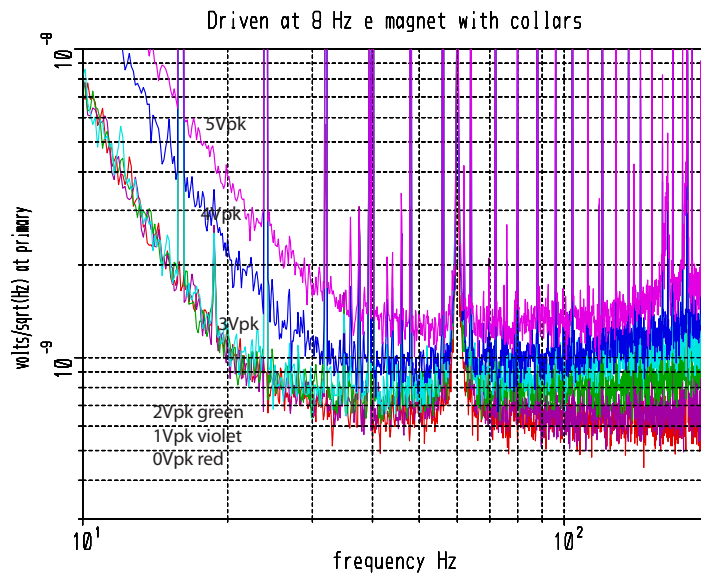


Figure 7 The up-conversion spectrum of magnet E with collars, the time domain is shown in **figure 6**. The spectrum is not the same as for the NdFeB magnets since it rises at frequencies higher than 100 Hz. The fact that the Barkhausen noise is large in pure iron sheet is, in retrospect, no surprise, maybe another material would have been better for a collar. One loses interest in collars seeing the data from SmCo.

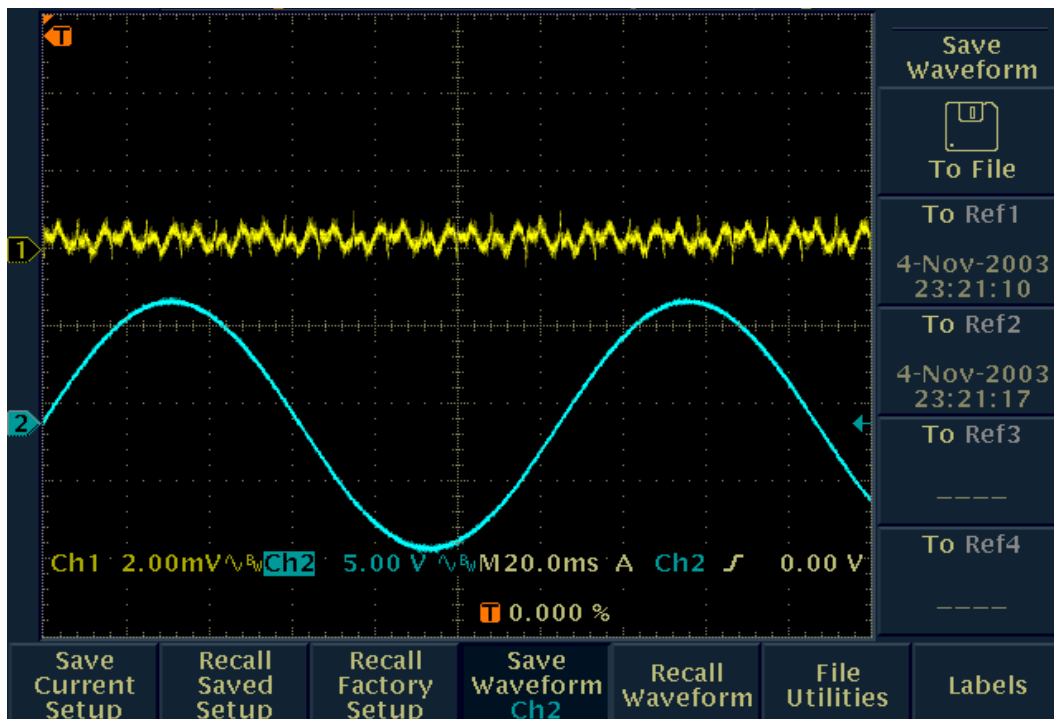


Figure 8 The times series of output and drive for a SmCo magnet. The drive is the maximum 5Vpk. The output is all derived from the 60 Hz line and its harmonics. There is no visible discontinuity nor broadband Barkhausen noise. The next figure shows this better.

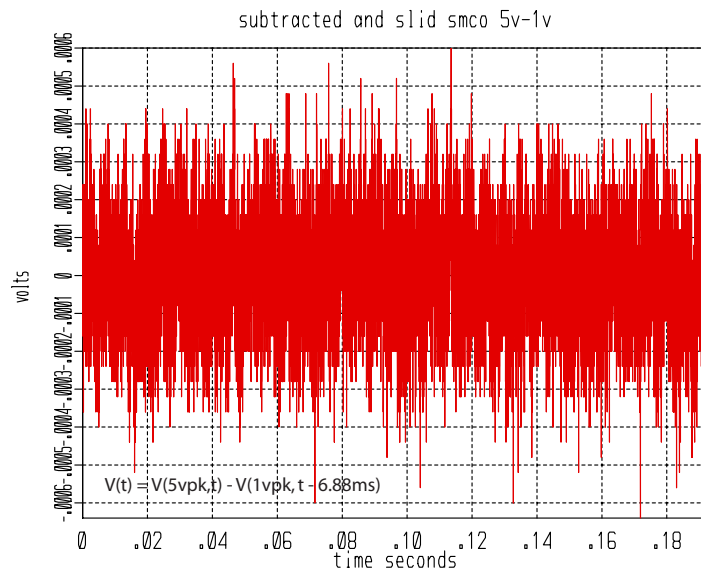


Figure 9 Difference of the 5Vpk and 1Vpk drive data for the SmCo magnet. The vertical units are in volts at the input to the oscilloscope. The data had to be slid in time by 6.88 msec to eliminate the 60Hz and harmonic time series that dominated the time series. (The time slide was necessary due to my not triggering the scope from a fixed point in the drive for the two scans.) The data has no features at high multiples of 8Hz, shows no Barkhausen noise above standard electronics noise nor any discontinuity. The power spectrum of the 5Vpk and 0Vpk data are shown in the next figure.

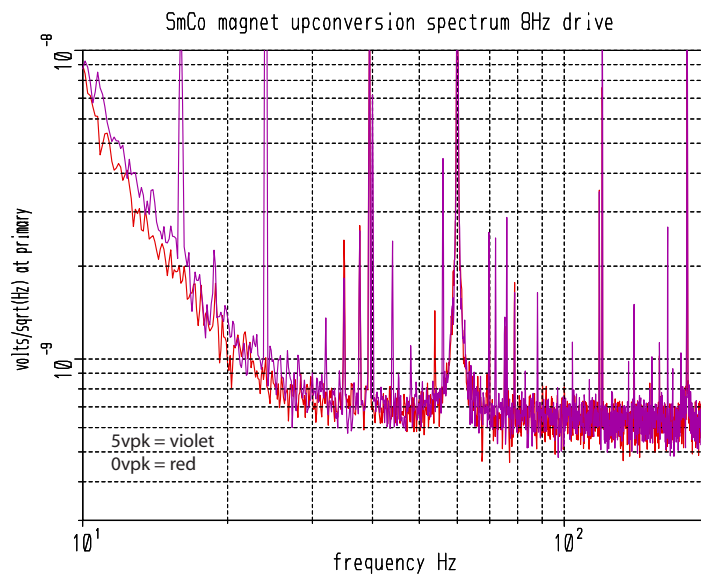


Figure 10 The up-conversion spectrum of the SmCo magnet at 5Vpk drive at 8Hz and without drive. There is a little pickup of the 8, 16 and 24 Hz drive which is in the data even when there is no magnet in sample coil. The harmonics structure at higher frequencies, so evident in the NdFeB magnets, is absent and there is no broadband Barkhausen noise above the electronics noise.

Data analysis: comparison of the SmCo and NdFeB magnets

The table below shows the comparison between the two types of magnets. All the present noise measurements were made with SmCo magnets manufactured by Electron Energy Corporation. The part number of the magnets is D07412410750TISO and the lot # is 7355-1. The magnet material is SmCo₂:17-27 (the B vs H curve is in the appendix of the report). The magnet dimensions, 0.075" diameter and 0.125" in length, are identical to the NdFeB magnets that had been made by the same company and are used on all the initial LIGO large optics. Earlier tests on SmCo used magnets given to us by the Dexter Magnetic Technologies. These have dimensions .070" diameter and 0.135" length. The material of these magnets is SmCo type R26H . These magnets also showed no broadband Barkhausen noise and no discontinuity in the magnetization although the limits set are not as good. The force/current with an OSEM coil was measured for all the magnet types. The technique was to use a strip of copper foil with adhesive backing as a cantilever beam. The magnet is stuck on the adhesive about a magnet length above the coil. The current required to pull the magnet entirely into the coil is measured.

