Contribution to

FESTSCHRIFT FOR JERROLD R. ZACHARIAS

June 1966

Tobey Island, Cape Cod, Massachusetts

The paucity of slow atoms in the Zacharias fountain experiment.

Rainer Weiss

Note: This title page was reconstructed on January 7, 1999 from memory. More scholarly people would have gone back to the MIT Archives on Jerrold Zacharias.

In 1954 Zach conceived of an ingenious method for extending the observation time of atoms undergoing RF transitions and thereby hoped to achieve an atomic cesium clock with a resonance line width of $\sim$1 c/sec. The instrument was to be used in a terrestrial measurement of the gravitational red shift.

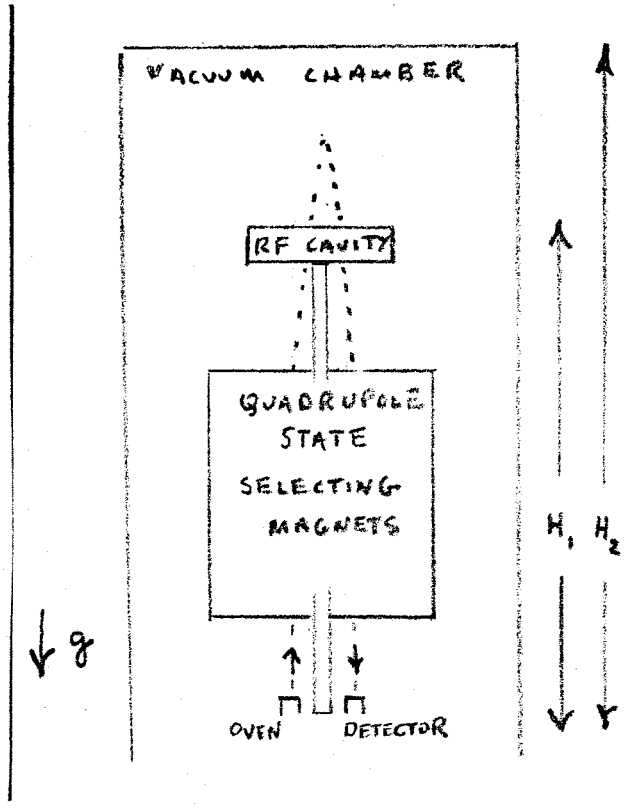


Figure 1.

The idea was to decelerate cesium atoms in the earth's gravitational field. The slow atoms in the tail of the Maxwell distribution could perform the trajectory, from oven to detector, shown in Fig. 1. The fraction of the velocity distribution that would be involved in these trajectories is given by

$$\frac{I}{I_o} = \left(\frac{2g}{\alpha^2}\right)^2 (H_2 + H_1)(H_2 - H_1)$$

The relevant parameters of the apparatus that was constructed are $H_2 = 594$ cm, $H_1 \sim 250$ cm, $\alpha \sim 2.3 \times 10^4$ cm/sec for an oven temperature of 100°C, so that $I/I_o \sim 3 \times 10^{-6}$. To overcome this small fraction, the areas of the detector and oven were made 10 cm^2 so that, at an oven pressure of 10^{-3} mm Hg, the detected beam would still have the very respectable intensity of 10^7 atoms/sec. These would then be atoms that leave the oven at $\sim$ 1/25 the most probable velocity in the beam and spend $\sim$ 2.2 seconds in free flight.

In the experiment we never saw these atoms, and it was not for lack of looking. The detector had a background noise of ~10 atoms/sec and a combined detection and collection efficiency ~ 1/5. In other words, the slow atoms suffered an attenuation of at least 5×10^{-6}. The search for these atoms was extended over a factor of 100 in oven pressures, two kinds of ovens -- krinkle foil and an open slit -- and apparatus vacuum ultimately as good (in those days) as 2×10^{-9} mm Hg over most of the beam path and 5×10^{-10} at the top of the trajectory.

Since little information concerning where the atoms were attenuated could be derived from the null result, we decided to adapt the apparatus to perform a measurement of the velocity distribution in a pulse of the vertical beam.

