Gravitational radiation: theoretical insight to observation

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Round table discussion

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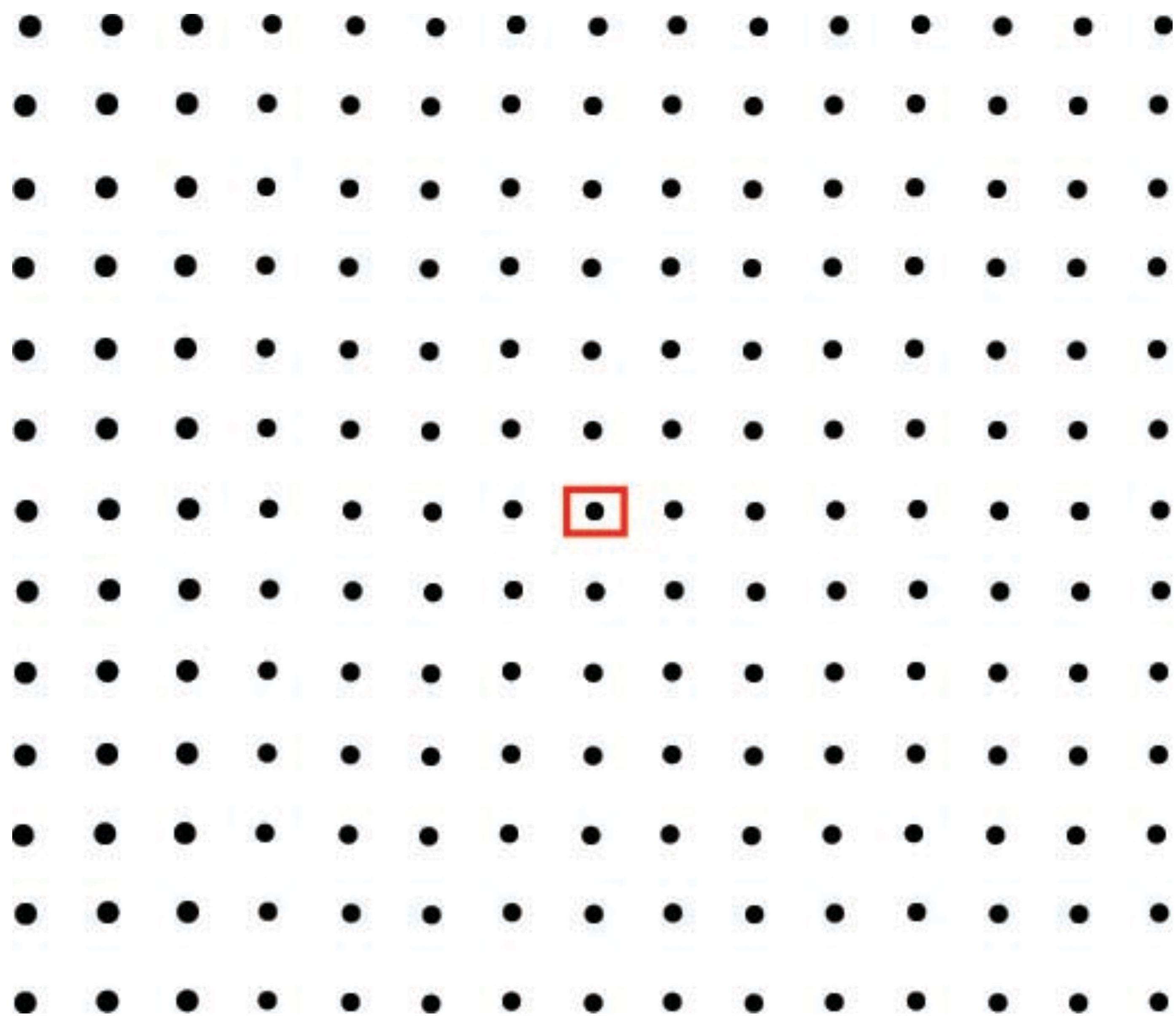
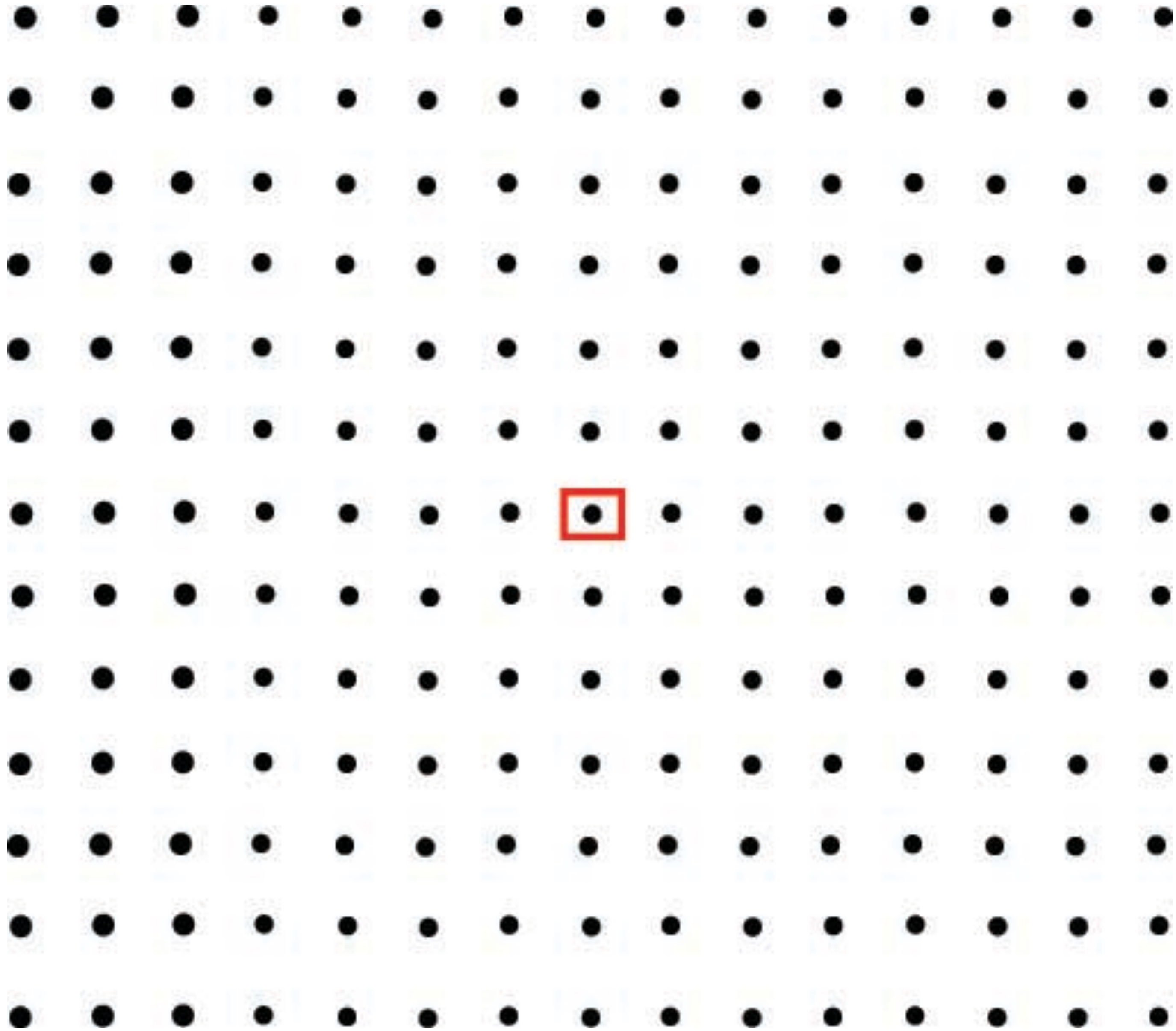
Basic concepts

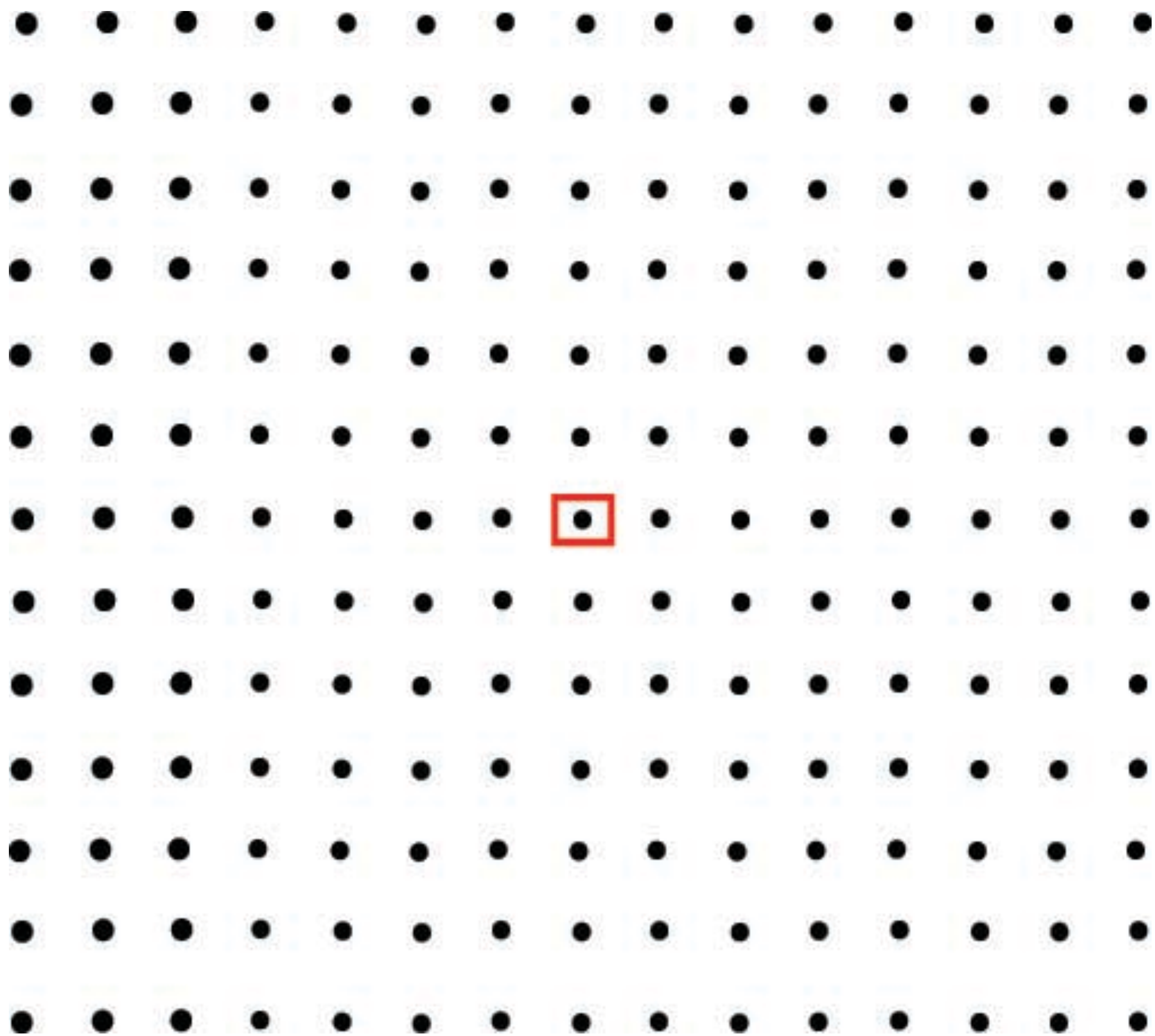
- Gravitational waves are required by all relativistic theories of gravitation
  - Sources are accelerated masses
  - Propagation velocity is assumed as c
- General Relativity
  - Lowest order source is a quadrupole
    - One sign of mass
    - Momentum conservation at the source
  - Weak field approximation for the waves

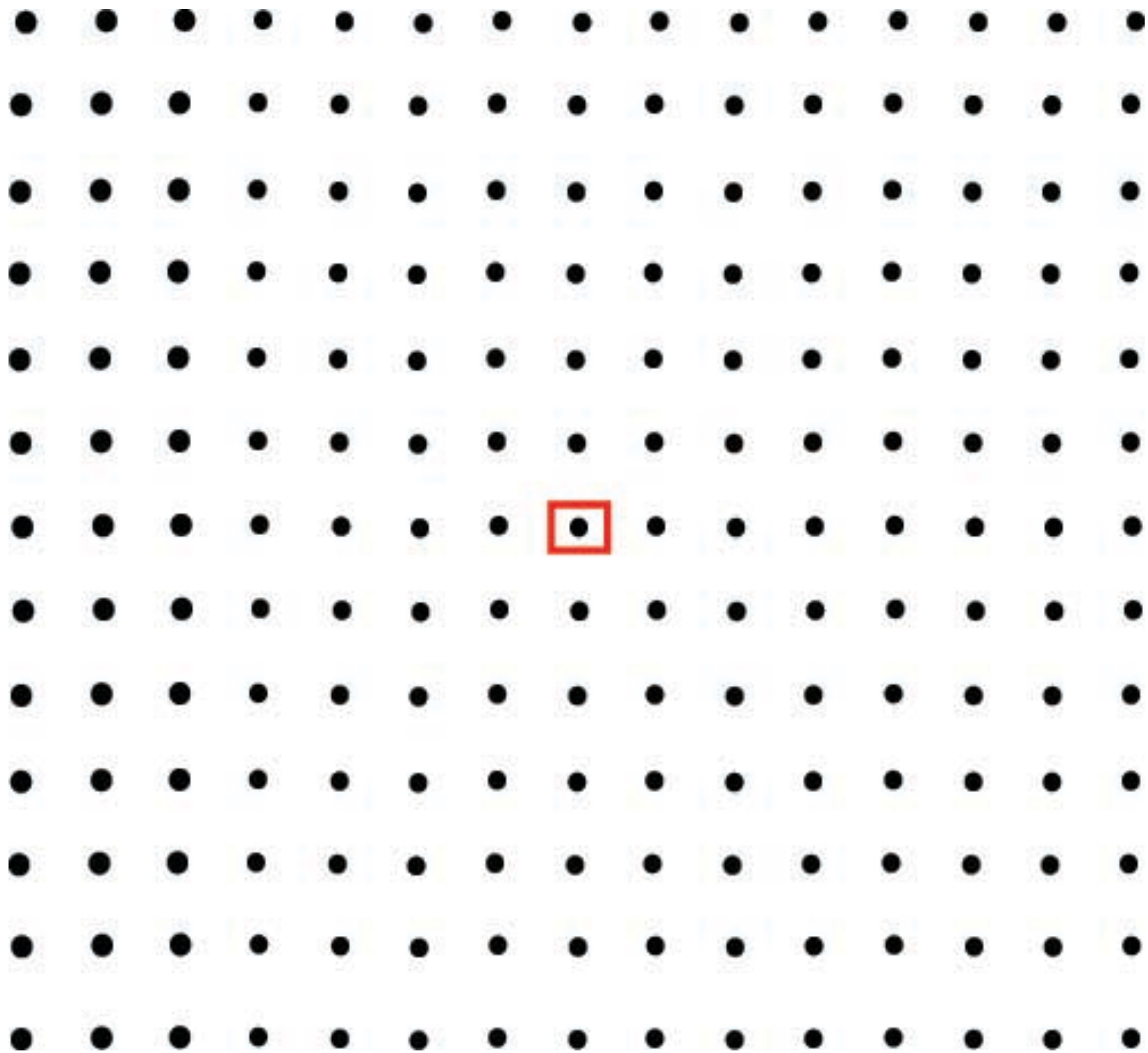
$$\nabla^2 h_{\mu\nu} = \frac{\partial^2 h_{\mu\nu}}{c^2\,\partial t^2}$$