Table 1: Comparison of NdFeB and SmCo

Property	NdFeB	SmCo	SmCo/NdFeB
Discontinuity in M after removal of 60Hz multiples	3.2 mV	< 0.02mV	< 1/160
Up-conversion spectrum 5Vpk - 0Vpk at 40Hz	2nV/ $\sqrt{\text{Hz}}$	< 0.1nV/ $\sqrt{\text{Hz}}$	< 1/20
Force/amp Newtons/amp in OSEM coil (Electron energy magnet)	0.018	0.017	0.932
Force/amp Newtons/amp (Dexter Magnetic Technologies)		0.0187	1.04

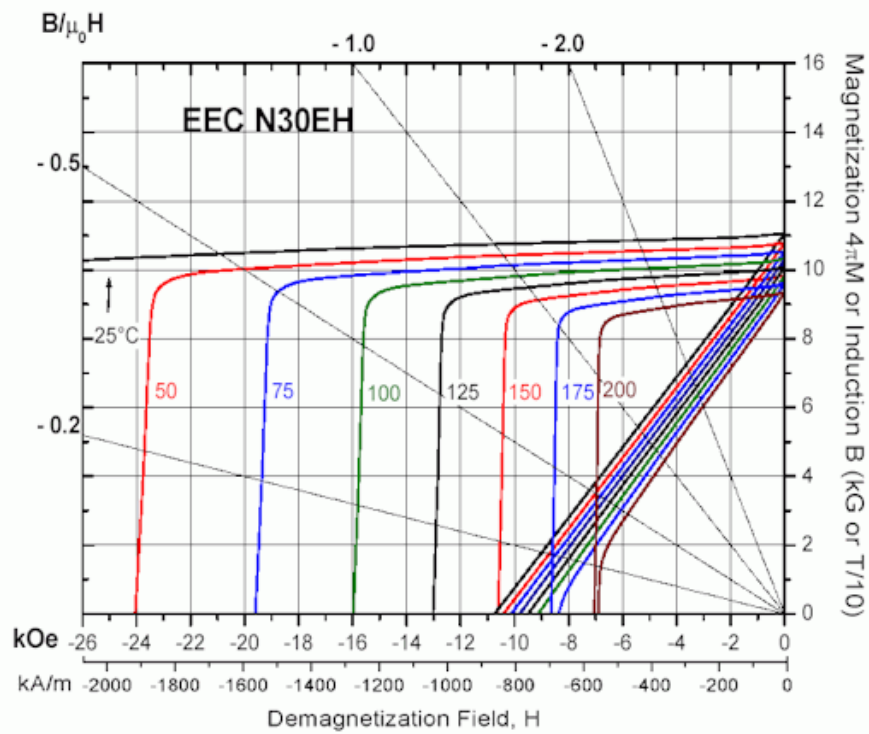


Figure 11 B vs H and energy product curves for Electron Energy Corporation NdFeB magnets close to the ones we have in initial LIGO. The material of our magnets NEO35 which has a slightly higher H_c and B_c at 25C

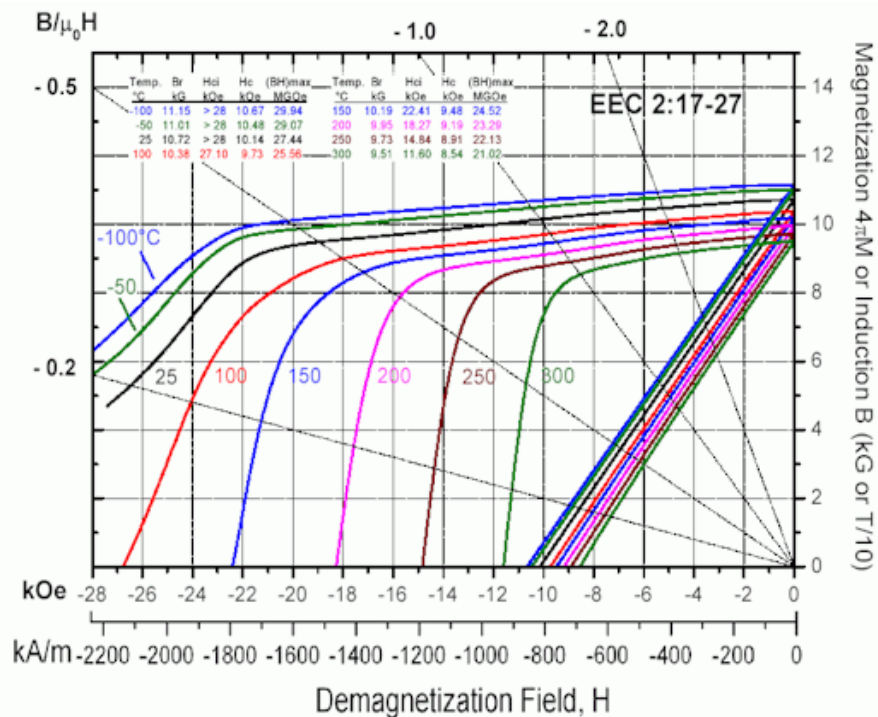


Figure 12 B vs H and energy product for SmCo 2:17-27, material used in tests.

Analysis: Table 2 lists the current parameters being used in the estimates and analysis. The end result is that it is not hard to explain the magnitude of the up-conversion with the parameters determined by the bridge experiment.

Table 2: Estimation parameters

OSEM coil N	200	magnet material	NEO-35 NdFeB
OSEM coil radius	0.953 cm	magnet radius	0.095 cm
OSEM coil length	0.476 cm	magnet length	0.317 cm
OSEM B/i	530 gauss/amp	coil/magnet force	1.6×10^{-2} newton/amp
OSEM max field grad.	1.19 i Tesla/m	magnet magnetic moment	0.014 Joules/Tesla
Small Coil N	626		
Small coil radius	0.23 cm	Small coil coupling efficiency g	$0.17 > g > 0.07$
Small coil length	0.4 cm	rms OSEM current and B	20 mA, 10 gauss (AC)
Small coil resistance	22 ohms	number of correlated magnets	$4 < Nm < 8$
Small coil B/i	1900 gauss/amp	up-conversion V(f) scaling	$i^{3/2}$
		V(f) for 10 gauss in small coil	$\frac{9.4 \times 10^{-9}}{(f - f_0)} \text{ volts}/\sqrt{\text{Hz}}$

To relate the observations made with the bridge experiment to those in the interferometer need to determine the fluctuations of the magnetic moment associated with the up-conversion voltage spectrum induced in the small coil. With the fluctuations in the magnetic moment it is easy to estimate the fluctuating force if one assumes knowledge of the magnetic field gradient from the coil at the magnet. The steps are the following (using the hateful MKS units). The fluctuations in the magnetic moment and the magnetic field are related $B(f) = \mu_0(H + M(f))$ where M is the magnetization per unit volume in the material. The voltage spectrum at the output of the small coil is then determined by Faraday's law expressed in terms of the magnetization $V(f) = \omega \Phi(f) = \omega B(f) g A_{\text{mag}} N_{\text{small coil}}$. Remembering that the magnetic moment of the magnet is the magnet volume times the magnetization, the fluctuation in the magnetic

moment is then $m(f) = \frac{V(f) L_{\text{mag}}}{\mu_0 \omega g N_{\text{small coil}}}$. The magnetic field gradient at the maximum force point out-

side of the OSEM coil at $\frac{a_{\text{osem}}}{2}$ is $\frac{dB}{dz} = \frac{0.43 \mu_0 N_{\text{osem}} i}{a_{\text{osem}}^2}$. At last, the fluctuating force is

$F(f) = m(f) \frac{dB}{dz}$. Now some choices are made. I assume that the drive current scaling for the up-con-

version is indeed the 3/2 power and that a typical rms current in the OSEM coils when there is up-conversion is 20 ma, this current would provide about 10 gauss AC field to drive the domain rotations as well as to

push on the magnet in the OSEM coil. Furthermore, I assume that the coupling efficiency of the magnet field to the small coil is $g = 0.1$. This comes from the amount of the B field returned between the poles of the magnet around the outside of the magnet within the small coil. (The coupling efficiency of the magnet to the OSEM coil almost vanishes as the coil diameter is so large, this is why using the OSEM coils in situ cannot be used to carry out these fluctuation measurements). Assume also that all four magnets on one mass act coherently in displacement forces so that the up-conversion noise spectra add in amplitude (not in power). The final result with these assumptions is a displacement noise for the two end masses added incoherently (suspect that for darm signals they should be added coherently but this is only a square root of 2) at 50Hz of 5×10^{-18} and at 100Hz 2×10^{-19} m/ $\sqrt{\text{Hz}}$.