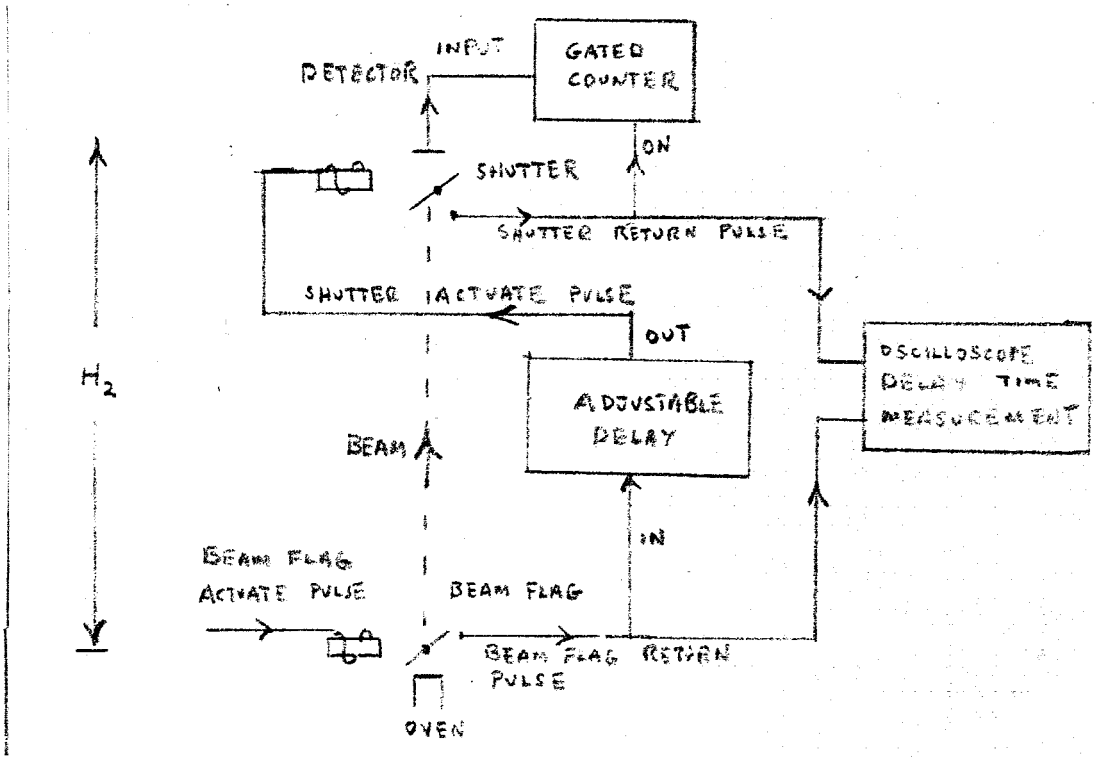
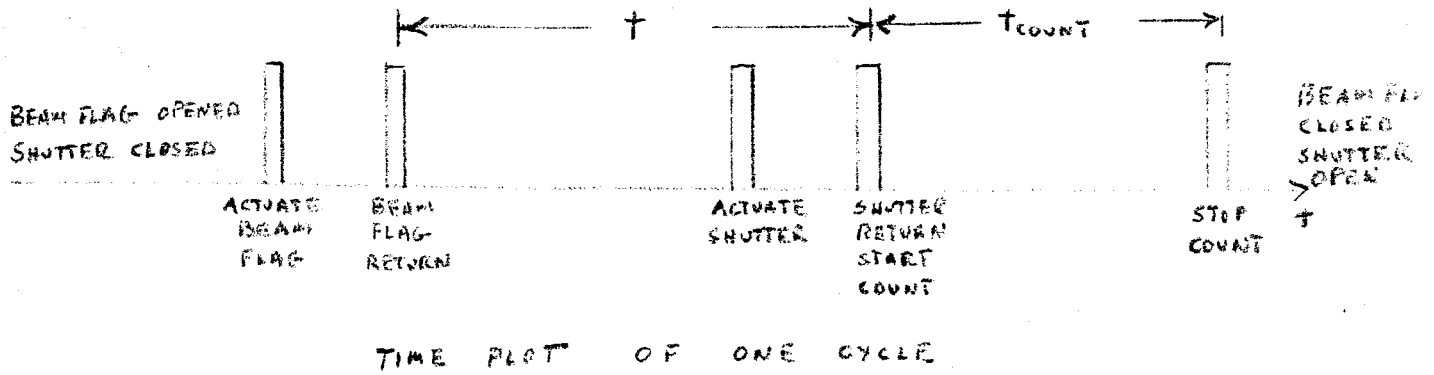


Figure 2a.



TIME PLOT OF ONE CYCLE

Figure 2b.

Figure 2 shows the experiment and the sequence of events. At the start of a cycle, a steady beam of atoms is established between the oven and a closed shutter in front of the detector. (The shutter is necessary to prevent the full beam from contaminating the detector.) The beam is interrupted by the beam flag at time $t = 0$. At time t later, the shutter is opened, the counter is turned on and continues to count for $\sim 1/2 \rightarrow 1$ second. The cycle is then repeated.

To analyze the experiment, consider a column of atoms that have velocity v and a small range of velocities Δv about v, an almost monoenergetic beam. The steady state intensity due to these atoms is $I(v)\Delta v$, where $I(v) = \frac{2I_o}{\alpha^4} v^3 e^{-v^2/\alpha^2}$. I_o is the total steady state beam intensity. Let this monoenergetic beam of atoms be interrupted at time $t = 0$ at a distance H from the detector. The intensity as a function of time will look like Fig. 3. The detector will not know anything has happened until a time H/v later.

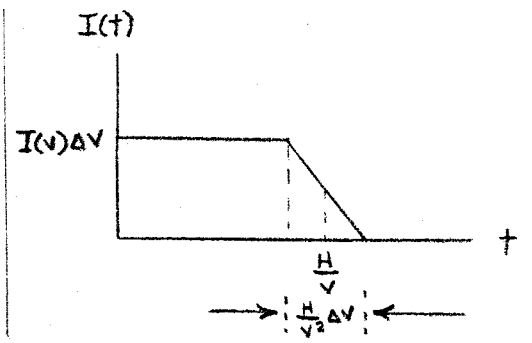


Figure 3.

Consider further three almost monoenergetic beams with velocities $v_1 < \alpha < v_3$ but all with the same Δv; the intensity, as a function of t, will be as in Fig. 4