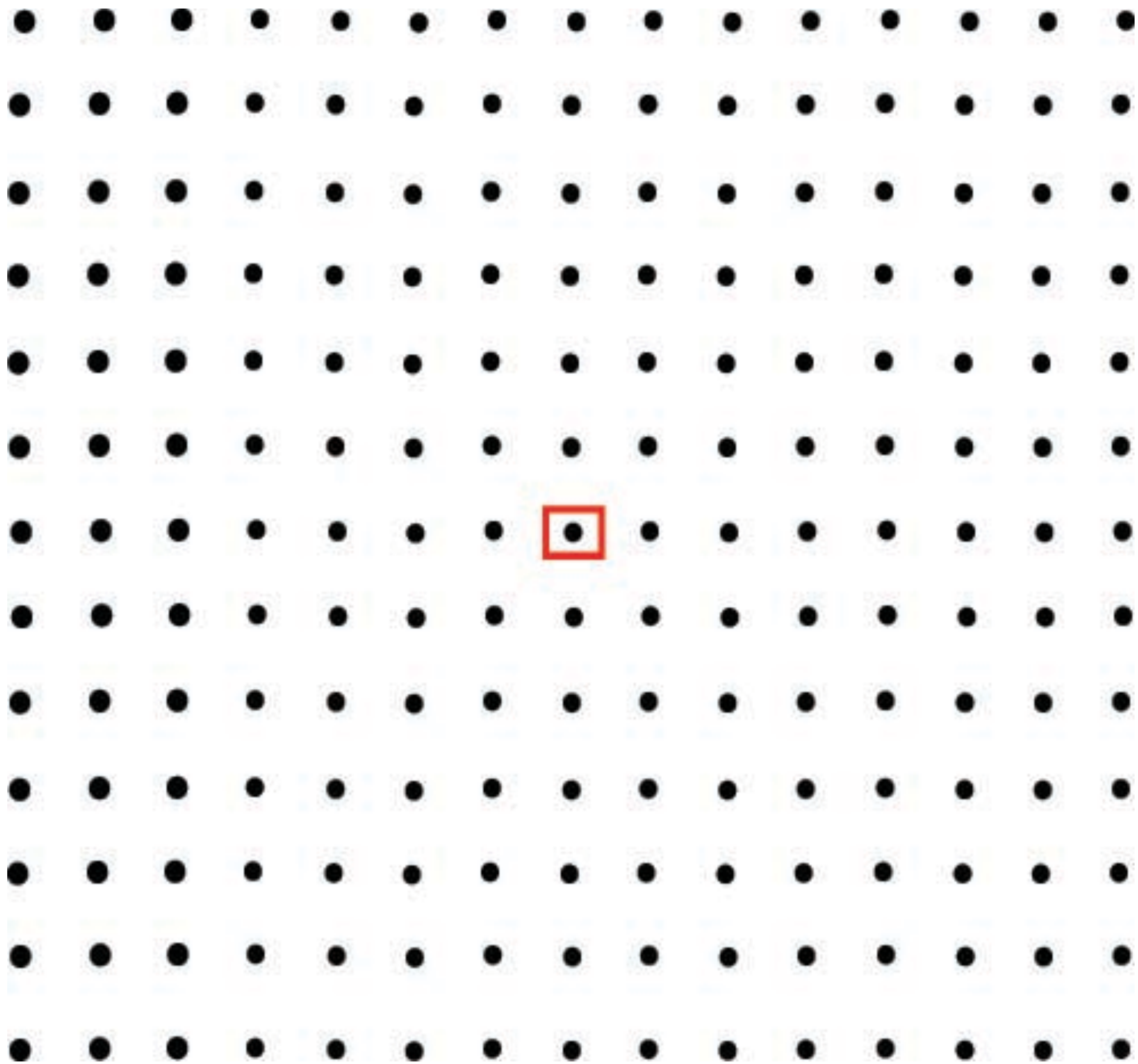
A. Einstein: Proceedings of the Royal Prussian Academy of Sciences XXXIII 688 1916, VI 154 1918