Further studies of the magnetization noise in the NdFeB and SmCo magnets

R. Weiss June 18, 2007

Summary:

1) The idea suggested by Sam Waldman to place a SmCo magnet on top of a NdFeB magnet as a means of reducing the Barkhausen noise and the magnetization discontinuity amplitude, without having to unglue the magnet, was tried. The Barkhausen noise became larger and was symmetrized, occurring on both positive and negative field excursions. The magnetization discontinuity grew in amplitude but did not change in phase. The idea does not serve the purpose intended.

2) Establish with DC bias fields, both by passing DC currents through the drive coils and by bringing external magnets in the proximity of the NdFeB magnet, how the magnetization discontinuity and the Barkhausen noise change with B field. In part, this is a test of whether the PAM magnets affect the performance of the drive magnets as well to attempt to understand better the mechanism for the magnetization discontinuity. The results of this study indicate that with fields typical of the PAM magnets at the location of the drive magnets, fields as large as several 100 Gauss, neither the Barkhausen noise nor the magnetization discontinuity are changed by DC external fields. Fields of 1000 Gauss do cause new behaviour in the magnets.

3) The Barkhausen noise and the magnetization discontinuity became easier to observe with increasing frequency. The noise and non-linearity in the NdFeB magnets and SmCo magnets were once again measured at 33 Hz. Neither Barkhausen noise nor the magnetization discontinuity are observed in the SmCo. The upper limit on the Barkhausen noise in the SmCo is smaller than a factor of 1/500 of the NdFeB. The magnetization discontinuity in SmCo is smaller by 1/500 relative to the NdFeB magnets.

4) The magnetization discontinuity occurs shortly after the change in sign of the magnetic field derivative with time - shortly after the extremums of the magnetic field. The changes in magnetization discontinuity were measured with varying drive field at fixed frequency and with currents of fixed amplitude but varying frequency. The NdFeB material seems to be a dreadful non-linear magnetic system and one wonders if it is really worth understanding it.

Units and conversions

The dimensions of the coils and magnets are given in the report of May 16, 2007. As the apparatus has evolved the parameters that have changed are the drive coil magnetic field/current and the relation between the voltage developed by the sample coil to the magnetic field change from the magnet inside the coil.

The magnetic field in the drive coil is given by

$$B(\text{gauss}) = 30.8 \times \text{Volts}(\text{across single drive coil})$$

The magnetic field derivative in the sample coil (if coupled with $g = 0.1$, the magnet coupling) is related to the voltage at the oscilloscope input by

$$\frac{dB(\text{gauss})}{dt(\text{seconds})} = 1.6 \times 10^5 \text{V}(\text{oscilloscope})$$

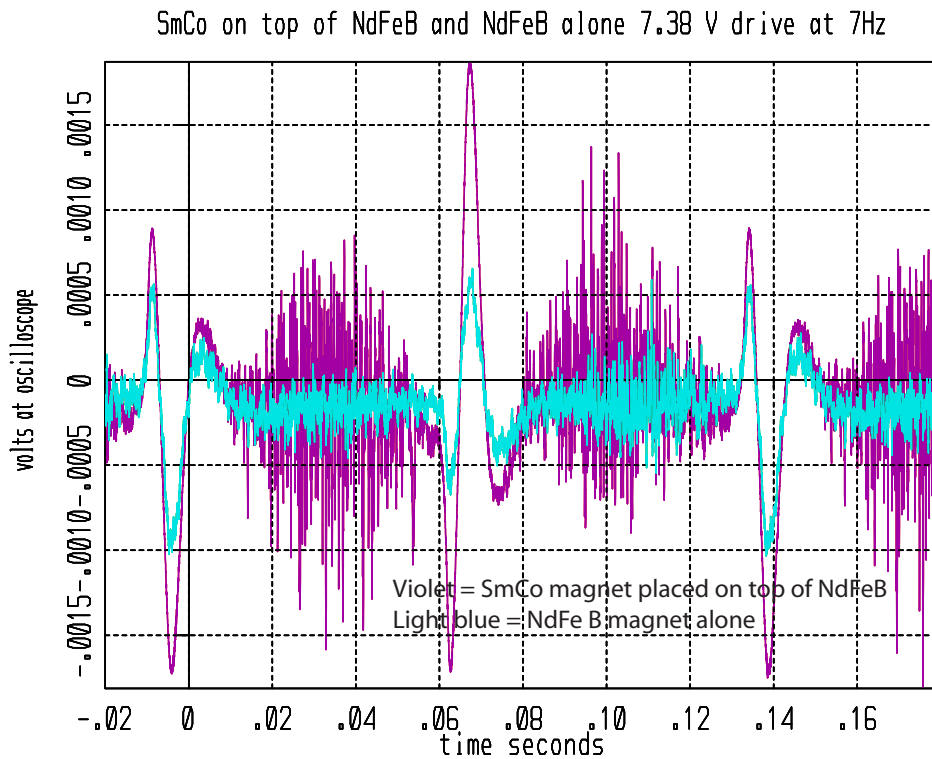


Figure 1 The Barkhausen noise and magnetization discontinuity in NdFeB magnet polarized in + direction (note asymmetry of the Barkhausen noise) with a drive of 7.4 volts peak (230 gauss) at 7Hz is shown in light blue. The noise and discontinuity with a SmCo magnet placed on top of the NdFeB magnet is shown in violet. The magnets are identical in size and dipole moment. The NdFeB magnet positive pole was attached to the SmCo negative pole. The drive field varies from a positive maximum at -0.02 seconds to a negative minimum at 0.05 seconds. The Barkhausen noise is symmetrized by placing the SmCo on top of the NdFeB magnet and the magnetization discontinuity is increased in amplitude but not changed in phase. This and subsequent figures were made by using the averaging feature of the Tektronix digital scopes and their ability to write a numerical file of the data displayed.

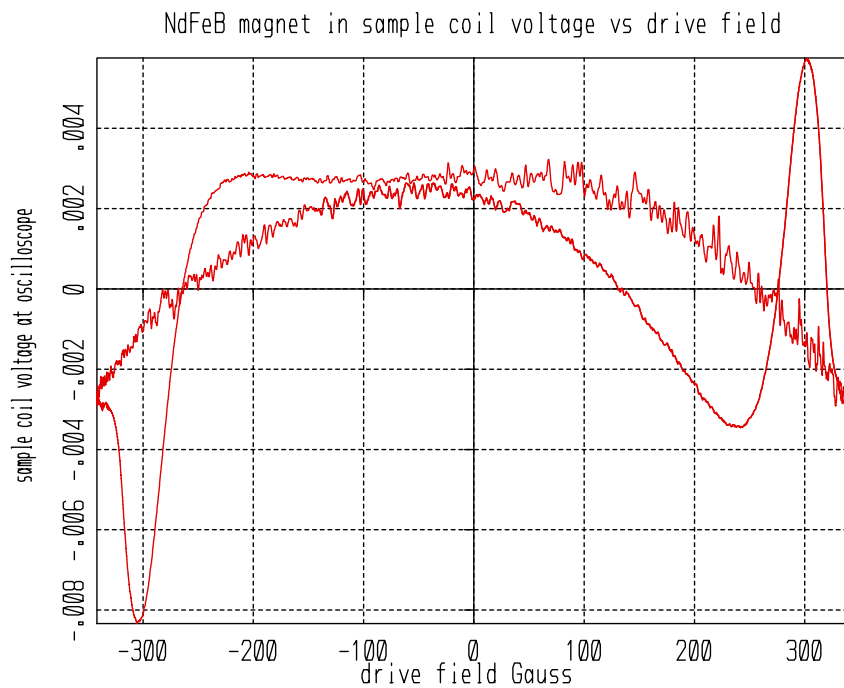
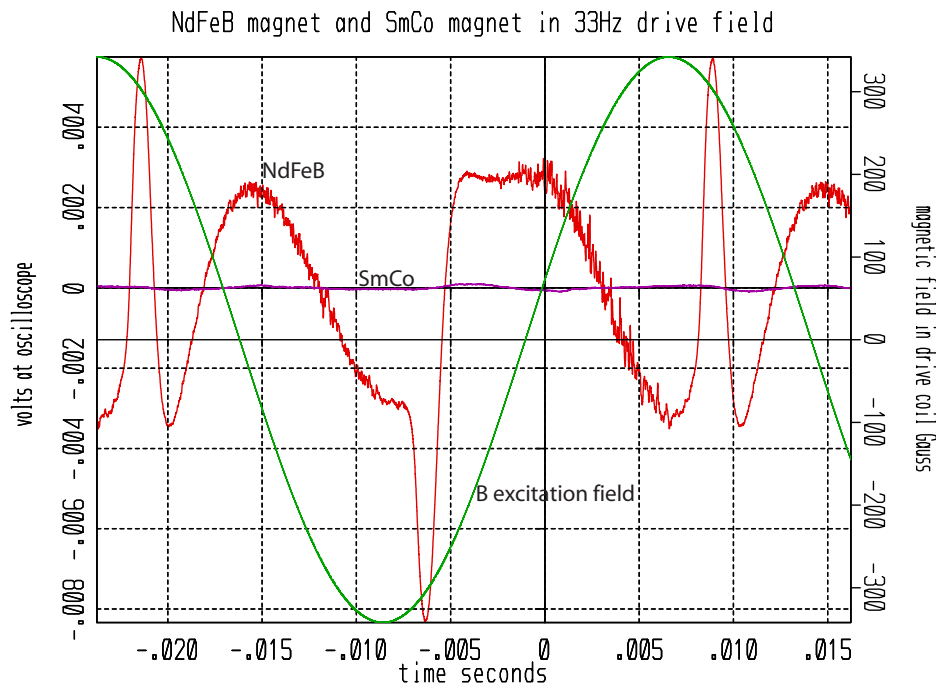