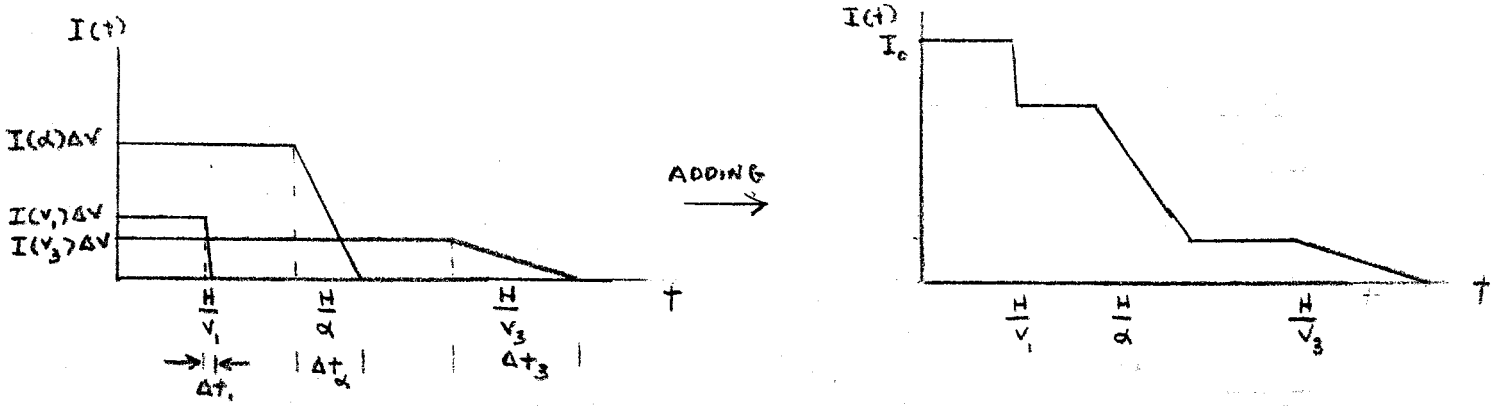


Figure 4

The beam intensity remaining after a time t will be deficient in all those atoms that take time $t \leq \frac{H}{v}$ to go from the beam flag to the detector. Analytically,

$$I(t) = I_o - \int_{oo}^{v=H/t} I(v)\, dv.$$

In terms of the time,

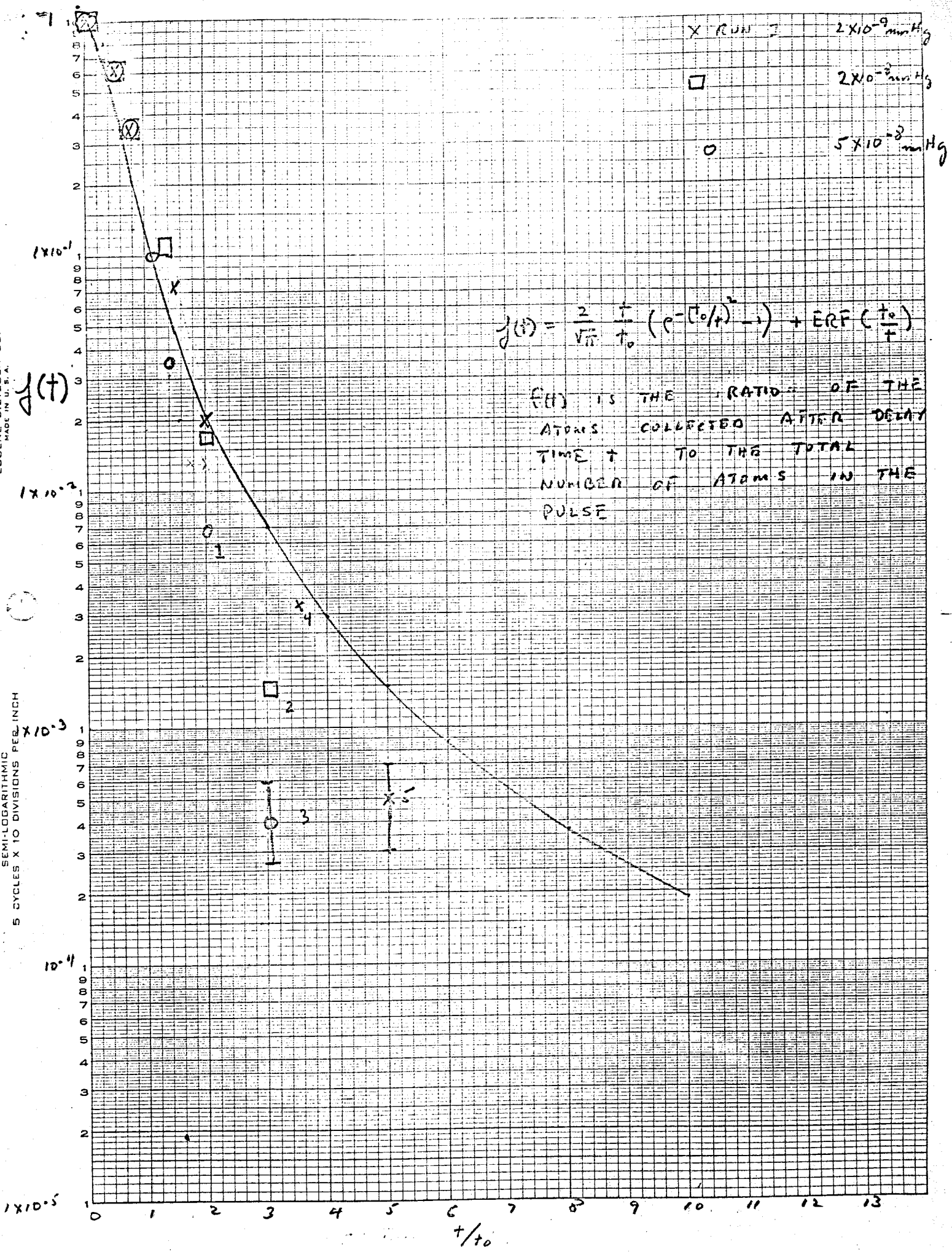
$$I(t) = I_o - 2I_o \int_o^t \frac{t_o^4}{t^5}\, e^{-t_o^2/t^2}\, dt$$

where $t_o = \frac{H}{\alpha}$. Integration gives

$$I(t) = I_o\left[1 - e^{-(t_o/t)^2}\left(1 + (\frac{t_o}{t})^2\right)\right]$$

The relevant quantity in the experiment is the number of atoms left in the beam after time t.