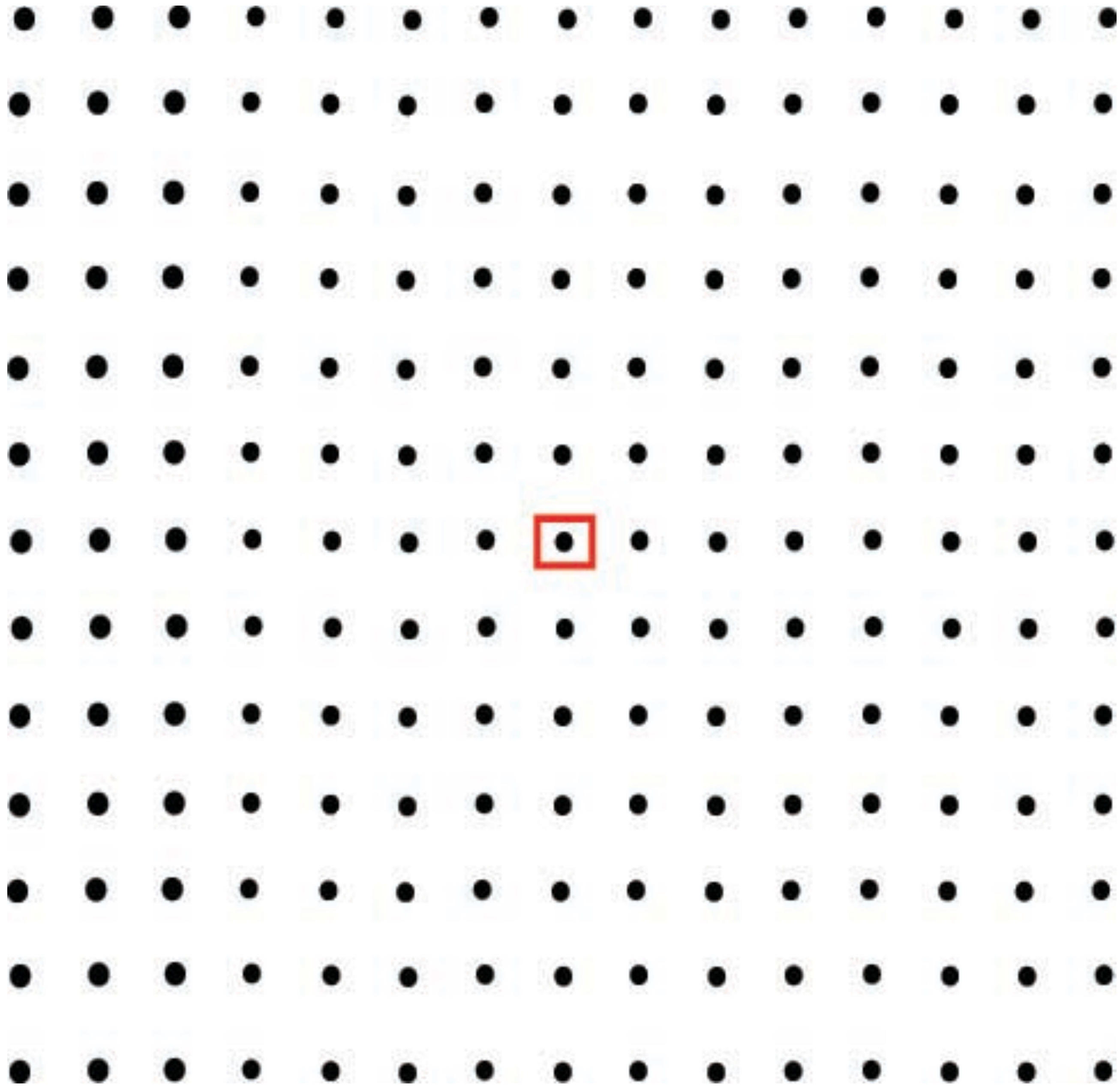


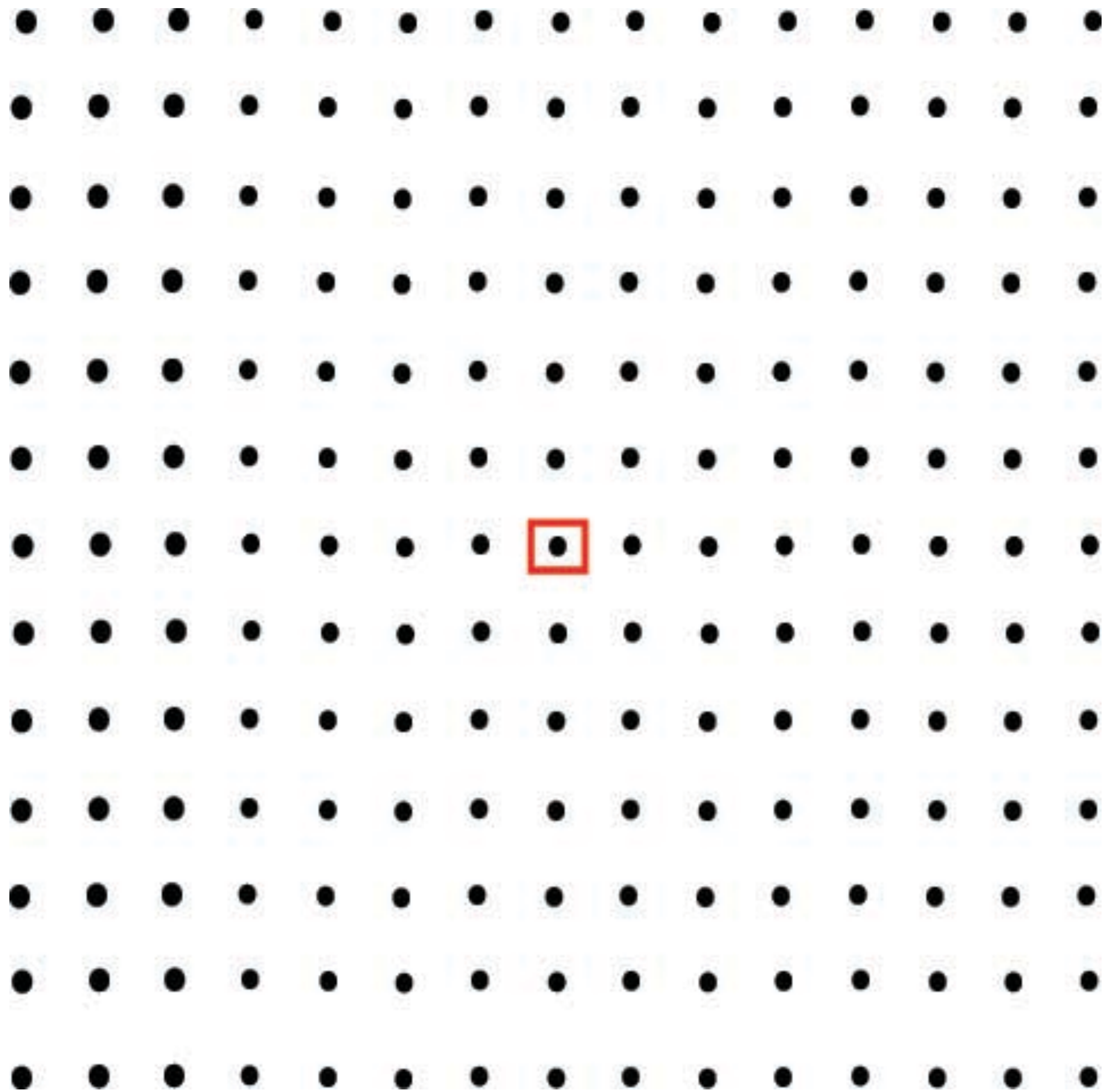


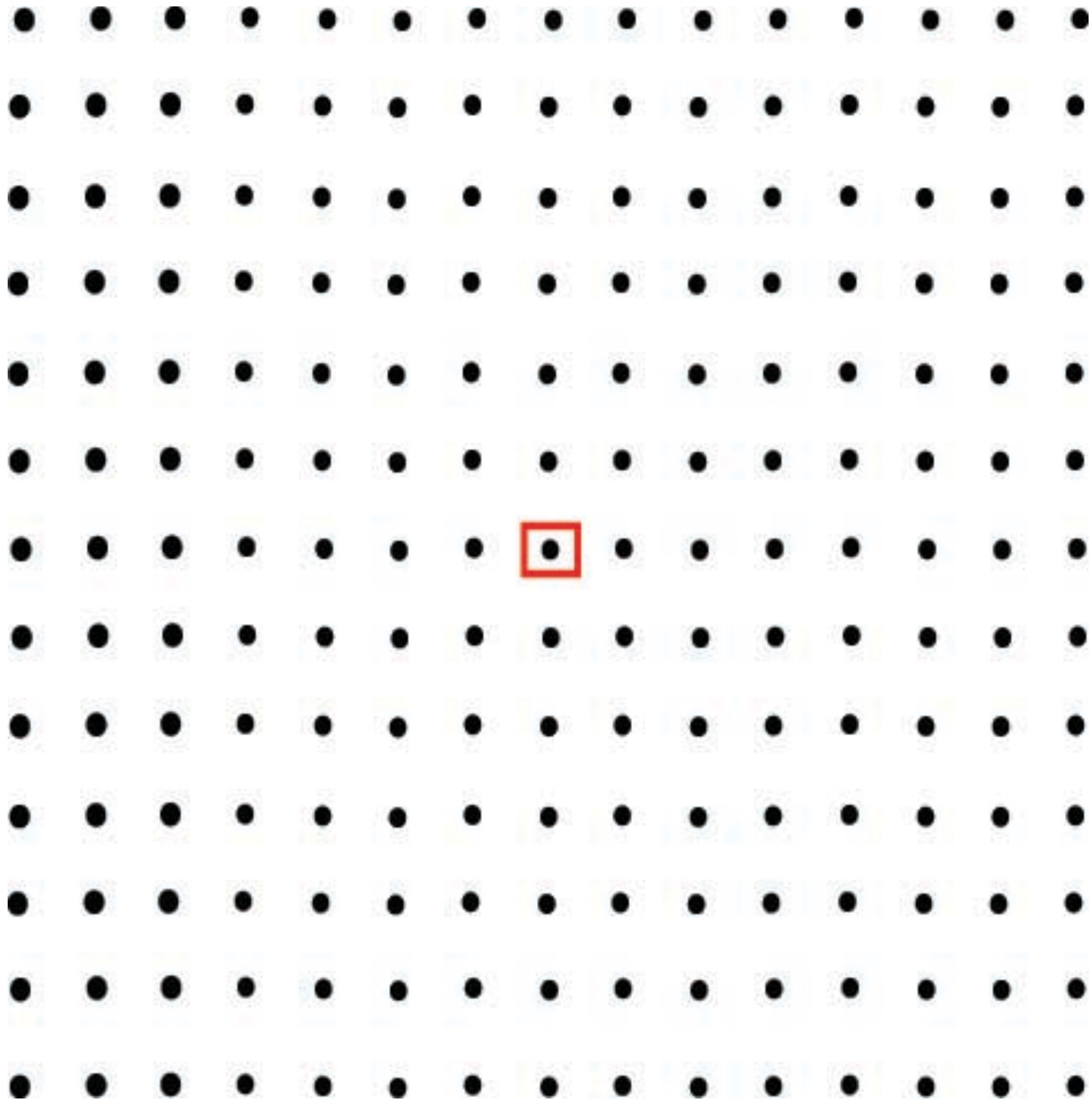


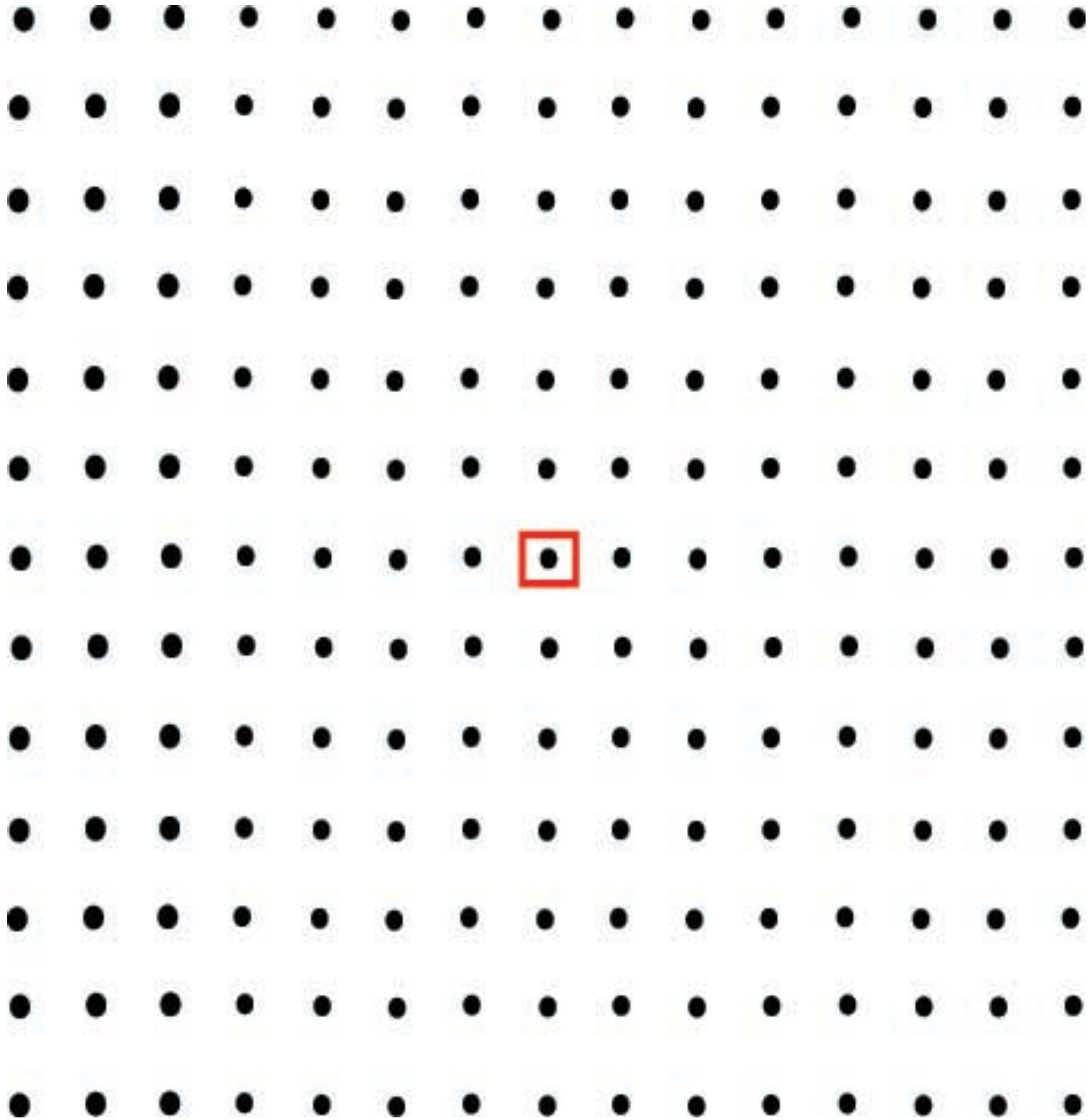


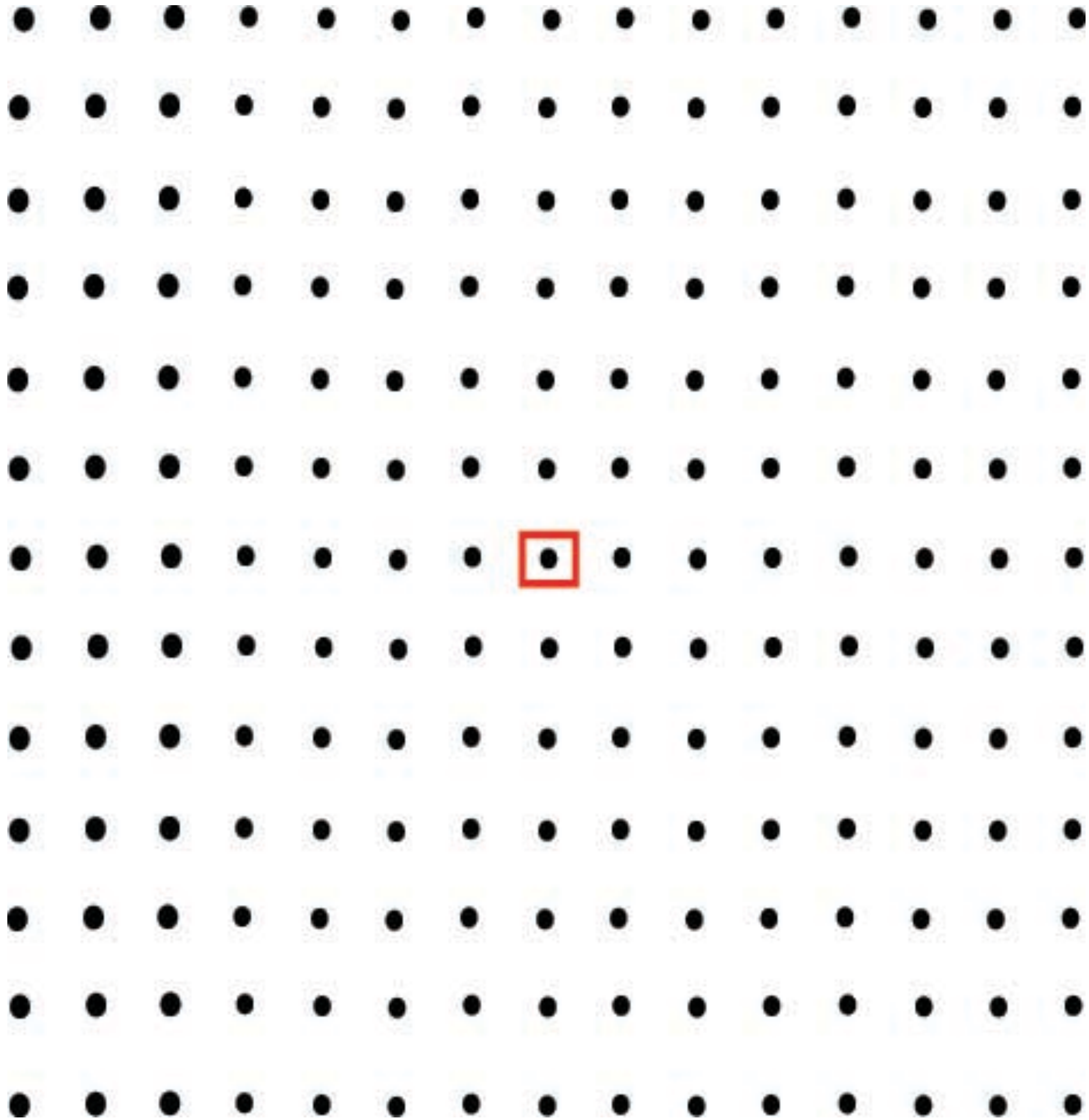