Figure 2a and 2b The complex magnetization discontinuities and the Barkhausen noise in NdFeB magnets as well as the behaviour of SmCo magnets with comparable magnet moment (consistent with amplifier noise) are shown with the time dependent exciting field in **Figure 2a**. The hysteretical behaviour of the NdFeB is seen most easily by replotting the oscilloscope voltage against the drive field. **Figure 2b**. The frequency of the drive field is 33Hz. The signal to noise on the magnetization non-linearities in these representations is so high that 33Hz is a good place to carry out go/nogo testing on the magnets. The magnetization discontinuities occur 35 degrees in phase after the field extrema, the Barkhausen noise occurs after the field zero crossings.

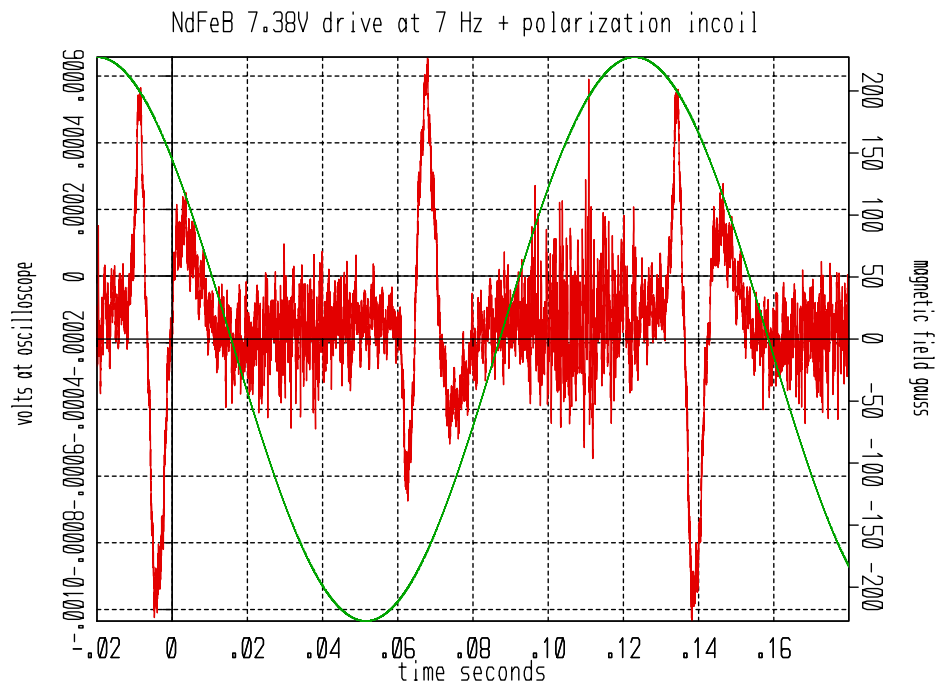


Figure 3a Barkhausen noise and magnetization discontinuity in NdFeB magnet with + polarization. 7Hz drive frequency.

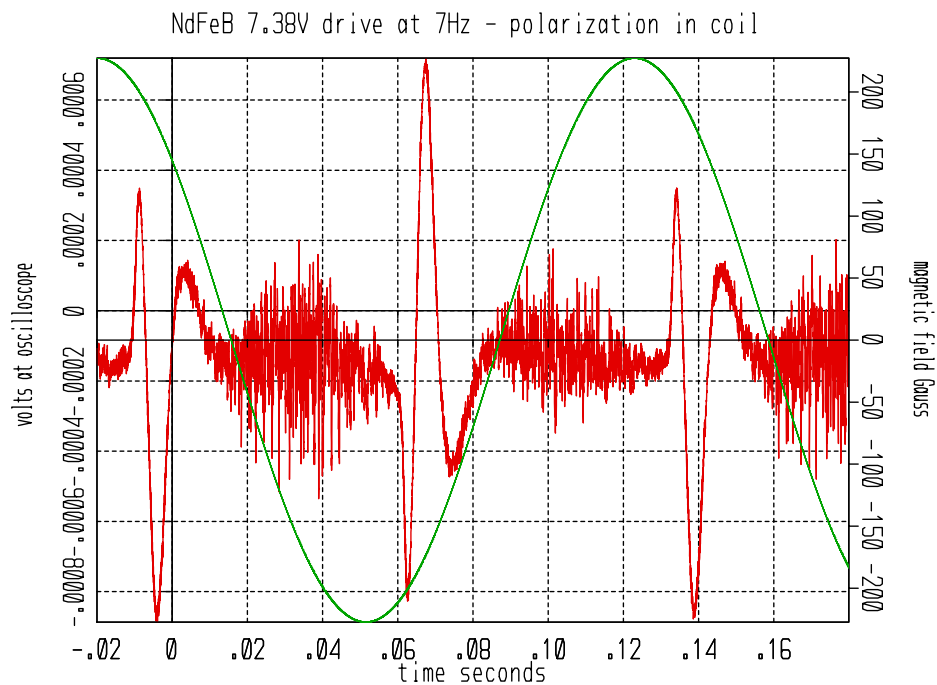


Figure 3b Barkhausen noise and magnetization discontinuity in NdFeB magnet with - polarization. 7Hz drive. Note the change in Barkhasuen noise with sign of the magnetic drive field.

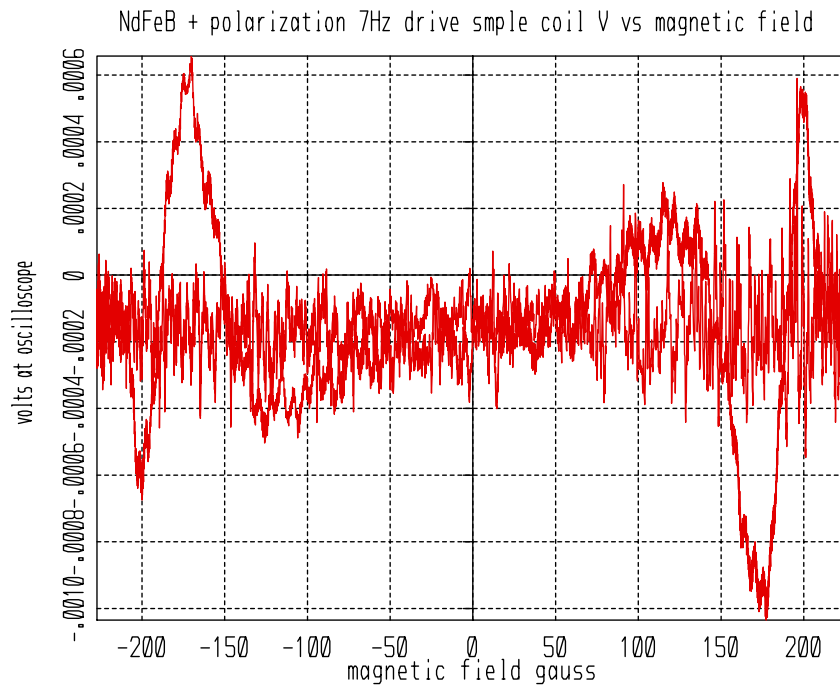


Figure 4a Replot of the NdFeB oscilloscope voltage with + polarized magnet as a function of the drive field. The signal to noise at 7Hz is much poorer than at 33Hz. The magnetization discontinuities again occur shortly after the extrema of the field where the field derivative with time changes sign.