$$N(t \to oo) - N(t) = \int_t^{\infty} I(t)dt.$$


X RUN 7 2X10⁻⁹ mm Hg
□ 2X10⁻⁸ mm Hg
○ 5X10⁻⁸ mm Hg
$J(t) = \frac{2}{\sqrt{\pi}} \frac{t}{t_0} \left(e^{-(t_0/t)^2} - 1 \right) + \mathrm{ERF}\left(\frac{t_0}{t}\right)$
J(t) IS THE RATIO OF THE ATOMS COLLECTED AFTER DELAY TIME t TO THE TOTAL NUMBER OF ATOMS IN THE PULSE
J(t)
1X10⁻¹
1X10⁻²
1X10⁻³
10⁻⁴
1X10⁻⁵
t/t₀
0 1 2 3 4 5 6 7 8 9 10 11 12 13
SEMI-LOGARITHMIC
5 CYCLES X 10 DIVISIONS PER INCH

Integrating (non-trivial),

$$N(t \to \infty) - N(t) = I_o\left[t\left(e^{-(t_o/t)^2} - 1\right) + \frac{\sqrt{\pi}}{2}\, t_o\, \mathrm{ERF}(t_o/t)\right]$$

where $\mathrm{ERF}(t_o/t) = \frac{2}{\sqrt{\pi}} \int_o^{x=t_o/t} e^{-x^2} dx$ is the normalized Gaussian Error function. The total number of atoms in the interrupted column is $N(t \to \infty) - N(o) = I_o t_o \frac{\sqrt{\pi}}{2}$; the fraction of these collected after time t is

$$f(t) = \frac{N(\infty) - N(t)}{I_o t_o \frac{\sqrt{\pi}}{2}} = \frac{2}{\sqrt{\pi}} \frac{t}{t_o}\left(e^{-(t_o/t)^2} - 1\right) + \mathrm{ERF}(t_o/t).$$

For large delay times, or small values of t_o/t

$$f(t) \cong \frac{2}{\sqrt{\pi}}\left[\frac{1}{6}\left(\frac{t_o}{t}\right)^3 - \frac{1}{15}\left(\frac{t_o}{t}\right)^5 + \frac{1}{56}\left(\frac{t_o}{t}\right)^7 + \ldots\right]$$

$f(t)$ is plotted in Figure 5 along with experimental points for three residual gas pressures in the beam path. The pressure changes were produced by the decomposition of Cesium Azide into Cs and N_2 in a separate oven, face down, separated from the velocity analyzed beam by the partition between the detector and oven side of the original apparatus.

The apparatus parameters in the velocity experiment were the following. The oven, a 1 mm x 2 mm slit, 1 mm deep; oven temperature 80°C; oven pressures 1×10^{-4} mm Hg. The detector, a platinum ribbon 1 mm x 1 mm, slanted at 60° to the beam. The source-detector separation $H_2 \sim 594$ cm. The detected beam intensity $I_o = 8 \times 10^6$ atom/sec, the mean time $t_o = H/\alpha = 2.7 \times 10^{-2}$ sec. The total number of atoms in

one pulse $I_o t_o \frac{\sqrt{\pi}}{2} \sim 2 \times 10^5$ atoms/pulse. The detector background count with shutter closed and beam flag opened ~ 30 atoms/sec. The residual gas atoms were at room temperature.

The data shows qualitatively that the residual gas pressure attenuates the slow atoms. The points for the fast atoms, for which the Maxwell distribution is probably valid, lie above the theoretical line because of storage in the detector structure. The storage increases the detector size as well as time constant; however its influence becomes less important for large delay times. In order to get further with the data, some theory of the scattering is necessary.

For a beam atom of collision cross section Q_{BG} and of velocity v, scattered by a Maxwellian gas with most probable velocity α_G, Kennard (Kinetic Theory of Gases) gives the following expression for the mean time between collision of the beam atom with any gas atom.

$$\bar{t}(\alpha_G v) = \frac{\sqrt{\pi}}{\rho_g Q_{BG} \alpha_G \left[e^{-(v/\alpha_G)^2} + 2\frac{v}{\alpha_G} + \frac{\alpha_G}{v} \int_o^{y=v/\alpha_G} e^{-y^2} dy \right]}$$

$$\bar{t}(\alpha_G v \to 0) = \frac{\sqrt{\pi}}{2 \rho_g Q_{BG} \alpha_G}$$

where ρ_g is the particle density of the gas. The limiting expression for $v \to 0$ is easily derived from elementary principle directly. The fraction of the beam atoms remaining after time t is then

$$\frac{N(t)_{experimental}}{N(t)_{theoretical}} = e^{-t/\bar{t}(\alpha_G v)}$$

Using the above expressions, one can calculate Q_{BG} from various of the experimental points. These are given in the following table

	Point	t	P mm Hg	$\frac{N(t)_{exper}}{N(t)_{theor}}$	Q_{BG} cm^2
Gas	1	$2.8\ t_o$	5×10^{-8}	.35	1.8×10^{-13}
	2	$4\ t_o$	2×10^{-8}	.21	3.0×10^{-13}
	3	$4\ t_o$	5×10^{-8}	.06	2.7×10^{-13}
Cs - Cs	4	$4.5\ t_o$	2×10^{-9}	.75	6×10^{-13}
	5	$6\ t_o$	2×10^{-9}	.3	2×10^{-12}

If this theory of scattering is at all applicable, it is evident from the table that two phenomena are involved. Points 1, 2 and 3 should give reasonable estimates for the cross section of $Cs \leftrightarrow N_2$ scattering. The cross sections derived from these points are ~ 4 times larger than previous measurements for $Cs \leftrightarrow A$ scattering, $Q(Cs,A) \sim 5.7 \times 10^{-14} cm^2$ (Rosin and Rabi). Whether the actual cross section <u>is</u> as large as the one measured in this experiment or if the large value is due to an error in ion gauge calibration (there is some doubt to a factor of 2) is not important in establishing if the vacuum, measured on the same gauges, was sufficient to perform the original experiment. Using the observed cross section, the attenuation of atoms taking 2.2 seconds to traverse the apparatus should have been 1/13 (2×10^{-9} mm Hg in the apparatus). Consequently, the vacuum contributed to the attenuation but it was <u>not</u> the dominant factor.