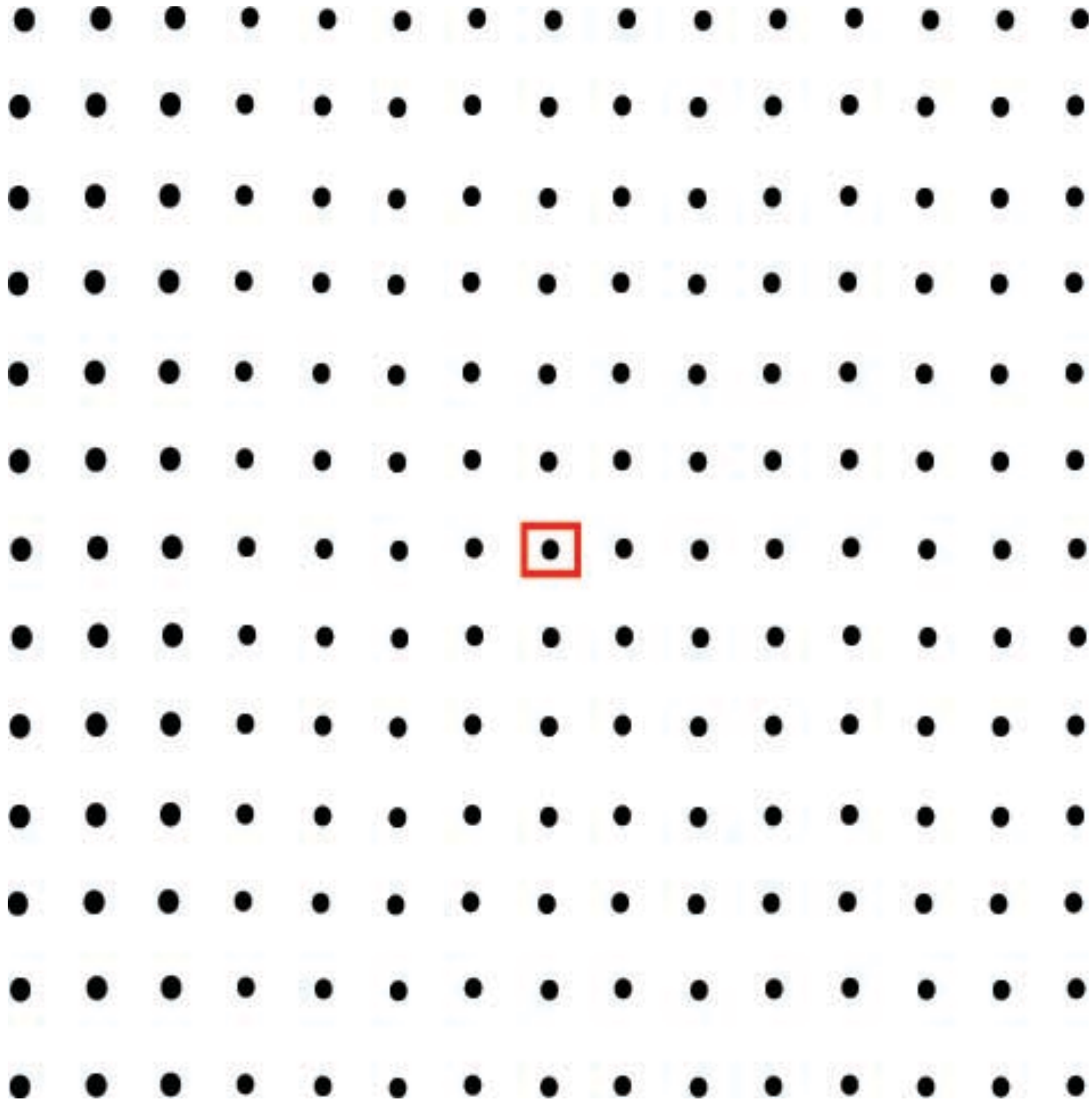


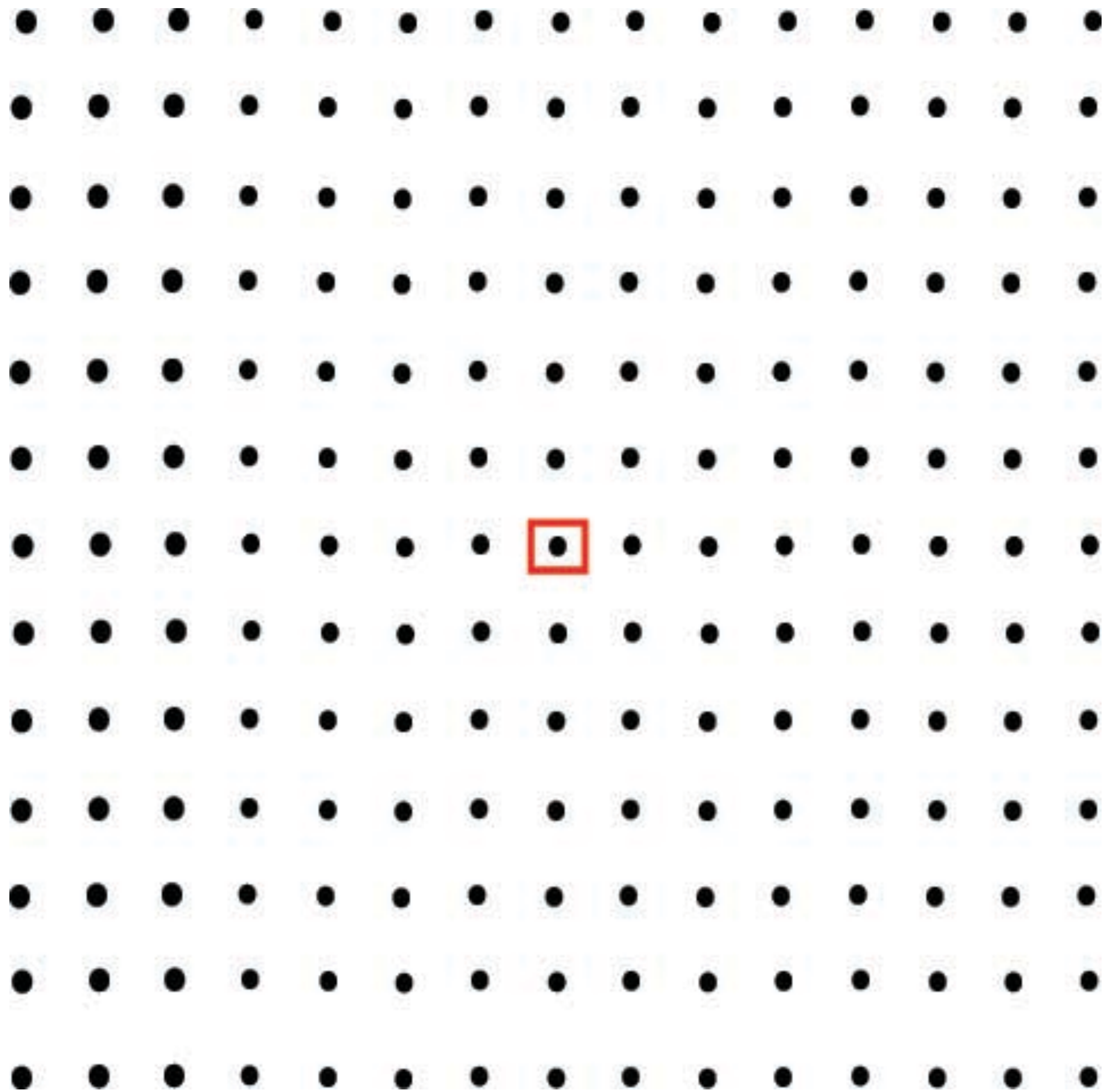


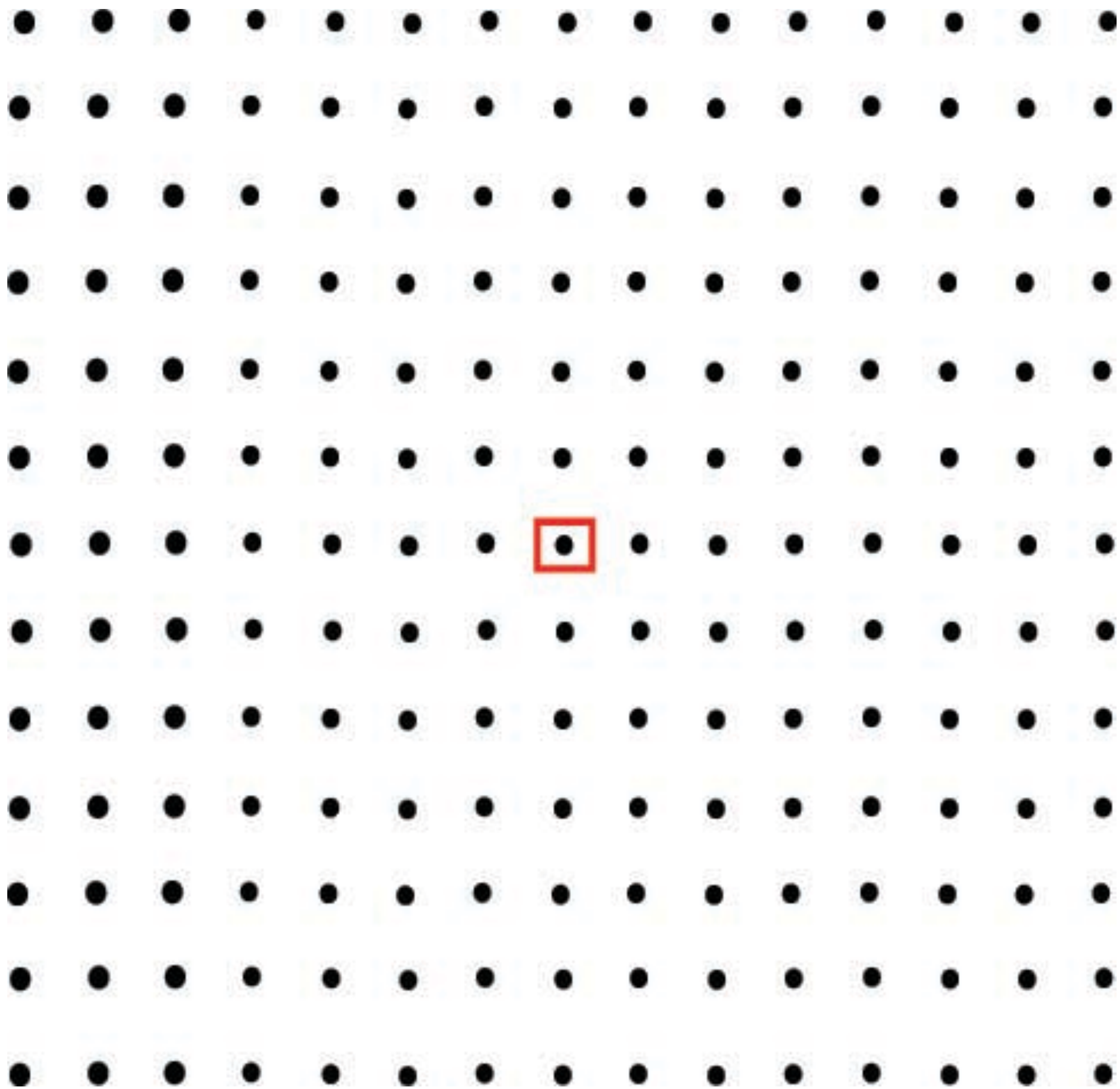


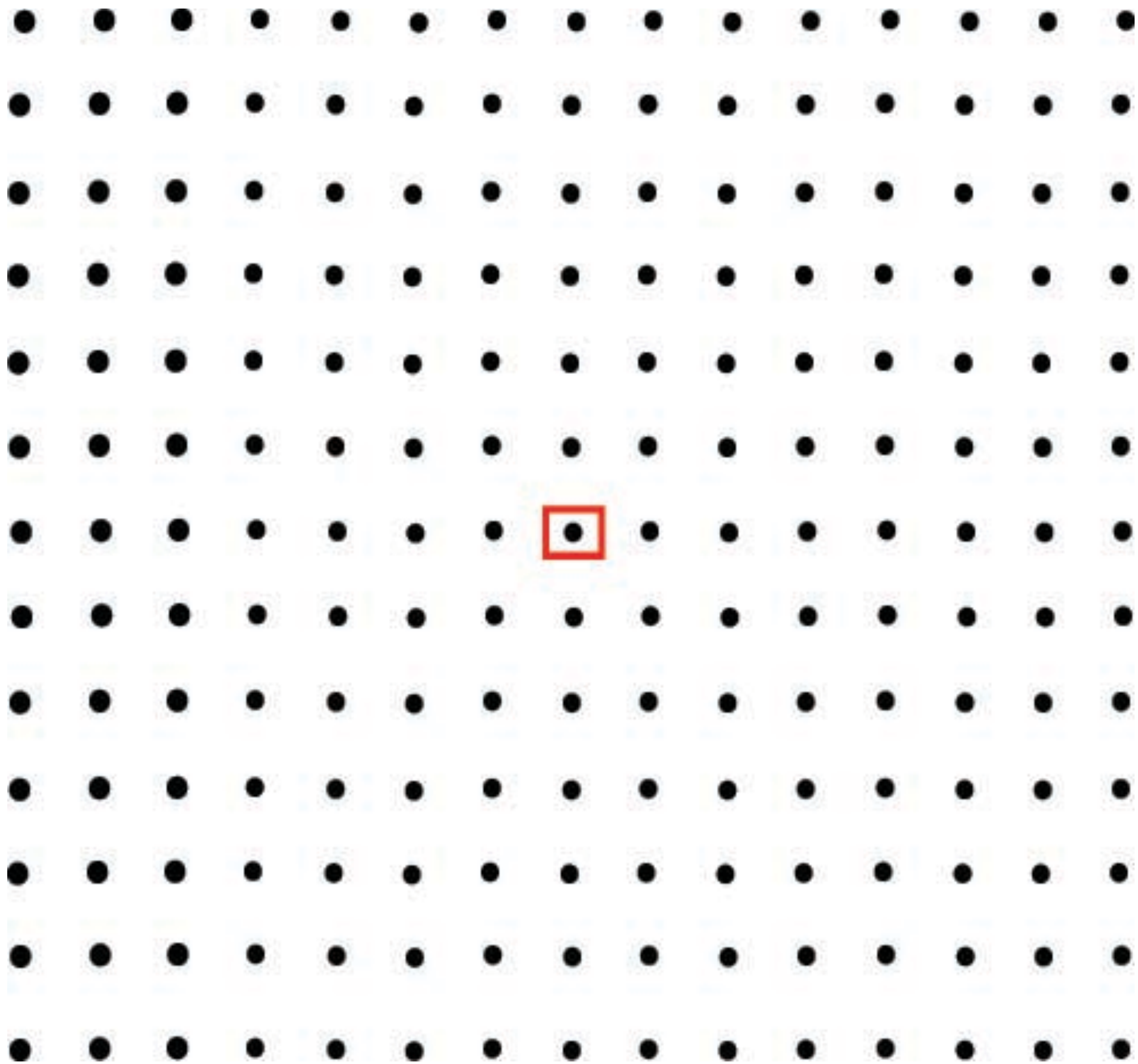


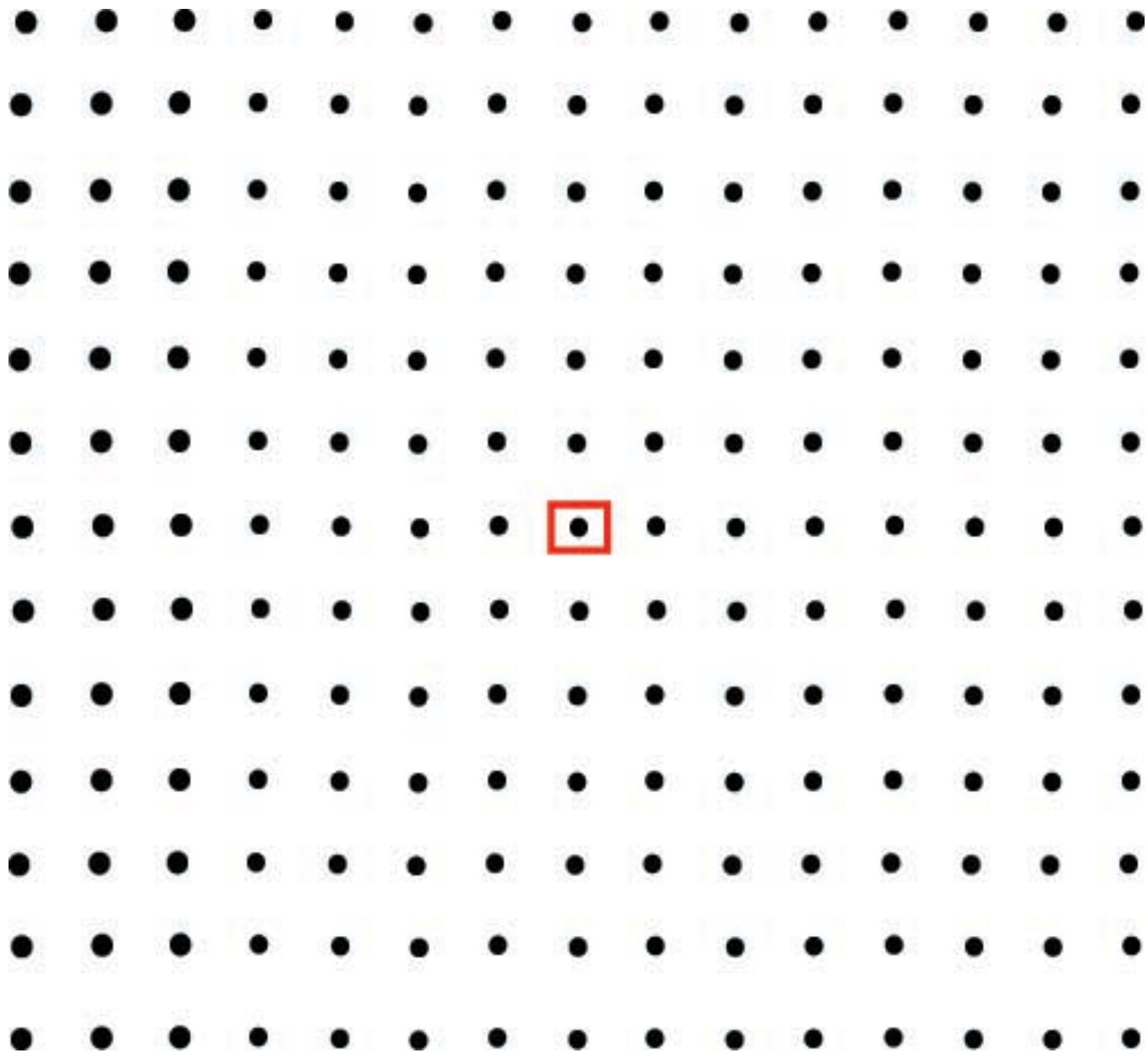