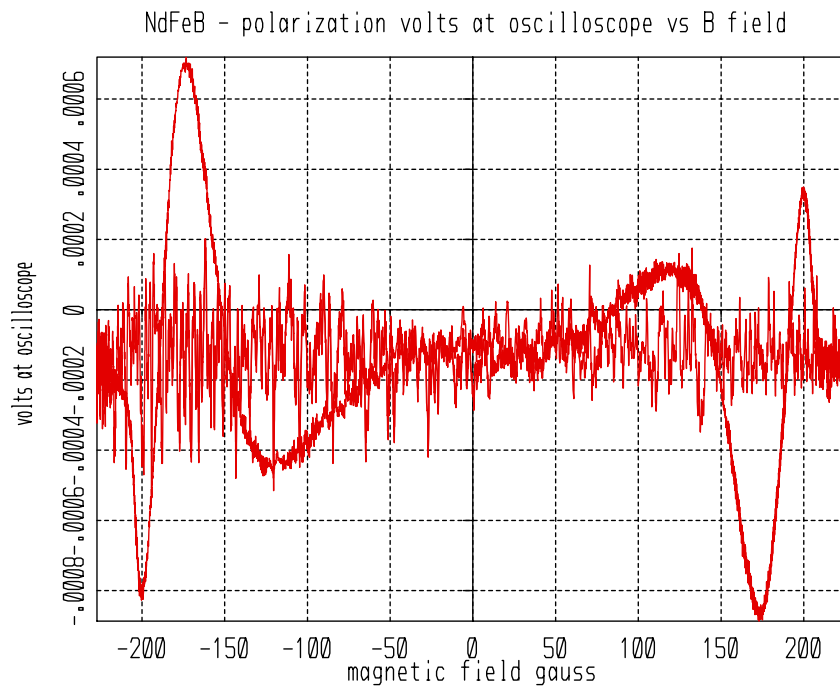


Figure 4b Replot of the NdFeB oscilloscope voltage with - polarized magnet as a function of the field strength.

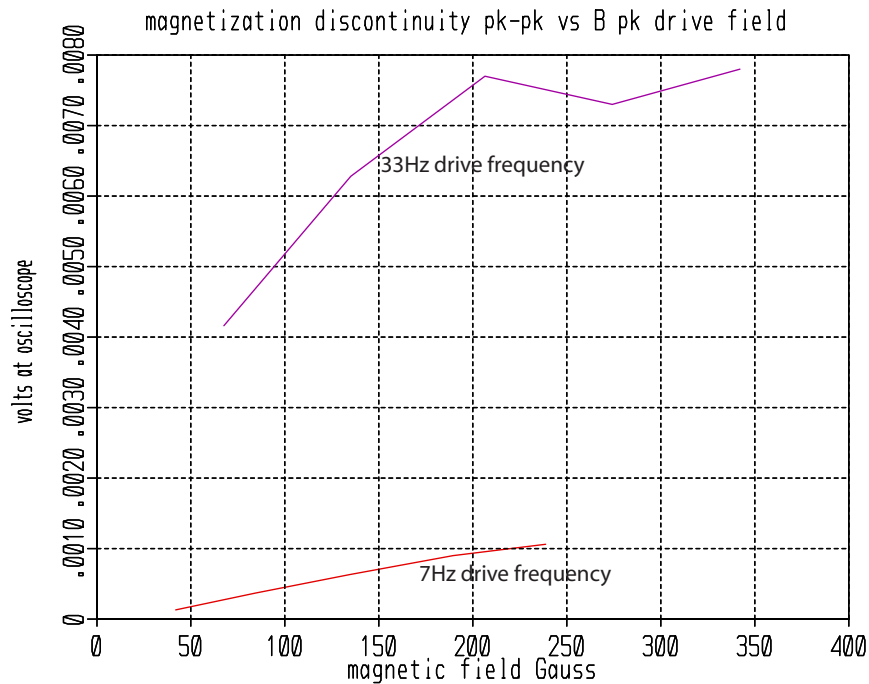


Figure 5 The pk to pk magnetization discontinuity voltage on the oscilloscope vs the drive field at 7Hz (red) and at 33Hz (Violet).

The magnetization discontinuity

Had hoped to understand this better but it is still mysterious. The properties of the discontinuity are:

1) The discontinuity occurs between 28 and 35 degrees after the field extrema almost independent of the drive amplitude and frequency. The width of the discontinuity is almost constant in phase again independent of drive amplitude and frequency. Typically the pk to pk magnetization change relative to the magnetization in the permanent magnet is $\frac{\Delta M}{M} = 7 \times 10^{-5}$ which seems a small quantity until one realizes that the Barkhausen contribution is even smaller and can cause so much trouble.

2) The amplitude of the discontinuity grows with drive amplitude but saturates as can be best seen in the 33Hz data. For magnetic drive fields less than 100 gauss, discontinuity pk to pk amplitude grows linearly with drive field amplitude.

3) The amplitude of the magnetization discontinuity grows faster than linearly with frequency. If the wave shape does not change one would expect a linear dependence on the amplitude due to Faraday's law.

It is very strange that we do not see the harmonic generation from this magnetization discontinuity in the LIGO displacement spectrum. It should be a wonderful way to make up-conversion independent of the Barkhausen noise contribution. I could talk myself into thinking that, with the

four magnets driven differentially and with identical non-linearities, the up-conversion from the magnetization discontinuity cancels - it doesn't sound right and the magnets are not that identical.

It almost looks like there is a collection of domains that rotate all at once when the field derivative with time changes sign. The number of domains partaking in this sudden transition after accumulation must be limited since the magnetic discontinuity saturates.

This latest study has made me even more uncomfortable with these magnets, they are simply too complicated to understand and they confirm my distrust of solid state physics as a horrible collection of uninteresting special cases.

Further studies of the magnetization noise in the NdFeB and SmCo magnets
D. Kelley July 18, 2007

Summary

SmCo magnets were proposed as a replacement for the current NdFeB magnets used to control the test mass mirrors. 43 of these SmCo magnets were tested. It was found that ratio of the magnetic discontinuities in SmCo to those in NdFeB was $3.4e-3 \pm 6.0e-4$.

Experiment

Six NdFeB and 43 SmCo magnets were tested in the magnet setup. The coil was driven at 33 Hz with a peak magnetic field of 350 gauss¹.

The dimensions of the coils and magnets are given in Rai Weiss' report of May 16, 2007.

Analysis

Each signal was processed by fitting a 33 Hz sine wave to the data then removing it. This signal was due to the 33Hz drive frequency, which remained because of our inability to perfectly balance the apparatus. Following this, the signal generated by the apparatus run without a magnet in it (33 Hz fit and removed) was subtracted from the signals. This second signal was due to harmonic generation in the source and the drive amplifier.

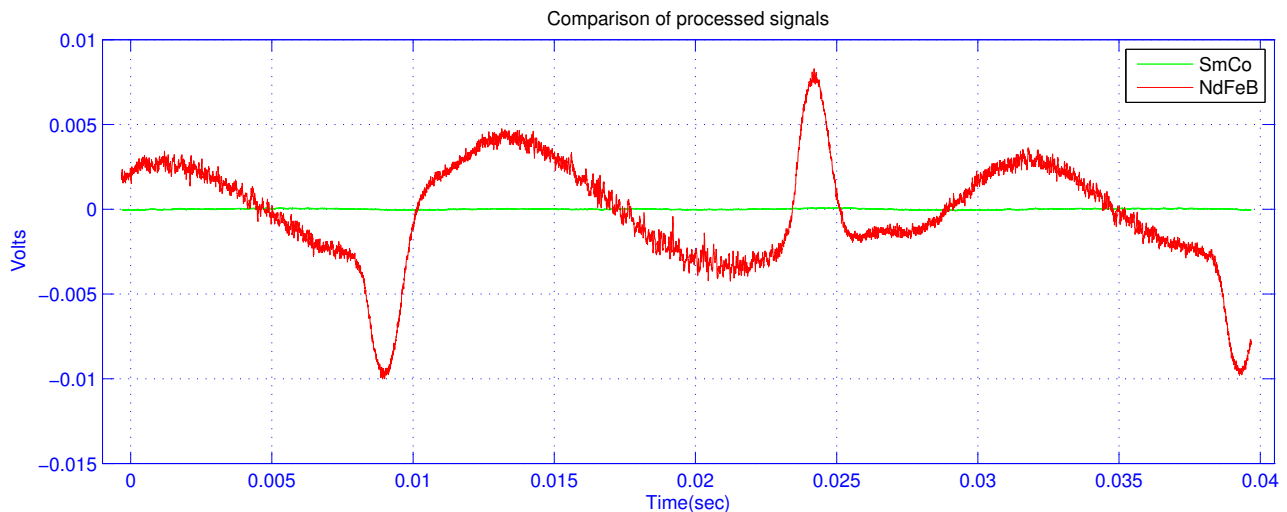


Figure 1. Processed NdFeB and SmCo signals.