The answer lies in points 4 and 5; these points should not be related to residual gas scattering but rather to $Cs \leftrightarrow Cs$ collisions near the oven. This hypothesis is partly borne out by the observation that atoms with $t/t_o > 3$ showed no directionality from the oven. A crude calculation of this follows.

For a particular oven and a fixed solid angle subtended by the detector at the oven, it is reasonable to assume that there is a characteristic length, ℓ_o, in the beam above the oven, and not too far from it, in which scattering can occur and still be influential in attenuating the detected beam. The probability of a collision between a particular slow beam atom of velocity v and any one of the average velocity beam atoms of velocity α will be given by an expression such as

$$\Phi \sim \rho_{beam}\, Q_{Cs\text{-}Cs}\, \ell_o \frac{|\alpha - v|}{v}$$

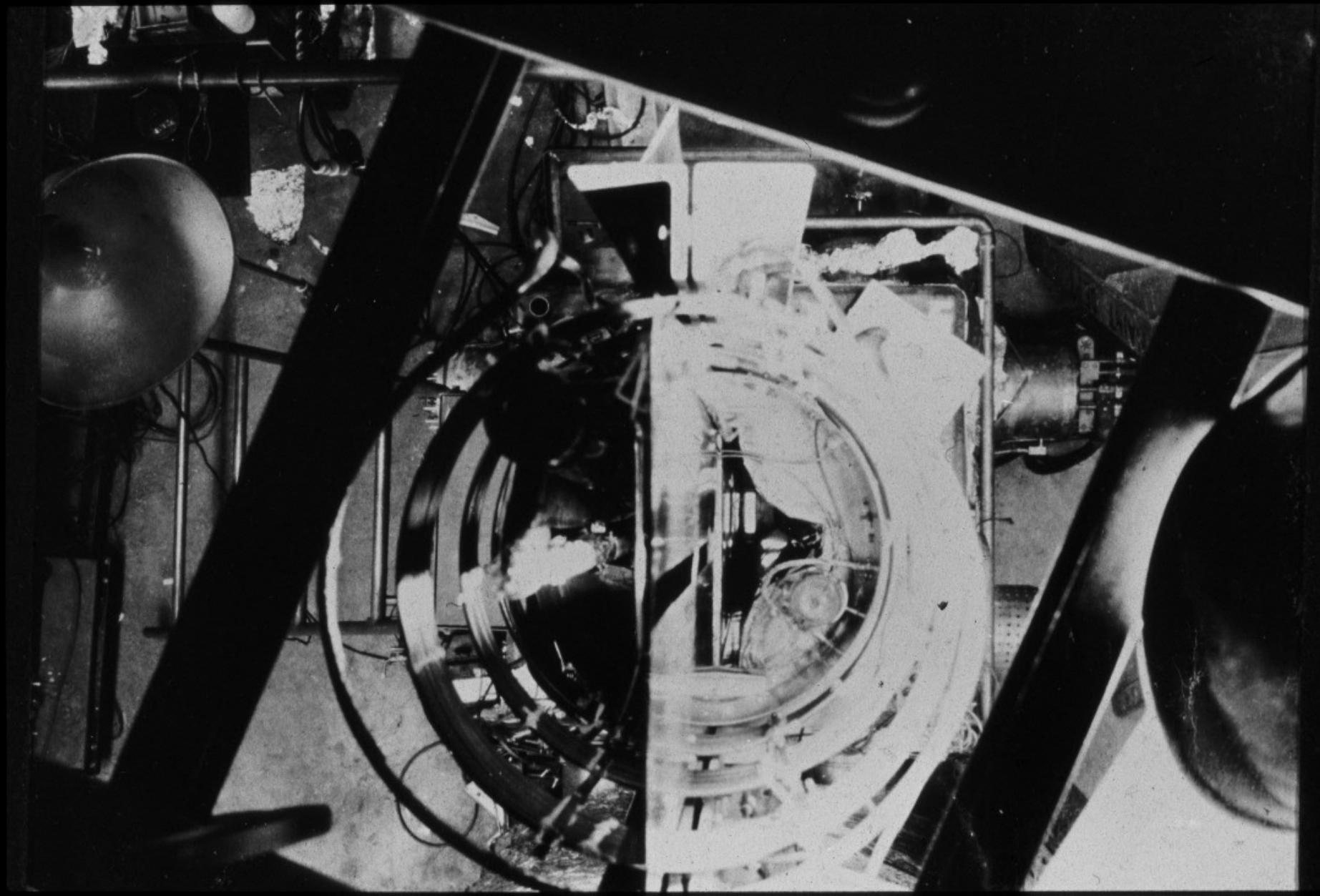
The attenuation of the slow atoms is then