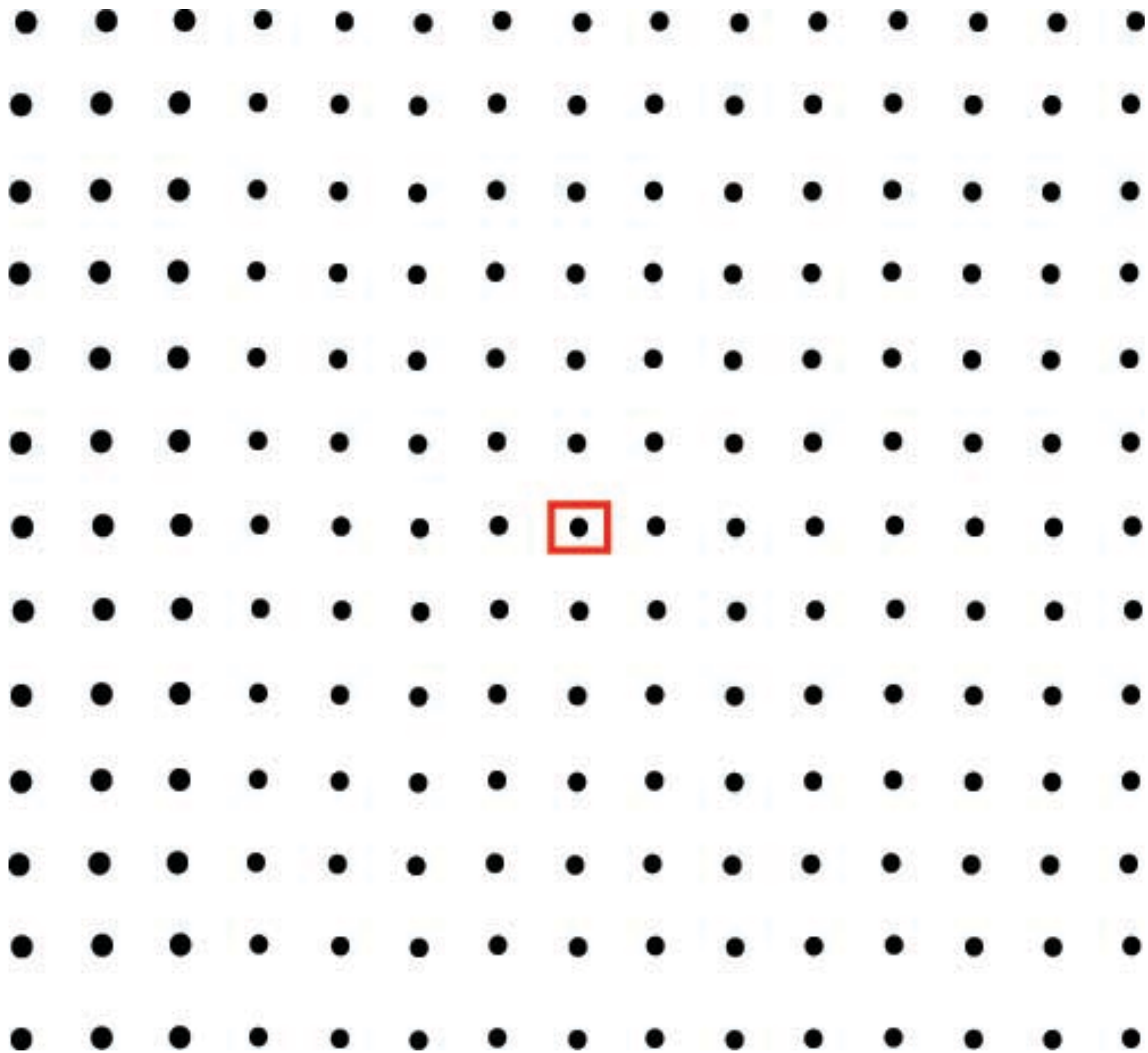


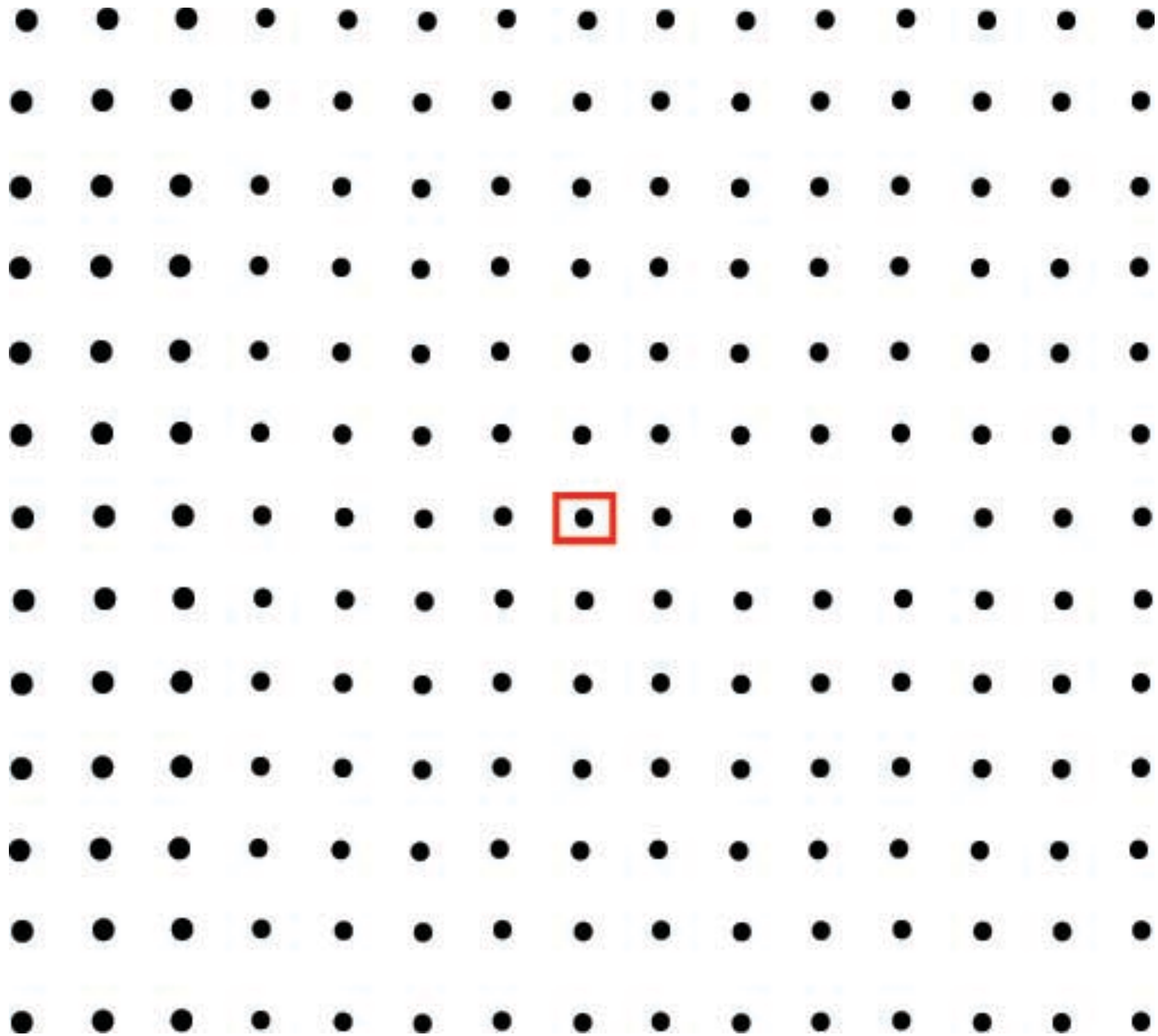


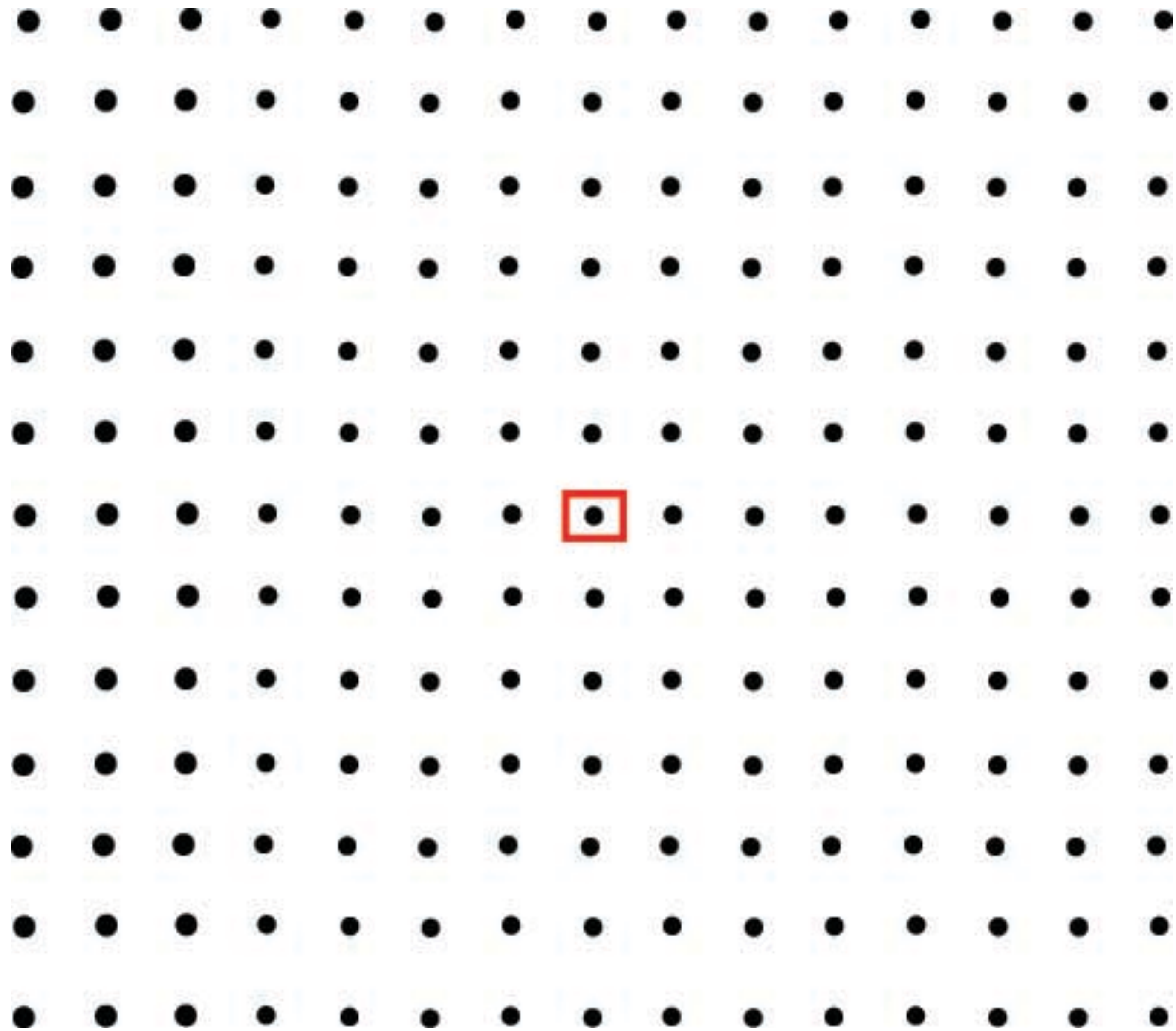


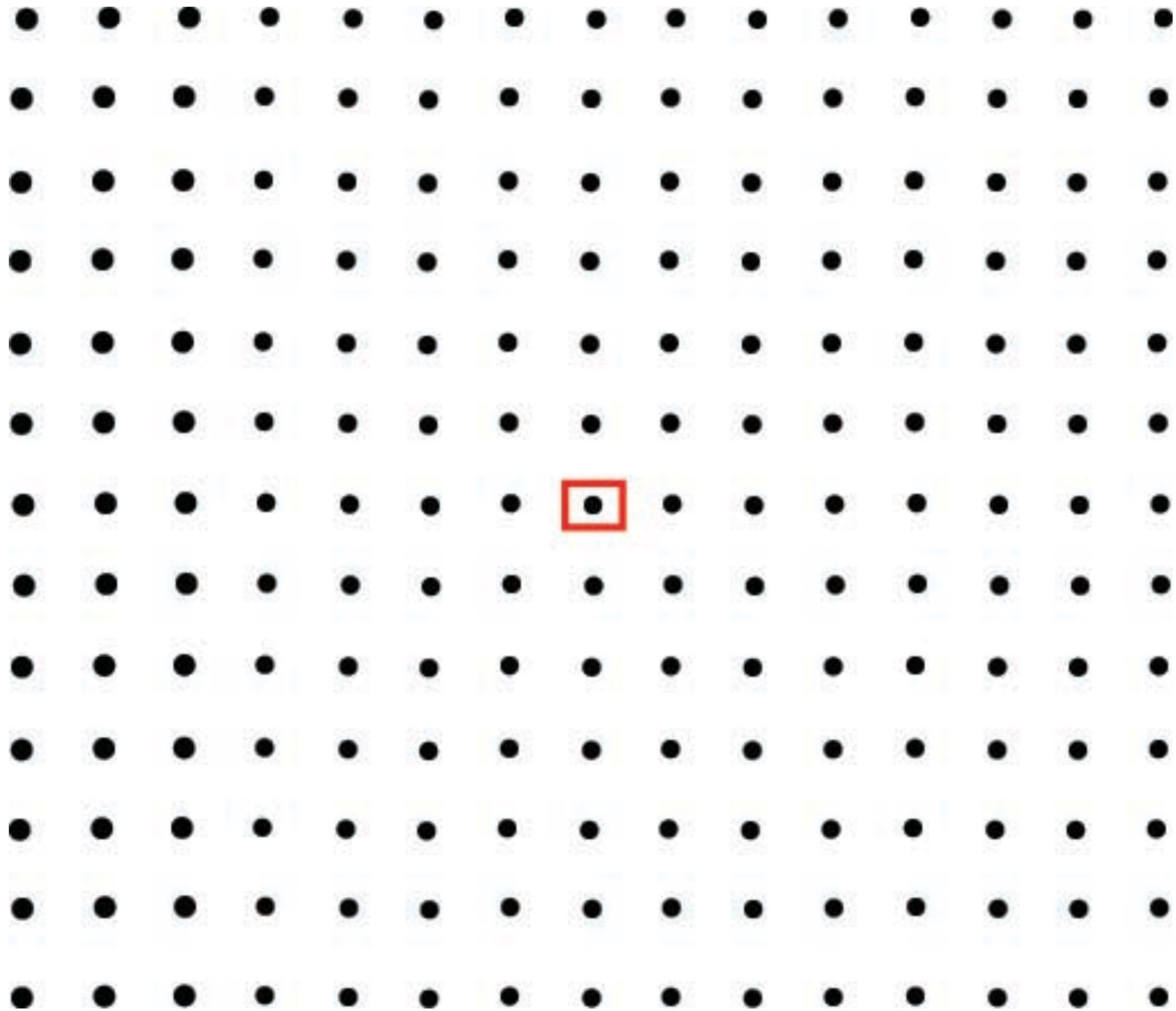


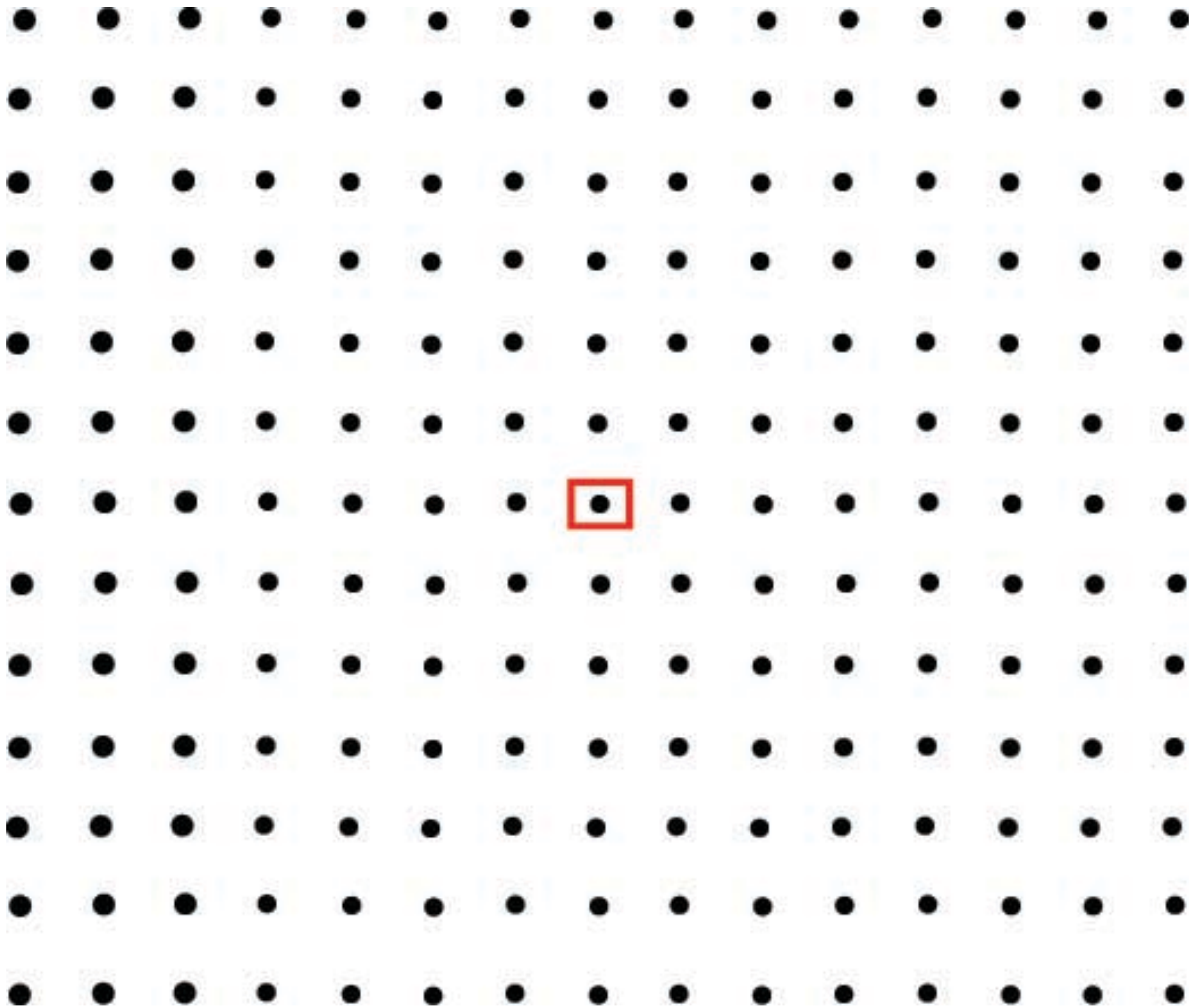