¹ $B_{\text{peak}} = 30.8 \times V(\text{peak in one drive coil})$

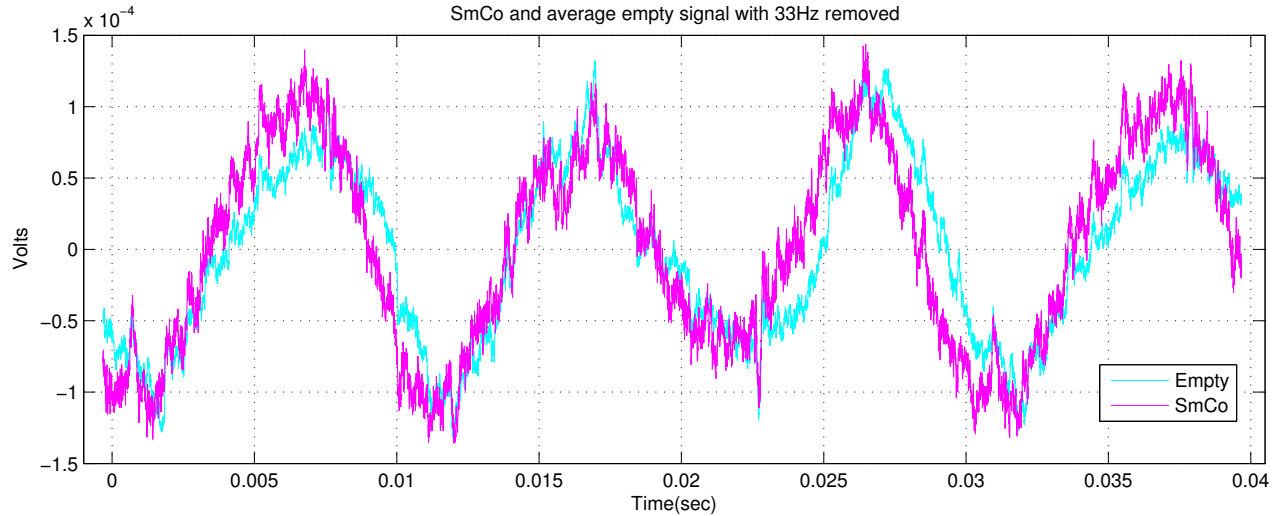


Figure 2. Average of seven signals from the apparatus run without a magnet in it compared with the SmCo signal. 33Hz removed from both.

In order to get an estimate of the ratio of the SmCo signal to the NdFeB signal, we will model the SmCo signal as a proportional function of the NdFeB signal plus a noise term.

$$V_{Sm}(t) = V_{noise}(t) + \alpha V_{Nd}(t)$$

α is plotted for the 43 SmCo magnets tested in Figure 3.

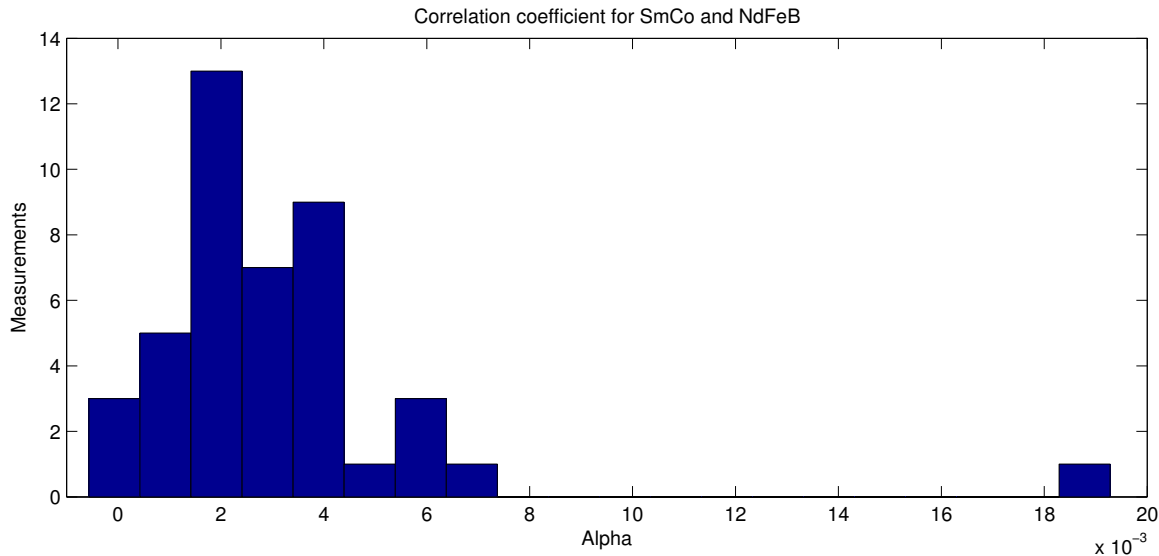


Figure 3. Cross-correlation coefficient between the two signals. The measurement on the far right comes from human error on a measurement.²

² For following calculations, we're ignoring that measurement on the right of Figure 3. I tested the magnet 3 times at a later time and it acted normally.

Another method to compare SmCo with NdFeB is to look for the magnetization discontinuity in the SmCo data. For this analysis, we found the peak-to-peak voltage by taking an average of each NdFeB signal from $3e-4$ seconds before the negative peak to $3e-4$ seconds after the peak, and subtracting that from a similar average around the positive peak. This gives us an average $\langle \Delta V_{Nd} \rangle = 0.017$ volts between positive and negative peaks. Next, the SmCo data are averaged for the same time period that the NdFeB data are, namely $3e-4$ before and after the NdFeB peak. This gives us a ΔV_{Sm} for each SmCo signal, the average of which is $\langle \Delta V_{Sm} \rangle = 5.4e-5 \pm 9.8e-6$ volts.

$$\frac{\Delta V_{Sm}}{\Delta V_{Nd}} = 3.4 \times 10^{-3} \pm 6.0 \times 10^{-4} \text{ Volts}$$

This is comparable to the alphas shown in Figure 3.

Barkhausen Noise and the Initial LIGO Detector Spectrum

R. Weiss September 11, 2007

Summary A new analysis of the magnetic moment fluctuations (the Barkhausen noise) in the NdFeB magnets when driven by time varying magnetic fields from the OSEM coils has been made. The interaction of the magnetic field gradient of the PAM magnets with the magnetic moment fluctuations of the drive magnets shows that in the frequency band between 40 to 100Hz, the displacement noise in the quiescent spectrum is most likely due to the Barkhausen noise. The new step in the analysis is the recognition that the PAM magnet field gradients are much stronger than the OSEM coil field gradients when driven by the typical bias currents. The up-conversion displacement spectrum depends strongly on the separation of the PAM and control magnets. The separation of the magnets is not well documented and it is possible that during initial angular alignment several PAM magnets were driven close to their associated control magnets. These would dominate in producing the up-conversion spectrum. The up-conversion from the etm has been established and one of the clues that the PAM magnets may be the dominant sources of field gradient comes from the almost comparable amount of up-conversion from the etmy and etmx at LLO even though there is a factor 3 difference in their bias currents.

Before we make final decisions on the magnet replacement, it would be worthwhile to establish if both the ITM and ETM are implicated. A comparison of the pendulum, pitch and yaw periods of the test masses could indicate if there are closely spaced PAMs. Better still, the up-conversion spectrum of the ITM should be measured and compared with those from the ETM.