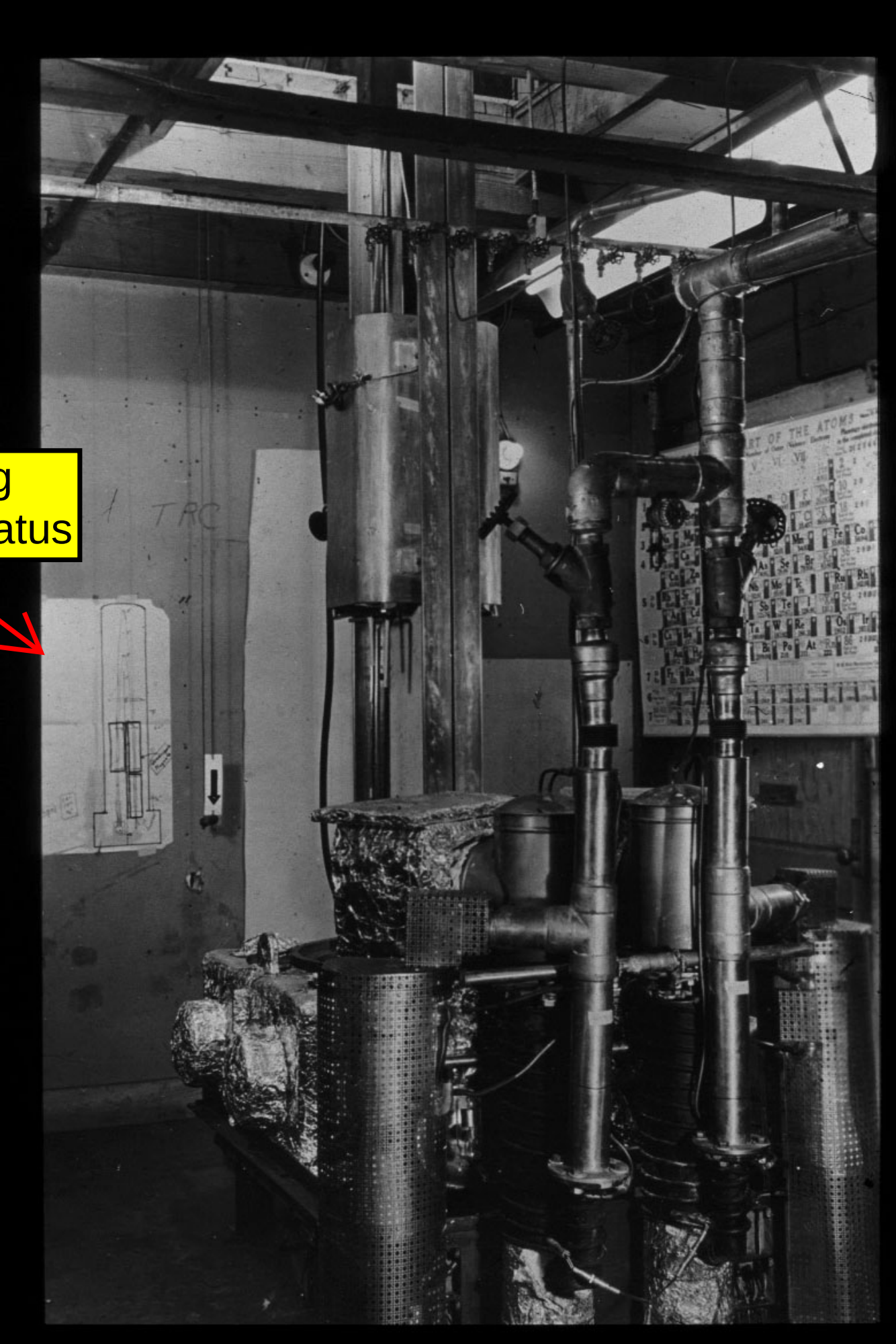
$$f\left(\frac{v_{atoms}}{|\alpha - v|}\right) = \rho^{-k \frac{|\alpha - v|}{v}}$$

The data of points 4 and 5 give $k \sim 0.2$. If this value of k is also applicable for the original experiment, the attenuation of atoms near the oven, going 1/25 the average velocity, would have been $\sim 4 \times 10^{-5}$ for an oven pressure of 10^{-3}mm Hg, so that the scattering of $Cs \leftrightarrow Cs$ near the oven can explain the null result of the first experiment. To really clinch it, the velocity experiment should have been extended over a range of oven pressures and with both of the ovens used in the original experiment.

Incidently, $\ell_o \sim 5$ mm, as calculated from these data using a $Q(Cs - Cs) \sim 10^{-13} cm^2$ and the known oven pressure of 10^{-4} mm Hg. A more rigorous calculation of the Cs - Cs scattering near the oven might be the subject of a senior thesis and possibly a guide to oven design.