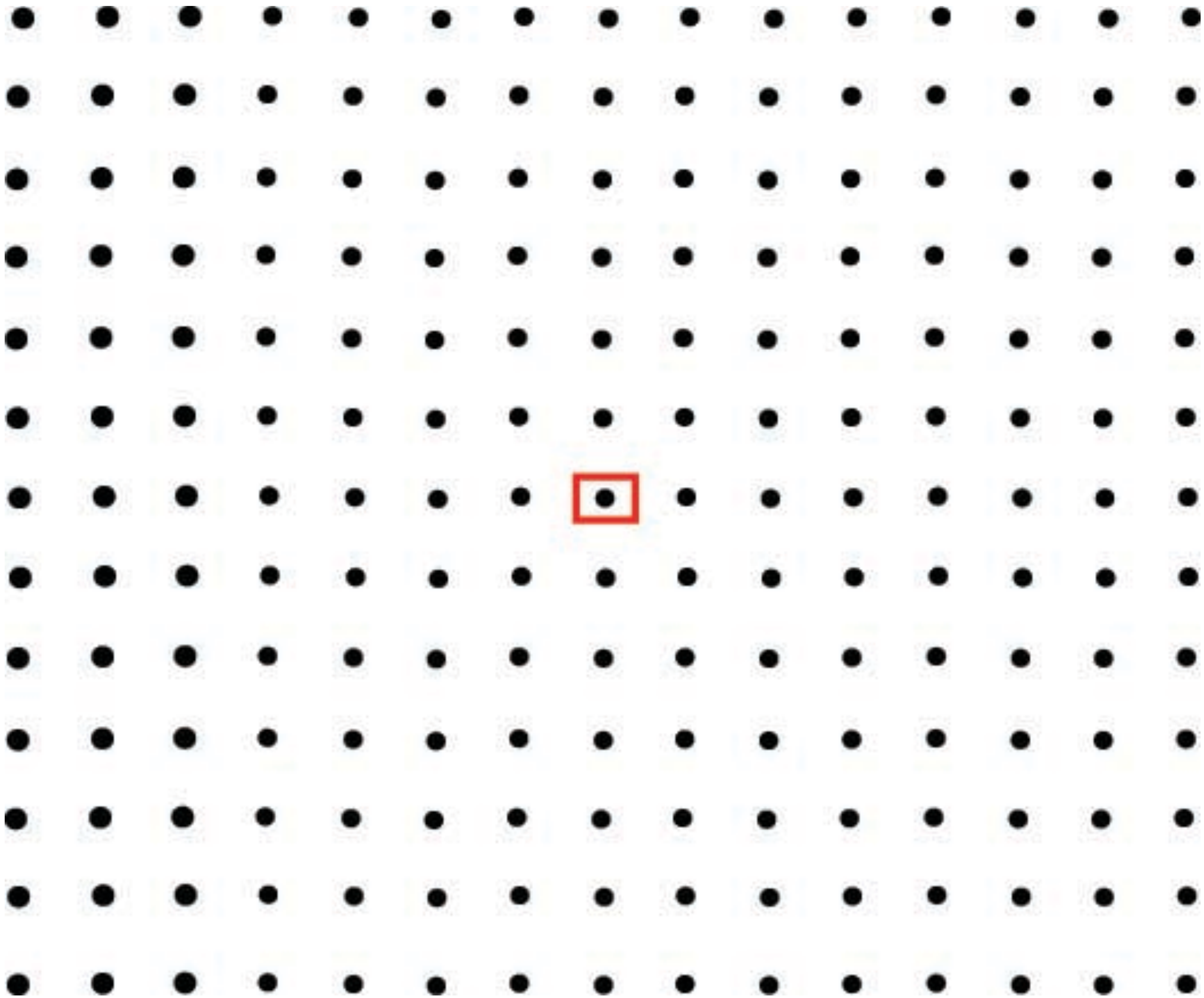


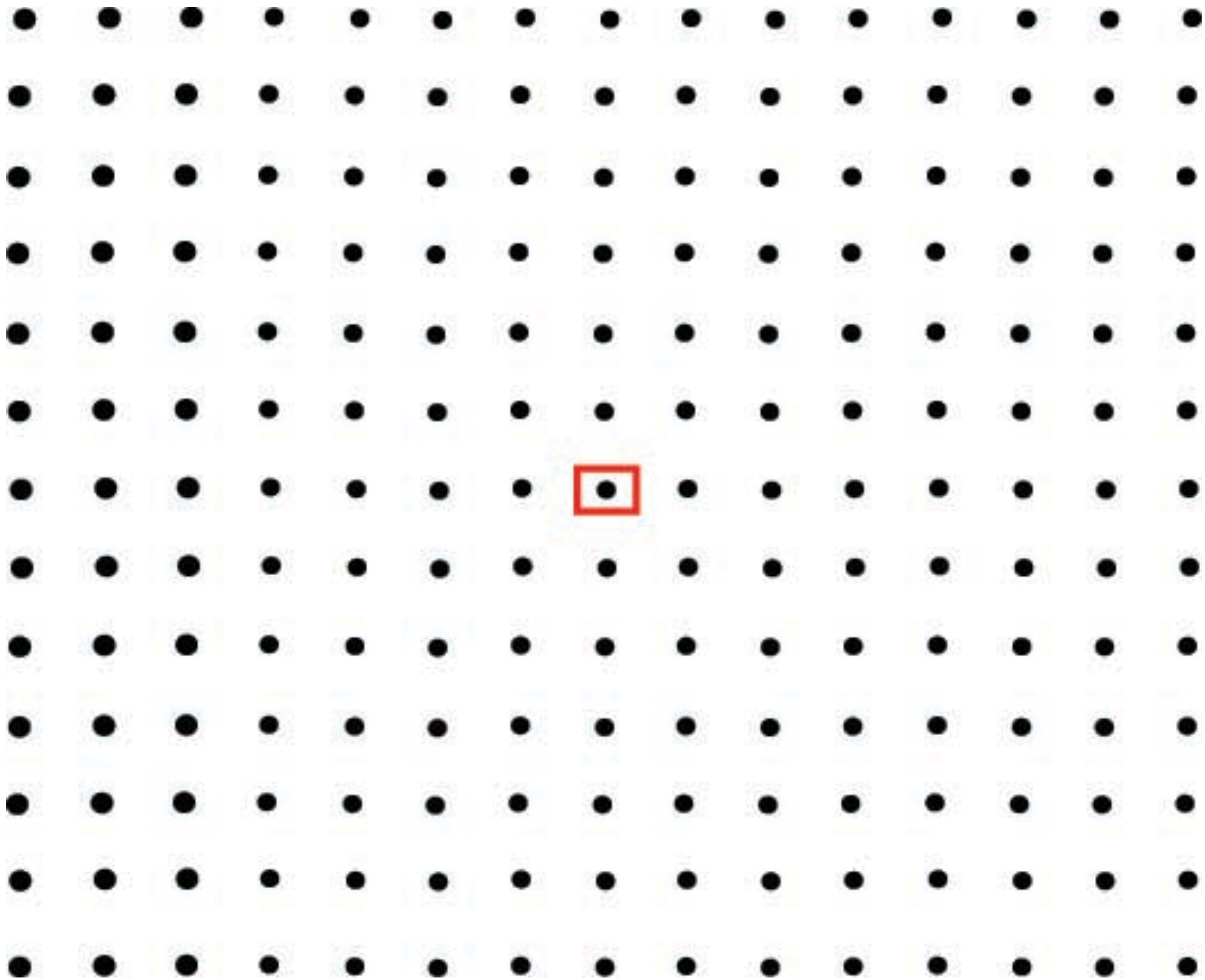


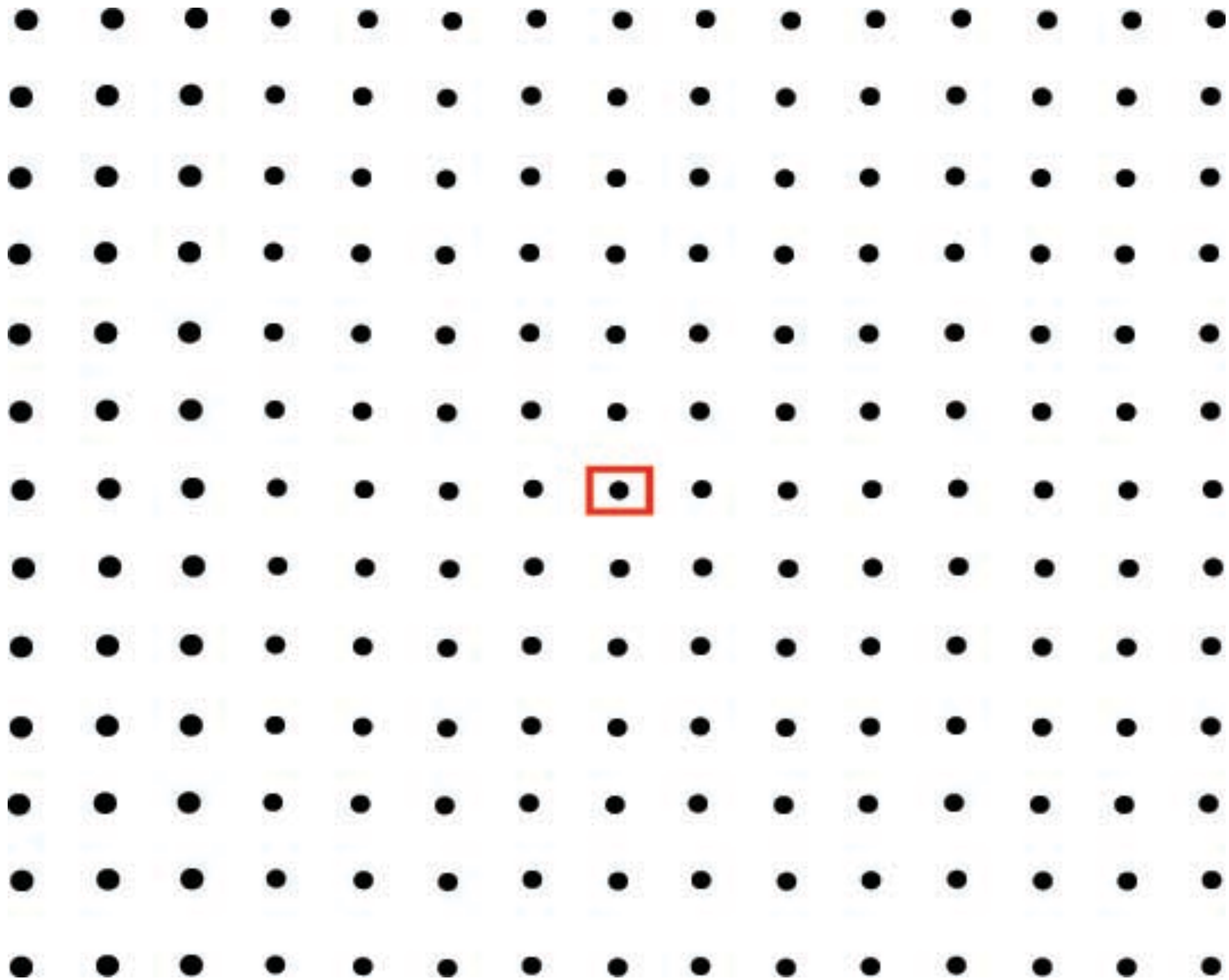


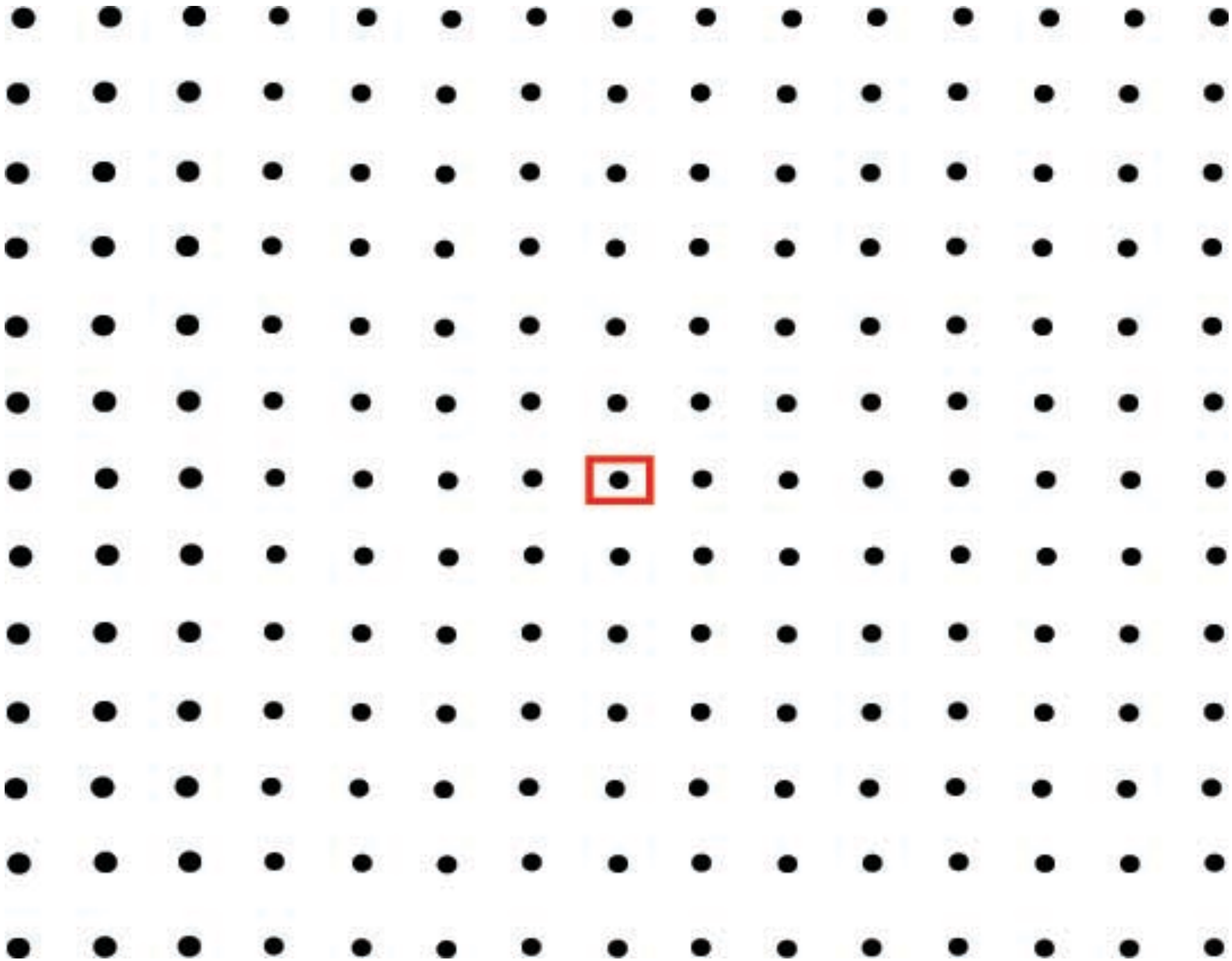


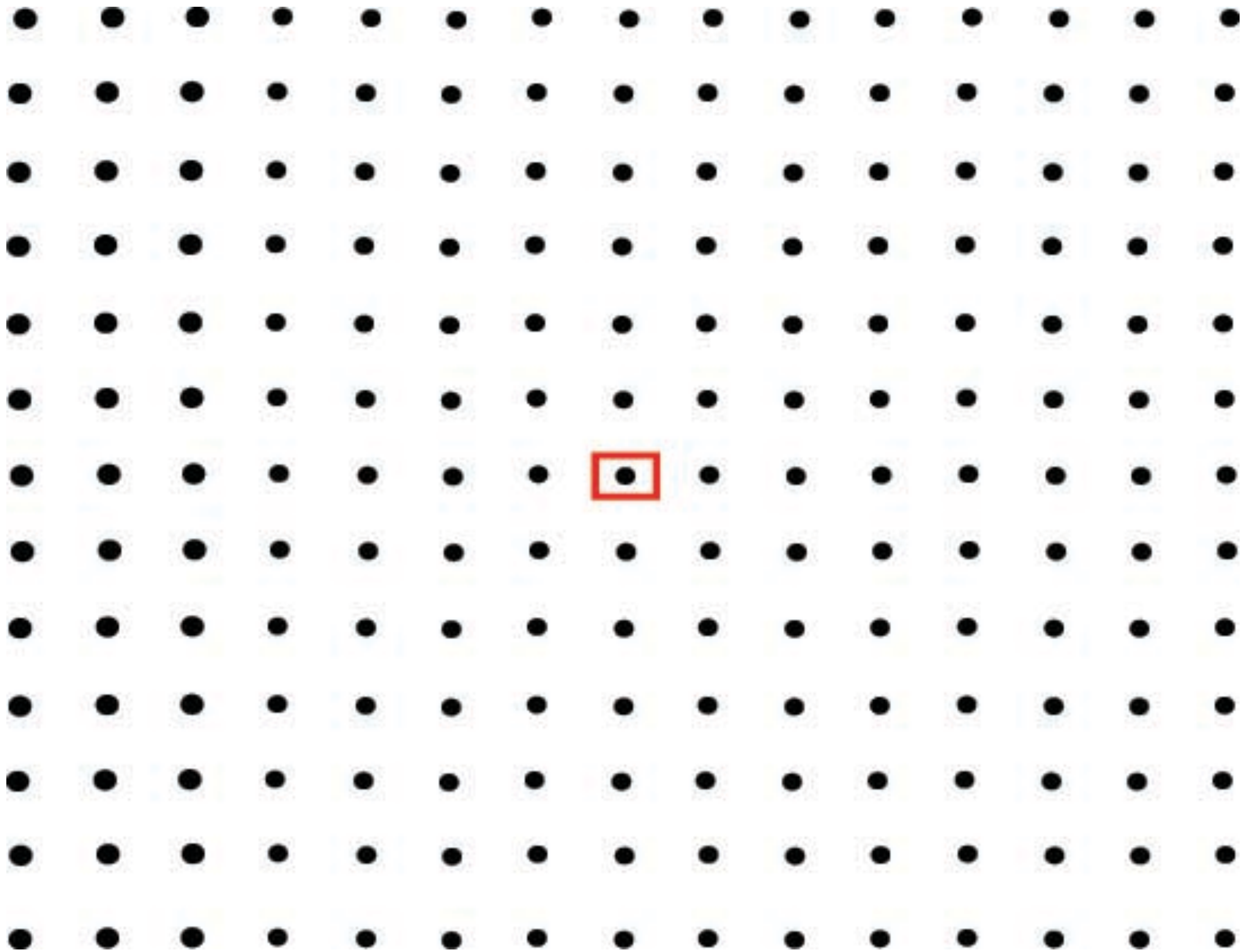