Revisited calculation The magnetic bridge measurements of the NdFeB magnets give the spectrum of the varying magnetic moment of the NdFeB magnets as a function of AC drive magnetic field and frequency. The relation is

$$\mu(f) = \frac{(2.3 \times 10^{-6} \pm 3 \times 10^{-7}) I_{\text{OSEM COIL}}^{\frac{3}{2}} (\text{amp})}{g f (f-f_0)} \text{Joules}/(\text{Tesla } \sqrt{\text{Hz}}) \quad \text{eq 1}$$

$I_{\text{OSEM COIL}}$ is the AC current in the drive coil, g is a means of carrying an uncertainty in the coupling of the magnet to the pickup coil in the bridge experiment, $0.17 > g > 0.07$. I use $g = 0.1$ in the estimates. f_0 is the drive frequency. The uncertainty in the numerical coefficient is associated with variation in the measurements from one magnet to another. It is assumed that the OSEM coil produces 530 Gauss/amp.

The force spectrum on the test mass from one magnet is expressed as

$$F_z(f) = \mu_z(f) \frac{dB}{dz} \quad \text{eq 2}$$

The gradient of the B field that produces the magnetic force depends on both the PAM magnet and the bias current in the OSEM coil. The gradients from the coil and the PAM magnet are

$$\frac{dB}{dz}_{\text{OSEM}} = \frac{0.43\mu_0 N_{\text{OSEM}} i_{\text{bias}}}{a_{\text{OSEM}}^2} \quad \frac{dB}{dz}_{\text{PAM}} = \frac{3\mu_0\mu_{\text{PAM}}}{2\pi z^4} \quad \text{eq 3a,b}$$

where $N_{\text{OSEM}} = 200$ is the number of turns in the OSEM coil, a_{OSEM} the radius of the coil in meters, μ_{PAM} the magnetic moment of the PAM magnet in Joules/Tesla and z is the separation of the PAM and control magnet in meters. The PAM and control magnets have the same magnetic moment $\mu_{\text{PAM}} = \mu_{\text{control}} = 1.4 \times 10^{-2}$ Joules/Tesla. **Figure 1** shows the magnetic gradient from both the OSEM coil at a constant bias current of 30ma and the gradient from the PAM magnet as a function of the magnet separation.

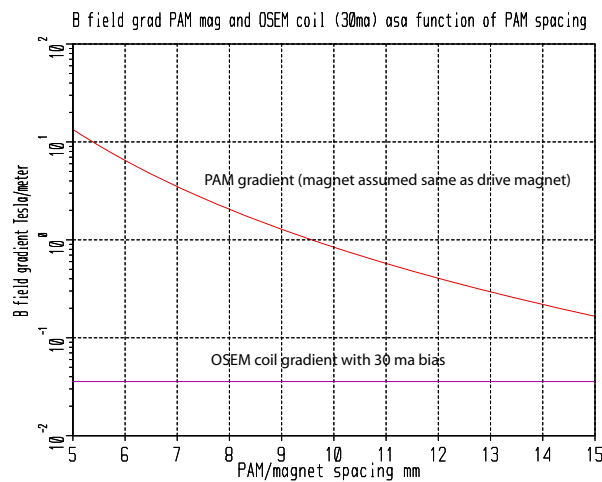


Figure 1 Comparison of the B field gradients of the OSEM coil at the optimum distance from the end of the coil $a_{\text{OSEM}}/2$ and the gradient from the PAM magnet as a function of the distance from the PAM magnet.

The PAM magnets are oriented so that the same poles are facing the control magnets while the control magnets are arranged to have opposite poles at each corner of the square on which they are placed. The intent is to have the PAM magnets produce no net force on the test mass if all are placed the same separation from the control magnets. The nominal separation is 1 cm. Under these idealized conditions the Barkhausen noise should also cancel as it is coherent in the four control magnets driven by a position control current. The two factors that break this symmetry are, first, that the Barkhausen noise in different magnets can vary by as much as 10% and, second, that the PAM magnets are not equally spaced from their control magnets.

Mark Barton has written two documents about the alignment of the test masses and the PAM magnets. They are “Use of Magnets in the Suspension Design LIGO T000119-00-D and “Pitch Adjustment Magnets for LOS and SOS” LIGO T970189-00-D. The strategy for the alignment and also many of the dimensions used in this estimate come from these documents. I know of at least one critical difference between the data given there and the as built values. The as built magnets have twice as large magnetic moments as those used in his experiments and in the data showing

the amount of rotation with PAM-control magnet spacing. The magnet-magnet forces are therefore 4 times larger than those in the documents.

During initial installation as well subsequent realignment at LHO, the instrument logs at LLO and LHO contain information about when PAM magnets were adjusted but do not state the number of turns taken on the PAM adjustment screws. In order to reconstruct the critical parameter, the spacing between the magnets, the best we have is oral history. Doug Cook and Betsy Bland describe the process they used at LHO which is much the same as that used by Gary Traylor at LLO. The PAM magnets were pulled back to about an intermagnet separation of 1.5 cm where the magnets had little influence on the pitch and yaw angles. Then little by little one was brought forward toward the control magnet usually going about 7 turns on the 32 threads per inch screw (5.5mm), the separation would then be 9 mm. Then the opposing magnet was brought in so that there would be control by both magnets. A final adjustment of less than a turn was made in most cases, about another 0.8mm. In many cases therefore one magnet could be as close as 8mm to the control magnet. In some extreme cases it would be even closer. Gary believes that one of the LLO ETMY magnet spacings could be as small as 6mm.

Another piece of the oral history is that on the average the PAMS were moved to adjust for 200 microradians of pitch and yaw misalignment. Using the known strength of the PAM and control magnets, the displacement of the support point from the mirror center of mass and the distance the control magnets are glued from the pitch axis through the center of mass; to correct a 200 microradian pitch would require a 9mm or smaller separation between the closest magnets. In summary the oral history would imply that the average magnet separation is between 8 and 9mm and the closest 6mm

ESTIMATES

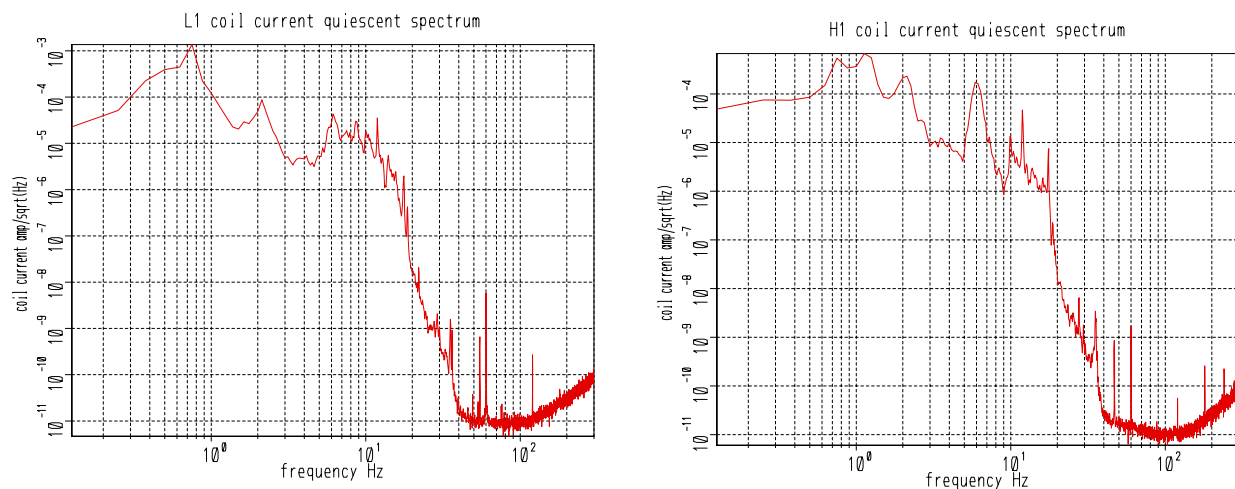


Figure 2a,b The control current into a single coil on an ETM at L1 and H1. The rms current at L1 is 1.5 ma while at H1 it is 2.1ma.