Best Wishes on your
61 st Birthday Rainer Weiss

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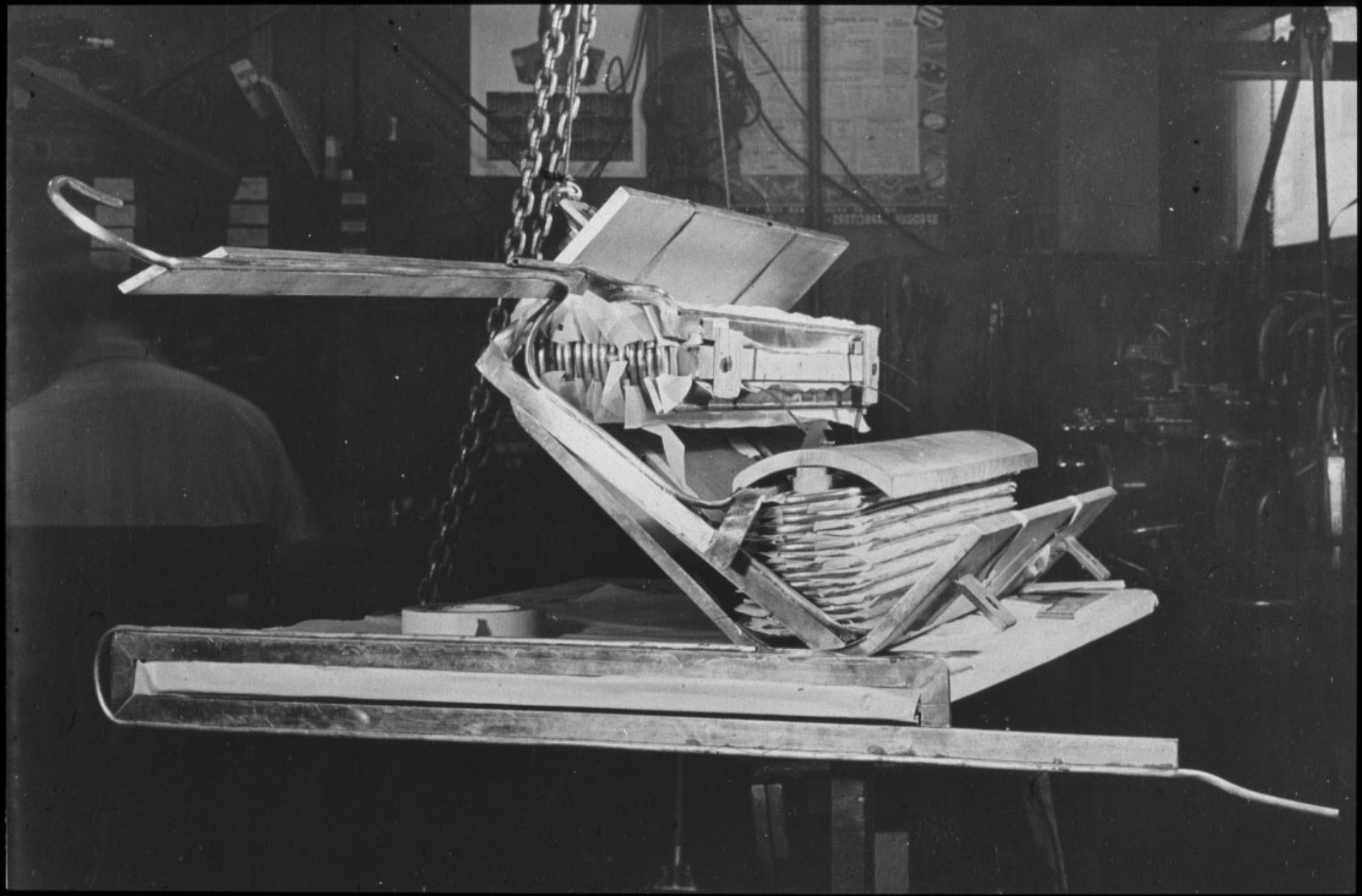