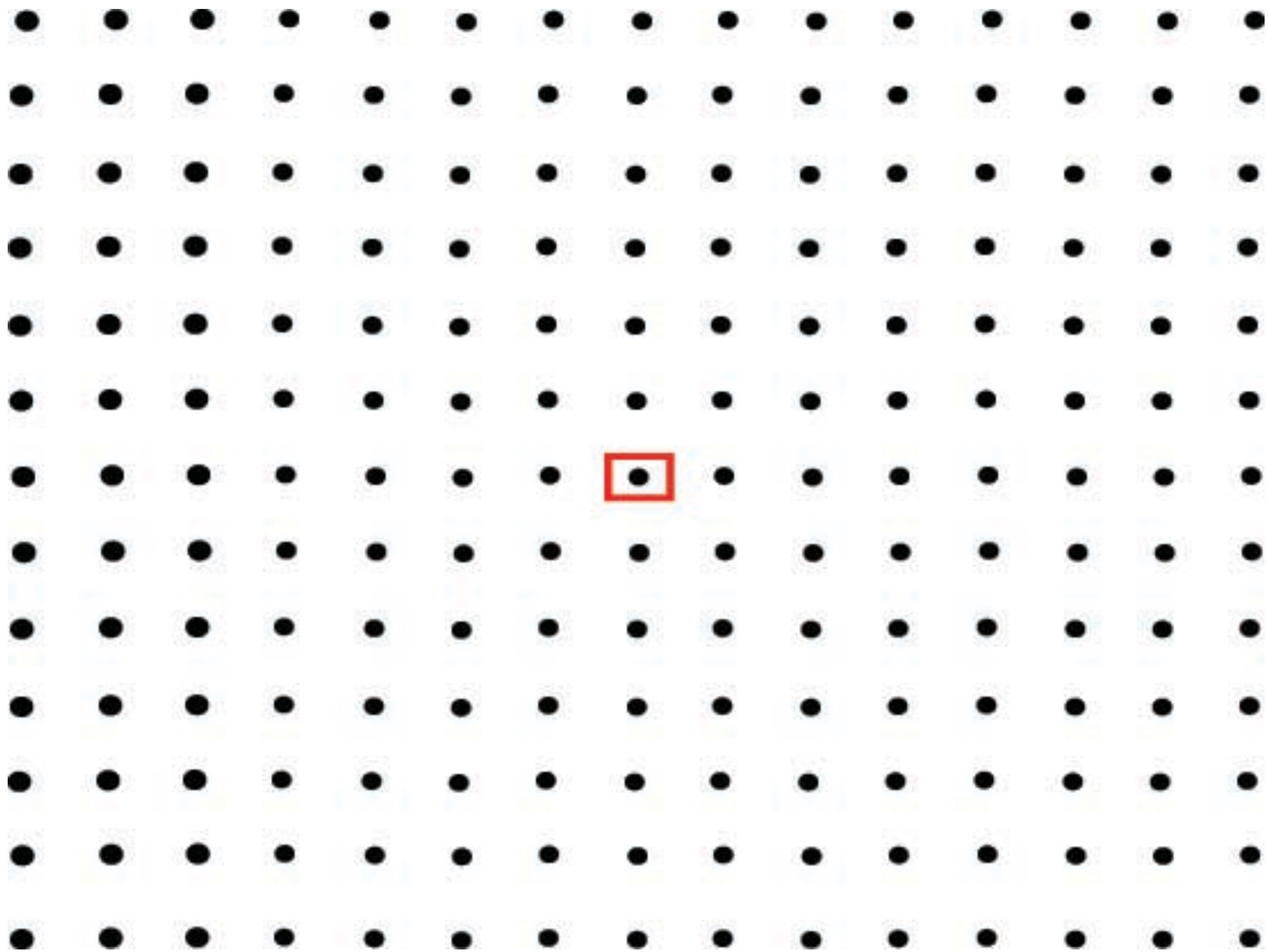


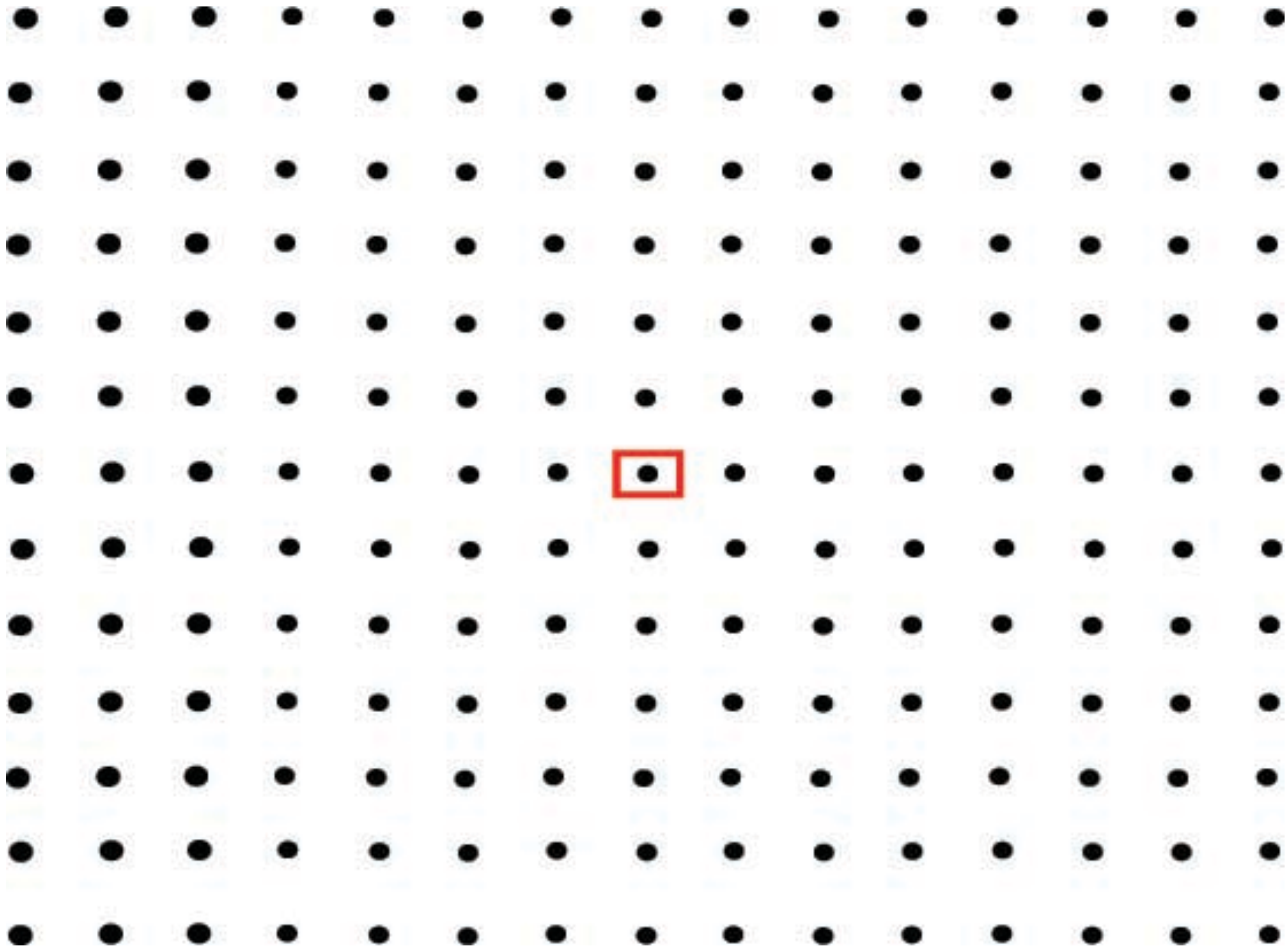


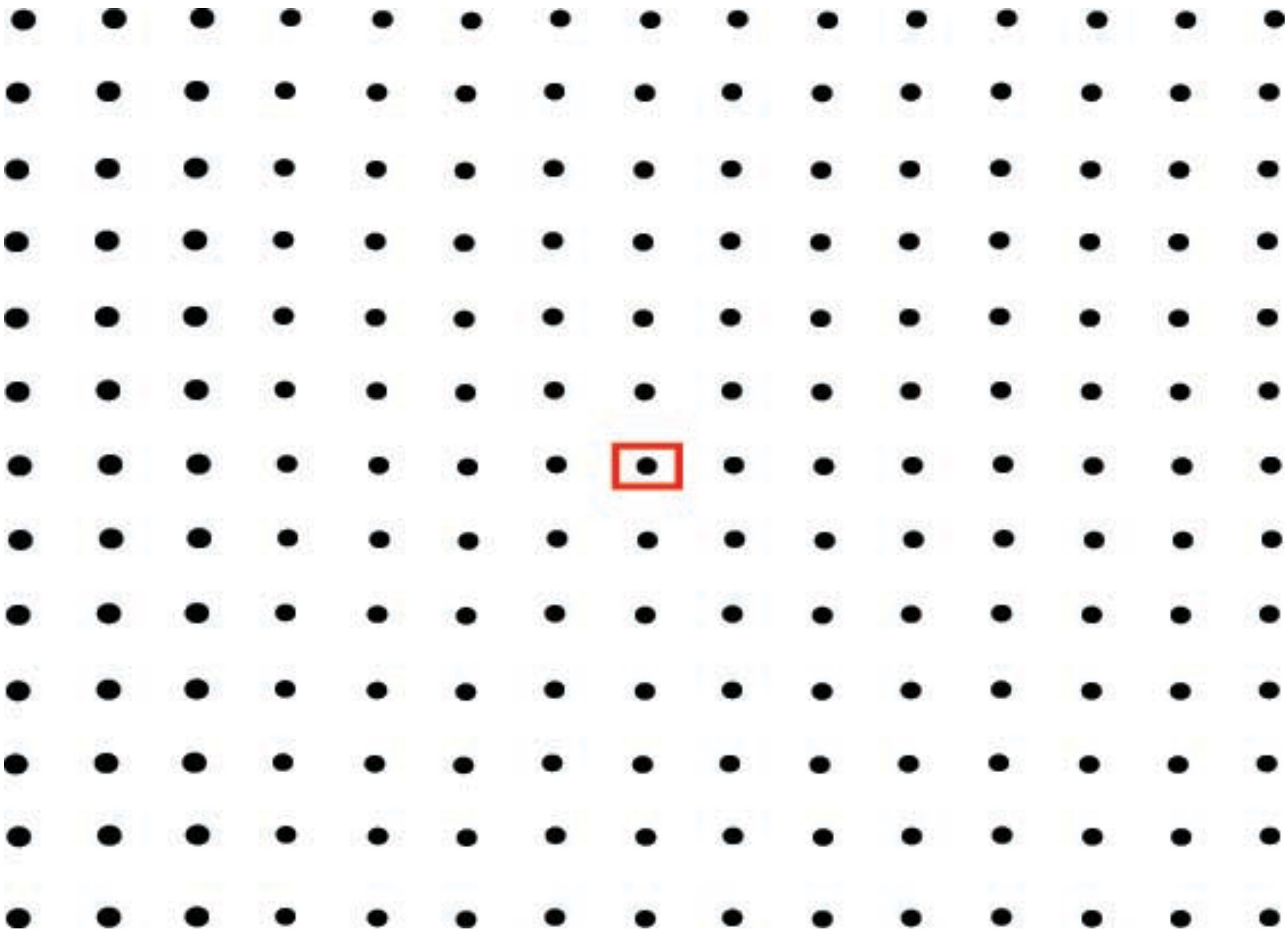


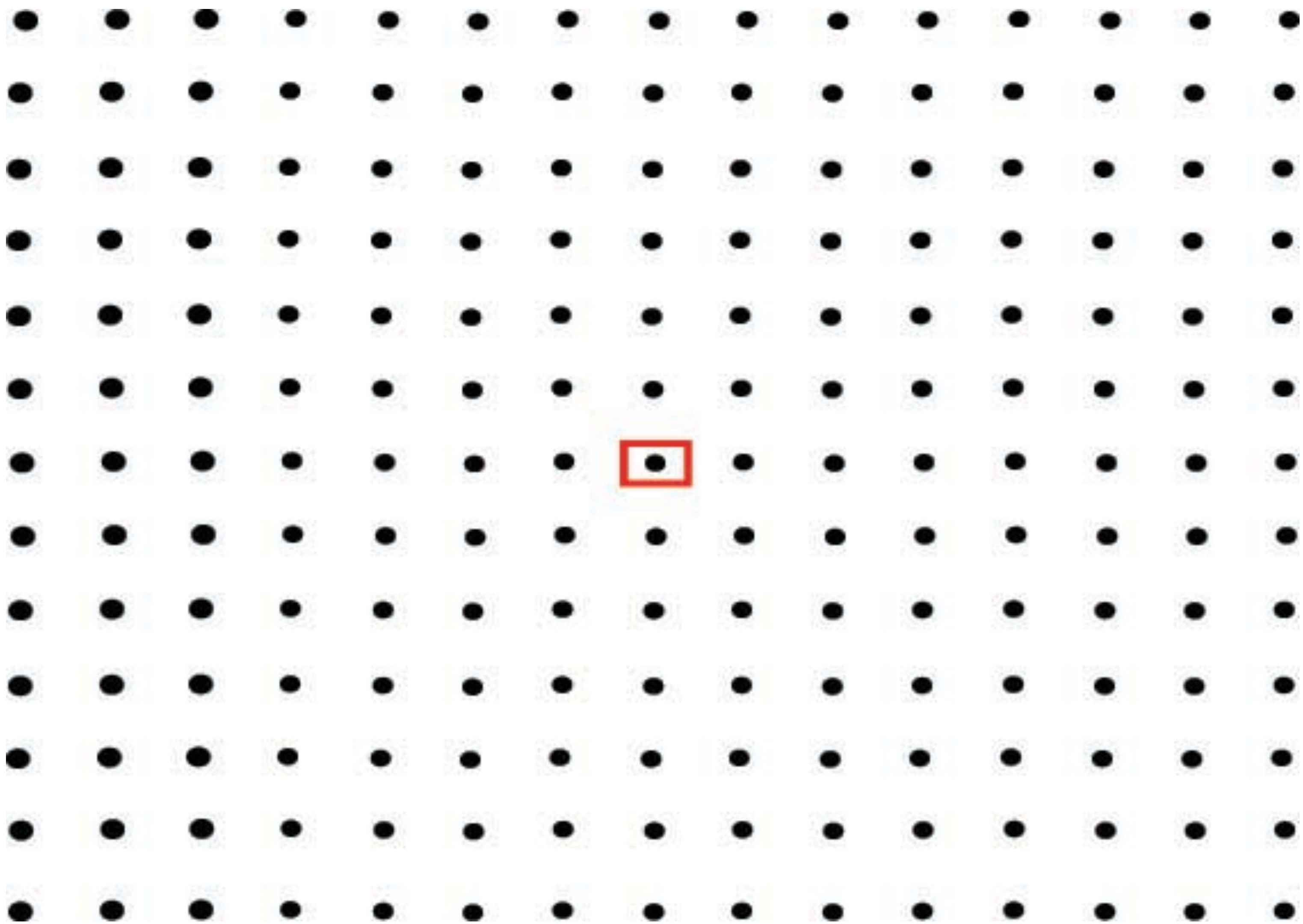


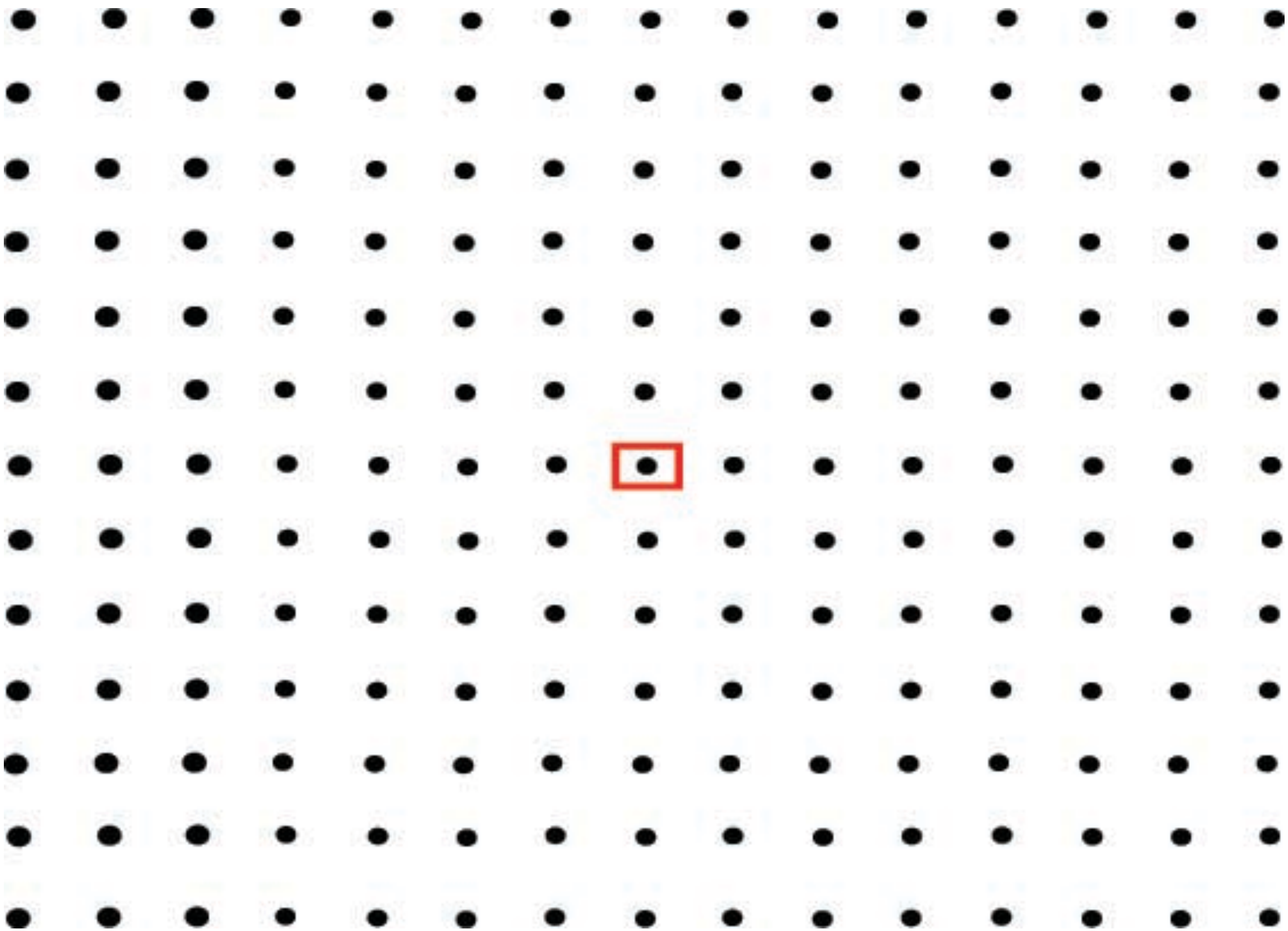