Table 1: Table of bias currents in ma from Rana LLO log entry 5/17/2000

test mass	L1	H1	H2
ETMX	20	13	11
ETMY	6	35	25
ITMX	3	28	29
ITMY	4	1	30

The bias currents are all small enough so that a PAM magnet will dominate the field gradient. The estimated up-conversion spectrum using the relations given in the beginning but with only the PAM field gradients included in the estimates are shown in **Figure 3a,b**

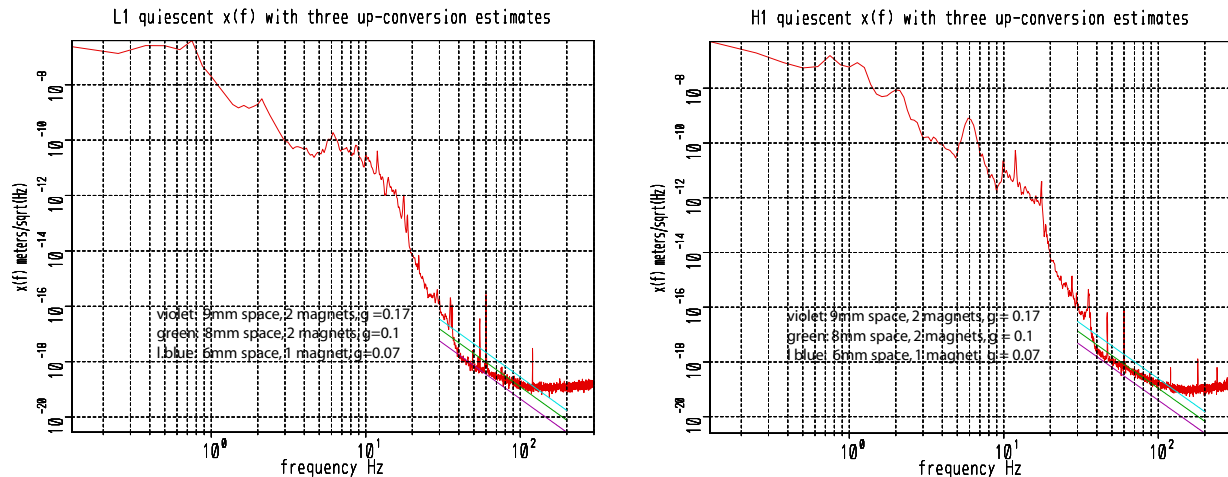


Figure 3a,b The quiescent displacement spectrum at L1 and H1 with three estimates of the up-conversion. The estimates for the up-conversion are made by a convolution of the current spectrum (**figure2**) with the product of **eq 1, 2** and **3**. The convolution is done with a 1 Hz bandwidth sampling of the current distribution to estimate the rms current. The bandwidth becomes relevant since the current dependence is non-linear. Choosing a smaller bandwidth reduces the estimate.

The estimates for the up-conversion use reasonable parameters given the oral history. The lowest estimates use a PAM to control magnet spacing of 9mm and assume that 2 magnets are this close. The bridge experiment magnet coupling constant of 0.17 is used which is the largest value expected (the smallest influence on the up-conversion). The middle estimate assumes 2 magnets

at a spacing of 8mm and the nominal magnet coupling constant of 0.1. The highest estimates demonstrates what happens as the inter magnet spacing becomes small .Here the spacing is assumed 6mm and only one magnet is this close. The coupling constant is assumed to be at the minimum value 0.07.

Discussion

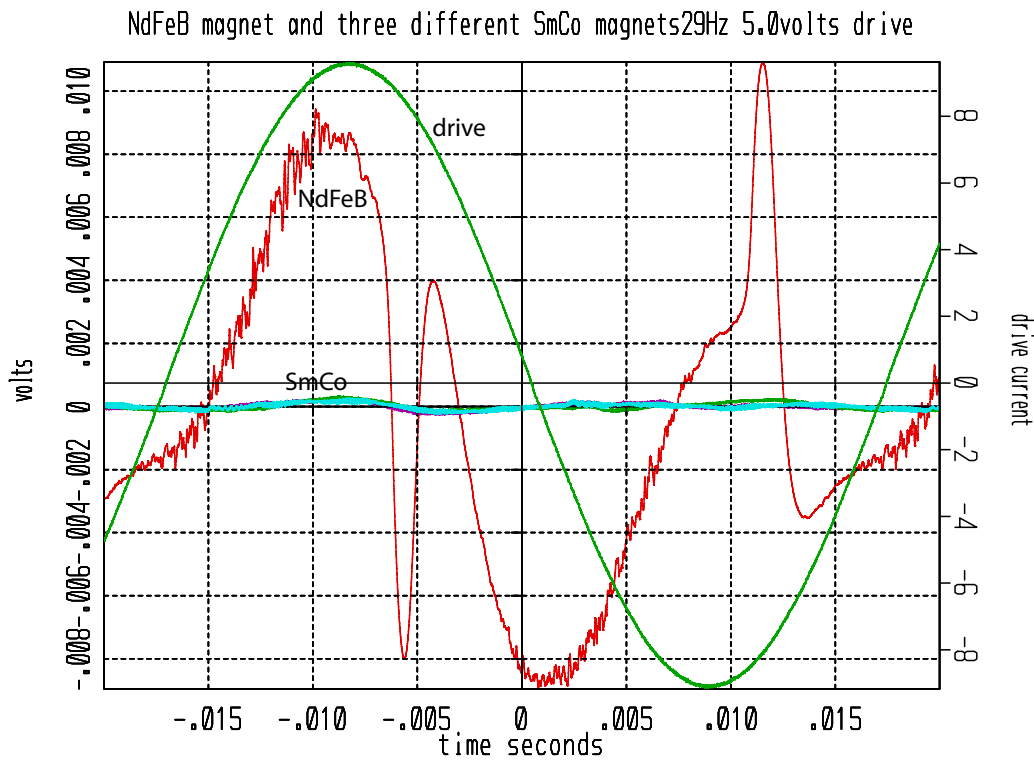
With the strong dependence on the inter magnet spacing, one cannot at the moment rule out the possibility that the ITM, even with their smaller control currents, could contribute to the quiescent up conversion spectrum. It takes only one PAM magnet to be in close to cause mischief.

Before we can decide if the ITM magnets also need to be replaced, suggest that we measure the up-conversion spectrum from the ITM and compare it to those from the ETM. One can also determine if there is a close magnet spacing by looking at the pendulum, yaw and pitch periods which will be different for a mass with a close magnet. A possible fix for up-conversion in the ITM would be to realign and pull back a close PAM magnet rather than replacing the control magnets.

Work on magnets at LLO November 14, 2007
R. Weiss

More work on the magnets for the test masses is reported here. A variety of measurements have been made.

1) A new batch of approximately 50 SmCo magnets manufactured by Electron Energy Corporation (Batch # 7666-1, part# D07412410750TISO) has been measured for Barkhausen noise. The noise in all the magnets is less than 0.02 of that in the NdFeB magnets (see the accompanying figure).



2) The strength of the SmCo magnets has been measured relative to the NdFeB magnets. The results are

Table 1: Magnetic moment ratio SmCo/NdFeB

batch 1	batch2
0.89 +- 0.09	0.97 +- 0.1

The uncertainty is mostly a systematic in the ability to set up the measurement apparatus the same way for each magnet. The setup used a cantilever spring to hold the magnet in front of the OSEM coil. The absolute force/ampere is $2.7 \pm 0.5 \times 10^{-2}$ Newtons/ampere for the magnet and OSEM spaced at the maximum force point.

3) 6 SmCo magnets from the first batch processed by Betsy Bland in the same way for mounting to the test mass was tested for Barkhausen noise. The magnets were glued to the aluminium stand-off and baked at 100C. There is no Barkhausen noise observed in any of the samples.

4) The SmCo magnets have a fine powder on the pole faces which needs to be removed before gluing. I find that scotch tape removes the powder after one sticking.

5) An interesting observation, which I should have reported on previously, is that all the NdFeB magnets detune the inductance bridge apparatus. SmCo magnets do not change the balance in the bridge and behave as though they have a magnetic permeability of 1. This is further evidence that the NdFeB magnets are not completely saturated while the SmCo magnets are saturated and is consistent with Barkhausen noise in the NdFeB magnets while none is observed in the SmCo magnets.

Reciprocity of the magnetic force and the magnetic induction in the coil/magnet drivers

In principle the force on the magnet per current in the coil should be related to the voltage induced in the coil by the magnet moving with a velocity. If they are not, one can make violations of the second law of thermodynamics and get energy out of the thermal noise of the pendulum. In trying to get an answer for which of the various F/i measurements that have been made is closest to being correct, I used the reciprocity by measuring the open circuit voltage induced in the OSEM coil when the magnet is dropped through it with a known velocity. The data is shown in the figure for several magnet drops from a height of 9cm (velocity at the coil 1.33 meters/sec). The results are: $F(\text{newtons})/I(\text{amps}) = \text{volts}(\text{open circuit})/\text{velocity}(\text{meters/sec}) = 1.7 \times 10^{-2}$ not far from the value of 1.6×10^{-2} Newtons/amp, the favored value.