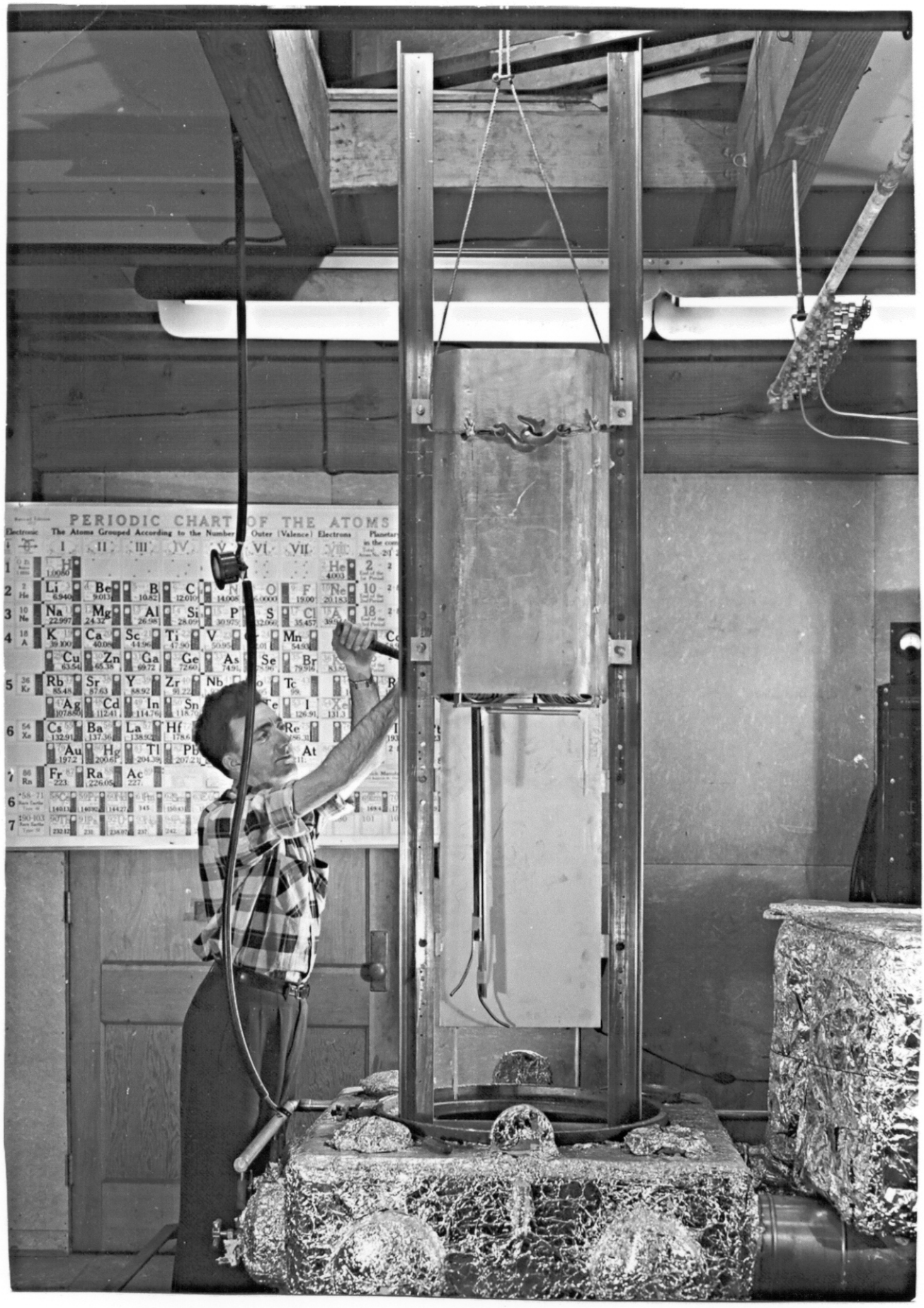

Frank O'Brien in 1956 adjusting the quadrupole deflecting magnets in the Zacharias fountain also called the "big clock" in the molecular beam lab at MIT.

Proposed Precision Magnetic Resonance Experiment

Jerrold R. Zacharias

As early as 1940, I. I. Rabi suggested that the hyperfine structure of the ground state of Cs^{133}, as observed by the molecular beam magnetic resonance method, 9192 mc/sec, could be used as a time standard and that one might be able to measure the gravitational red shift of an atomic clock on the earth. Recently Lyons (1) and others of the National Bureau of Standards have made a careful measurement of this frequency and developed equipment for using it as a reference standard for time. Their precision is limited by the time that the cesium atoms remain in the radiofrequency field, and their present curves are about 3 kc/sec broad. It is the purpose of the present note to indicate how this precision may be extended to linewidths of 1 cps or less with corresponding improvement in precision timing to several parts in 10^{13}.

Consider a beam of cesium atoms moving upward in a well evacuated vessel. Atoms with an initial velocity of about 10 m/sec will rise 5 m and fall back down onto a detector. In rising and falling they might pass through an inhomogeneous magnetic field which would serve as both deflecting and analyzing field in the molecular beam method. At the top of the path there could be located the collimating slit and a radiofrequency flopping field, perhaps 1 m long in which the atoms will remain for about 1 sec, allowing a linewidth of 1 cps.

It appears that this experiment can be performed without the loss in beam intensity that one might first expect because of gas scattering of

slow atoms and the low population of slow atoms in a beam.

For a molecular beam the expression for the distribution of speeds, v is given by

$$(v^3/\alpha^4)\, e^{-v^2/\alpha^2}\, dv, \text{ where } \alpha = (2kt/m)^{\frac{1}{2}},$$

giving approximately 300 m/sec for cesium at 400°K as the most probable velocity in a beam. We therefore consider working with $(0.03)^4$ or 10^{-6} of the total beam. Now the deflection of a beam in a molecular beam resonance experiment is proportional to $1/v^2$ so that we have $(30)^2$ more deflecting power than in the usual experiments. We can use this factor by making the area of the slit system (source, collimator, and detector) 1000 times greater in area (30 times wider and 30 times higher). Since the so-called intensity of a beam (total number of usable atoms) varies as the square of the slit area, we have a factor of 10^6 to cancel the loss mentioned above. The factor of 30 not used in slit width is used to increase the aperture in the deflecting and refocusing magnet, making the ratio of gradient to field 0.03 instead of about 1, as usual.

With regard to gas scattering, the mean free time in the apparatus would have to be a few seconds, which is to be expected at a pressure of 10^{-7} mm Hg. Naturally one should try to use lower pressures, which can indeed be obtained in clean systems with getter pumps or other devices.

If it really becomes possible to use this low-energy end of the Maxwell distribution, it should become possible for the first time to use highly effective (2,3) focusing of a divergent beam. For instance, if the source of beam is placed in a field of 10^4 gauss and the exit canal or

canals are situated at a point of high magnetic gradient, it will be possible to collimate the beam from a rather wide angle into a ribbon 1 cm wide, as proposed here. Chromatic aberration of the magnetic lens is of no great concern, since gravity itself produces velocity selection at the top of the path. It should also be mentioned that we have here an analogy with ion optics with the additional feature that the sign of the deflection can be reversed in the field by means of radiofrequency flipping.

It should be noted that for cesium atoms discussed here with one Bohr magneton in 10^4 gauss, $\mu_0 B \approx 10^{-16}$ ergs $\approx mgh \approx mv^2/2$.

Consideration has been given to the possibility of mounting the beam source on a wheel which rotates in such direction that it moves away from the detector when it faces it. Running at a peripheral speed of 3×10^4 cm/sec we would be using those atoms which have, in the oven, the most popular velocities. This process regains only two of the four factors of v and may not be worth the complication except in the case of molecules other than cesium to which this general precision technique may be applicable.

Experiments are now under way to find the low-velocity atoms, preparatory to building an apparatus for precision frequency control with cesium, and for precision measurements with other substances.

References

(1) Harold Lyons: NBS Report 1848, August 8, 1952, contains a complete discussion of this subject with many references.

(2) M. I. Korsumskii, Ya. M. Fogel: J. Exp. Theor. Phys.(Russian) 21,25, 1951.

(3) H. Friedburg & W. Paul: Naturwiss 38, 159, 1951.