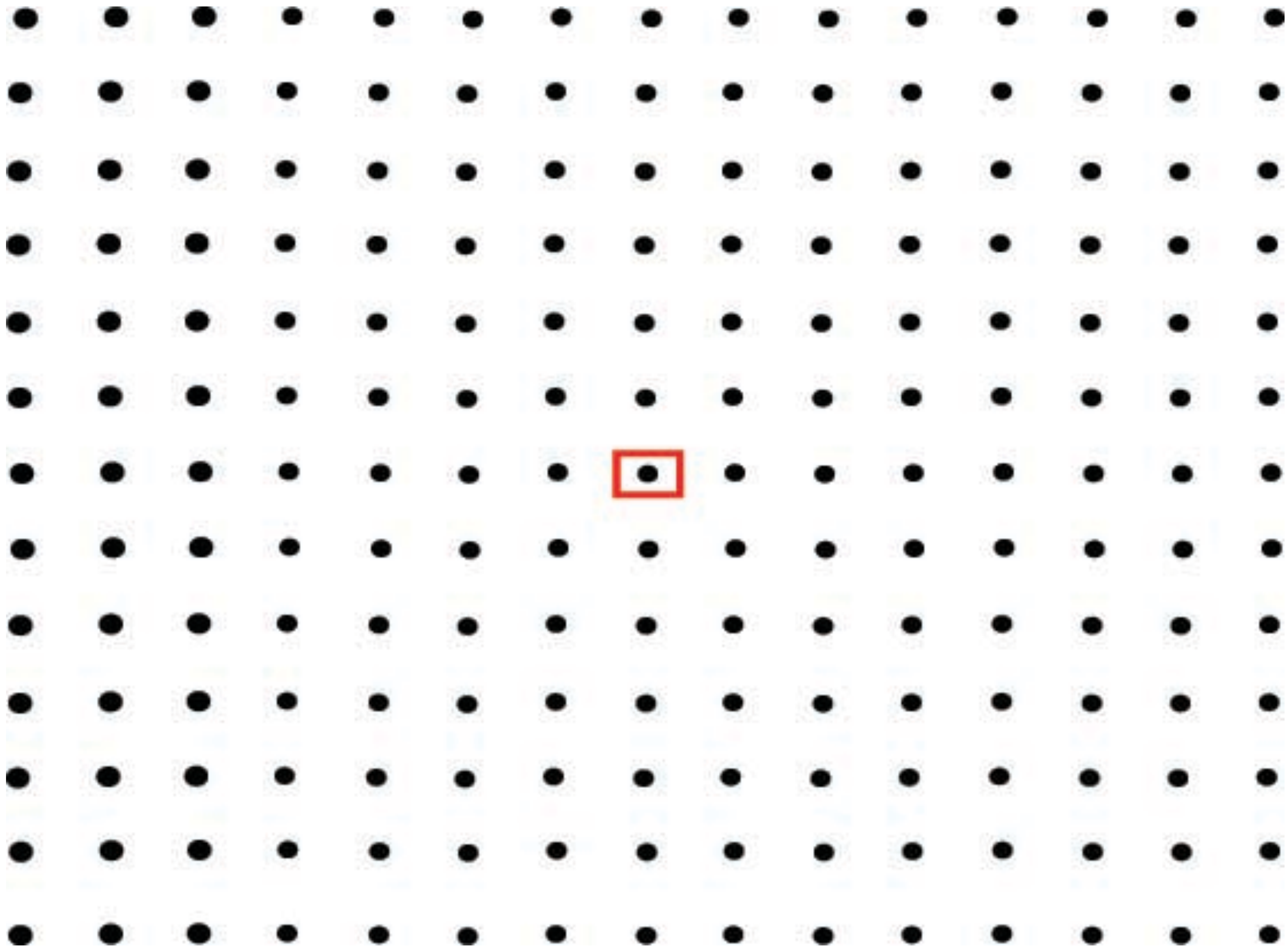


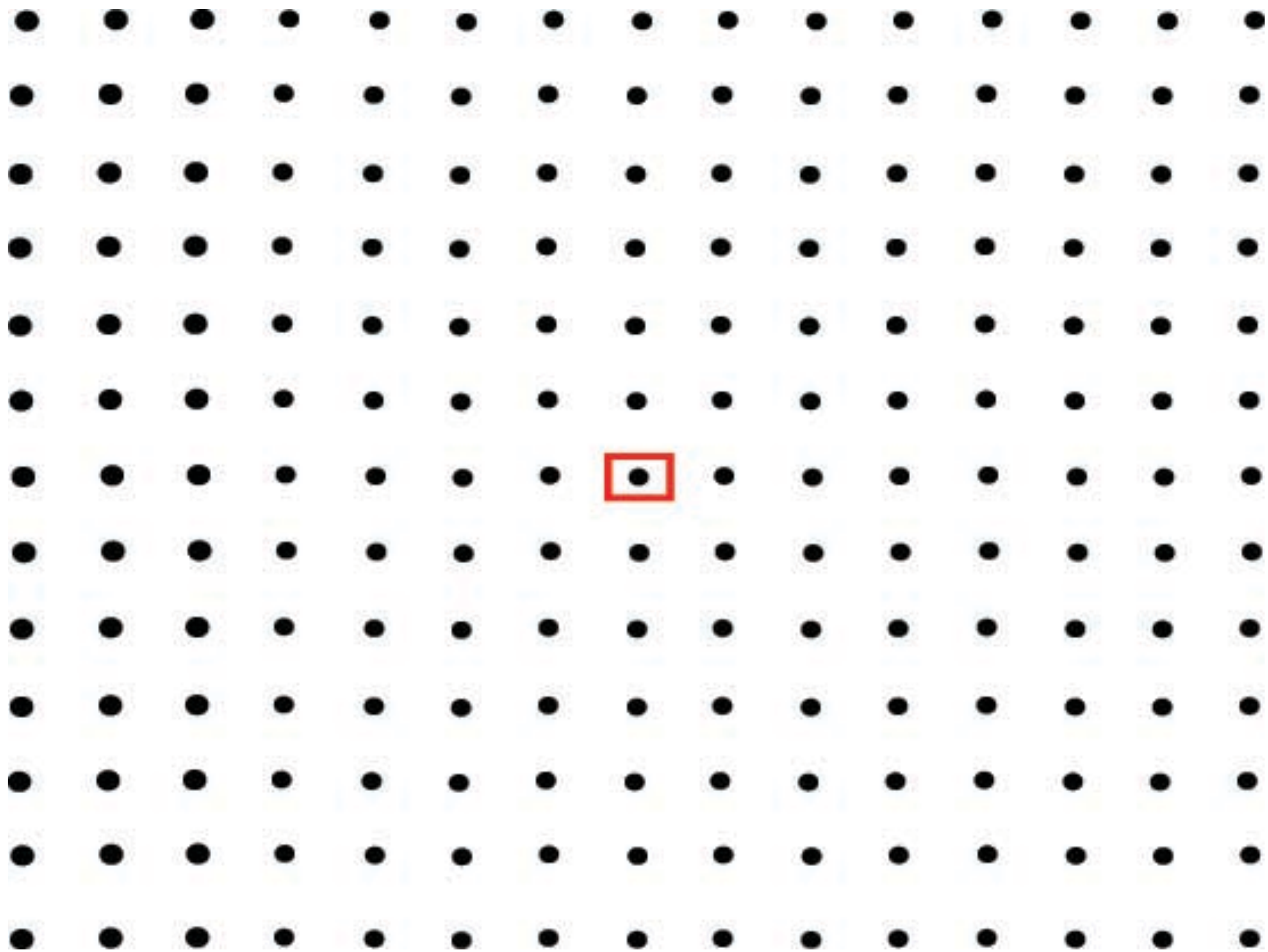


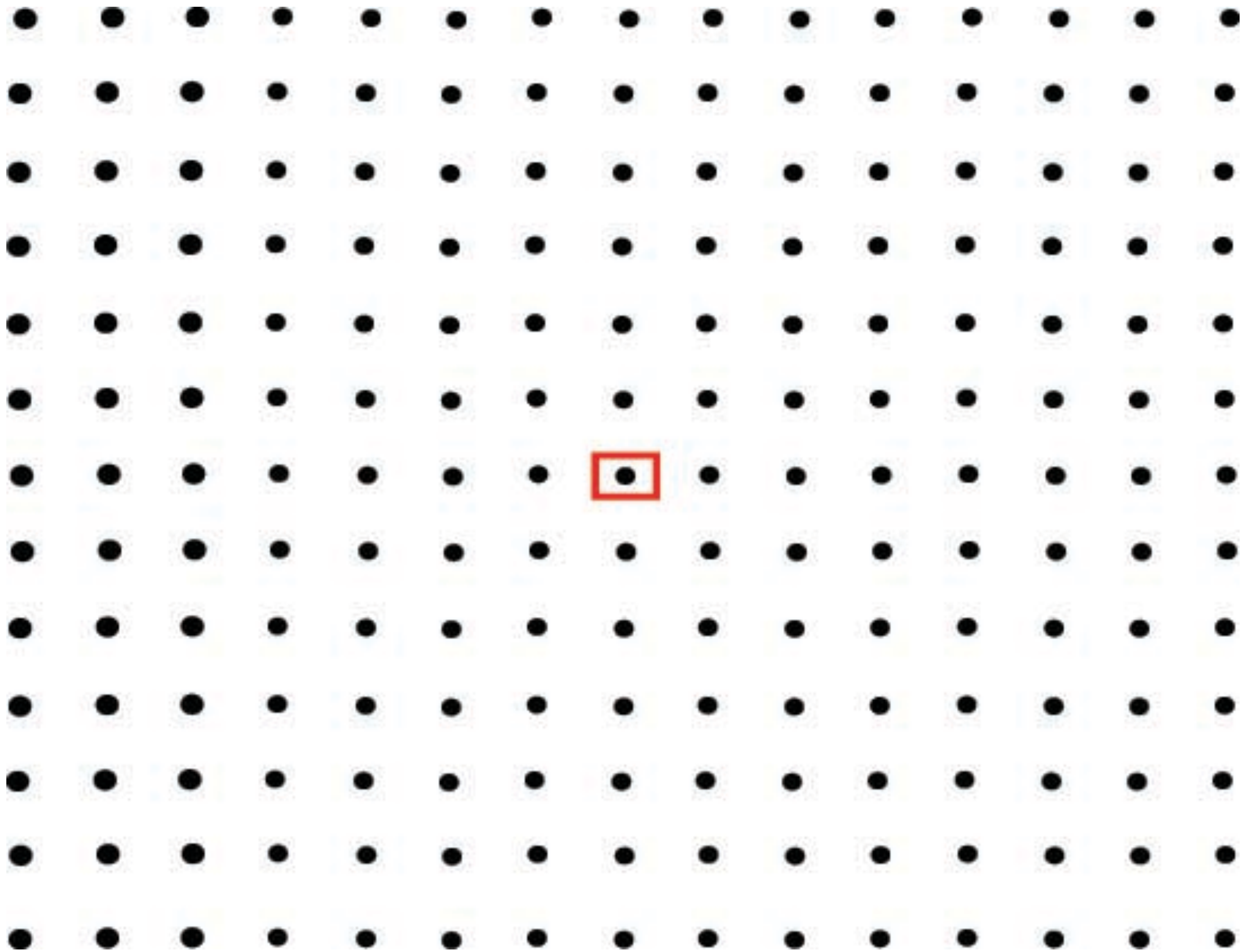


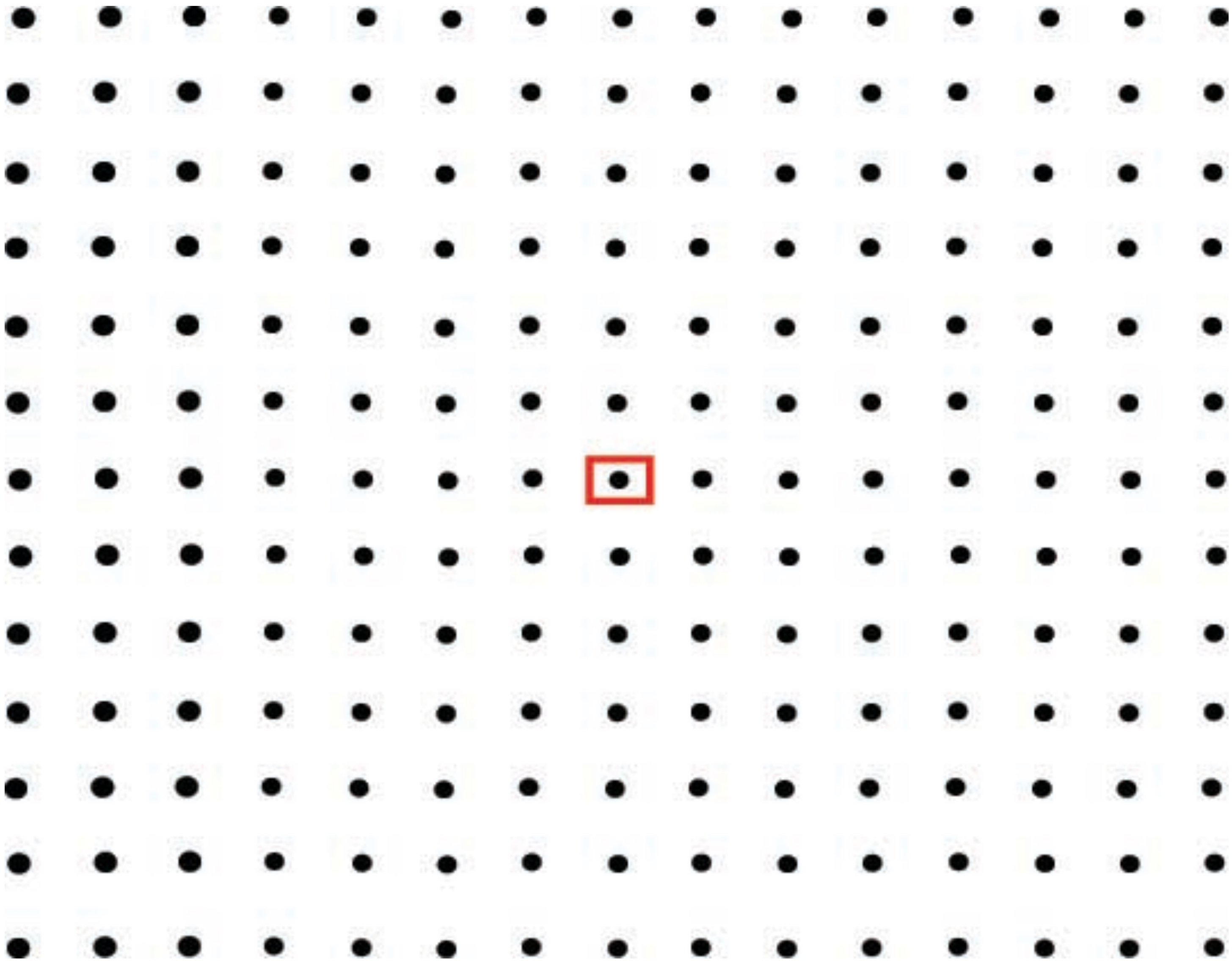


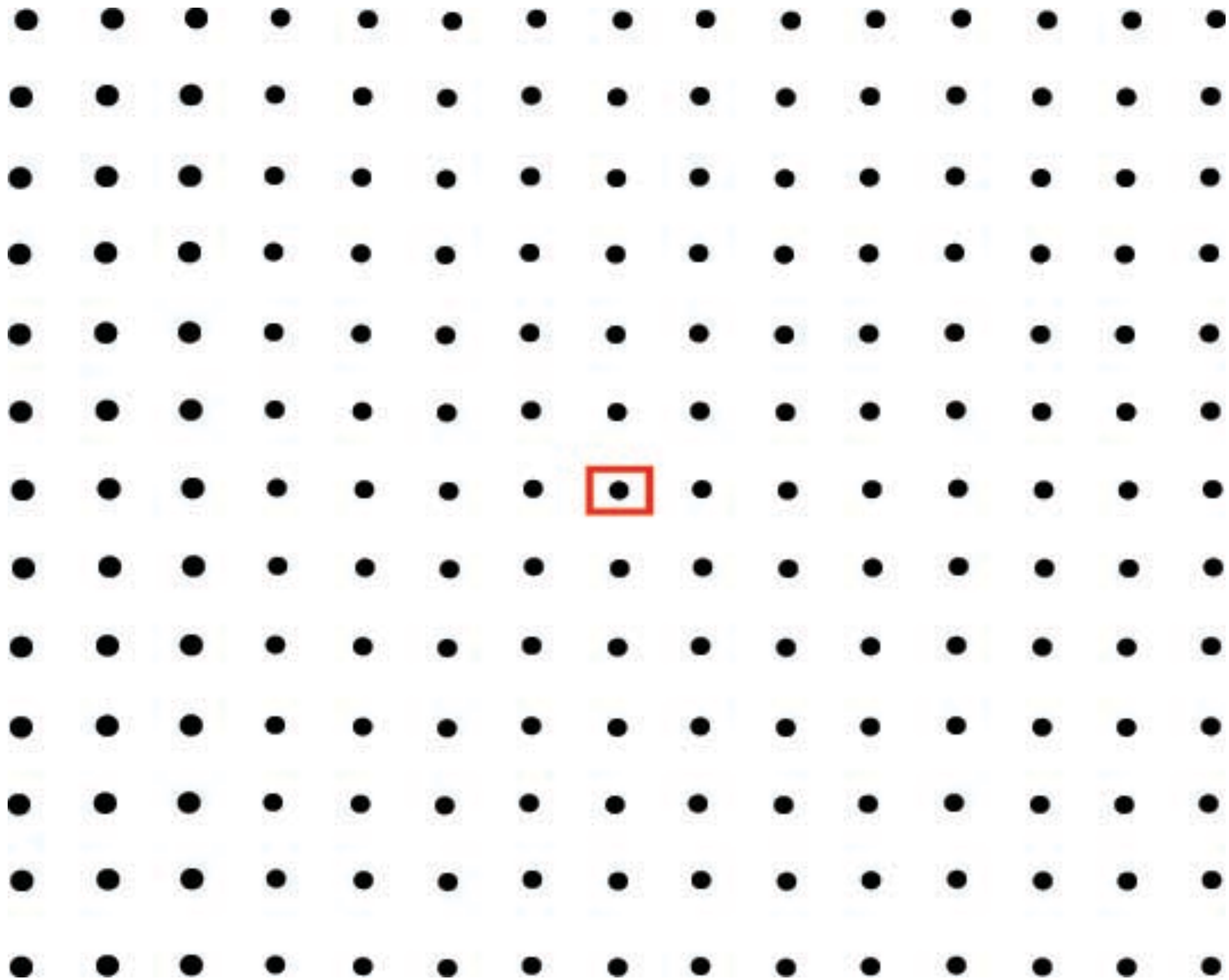


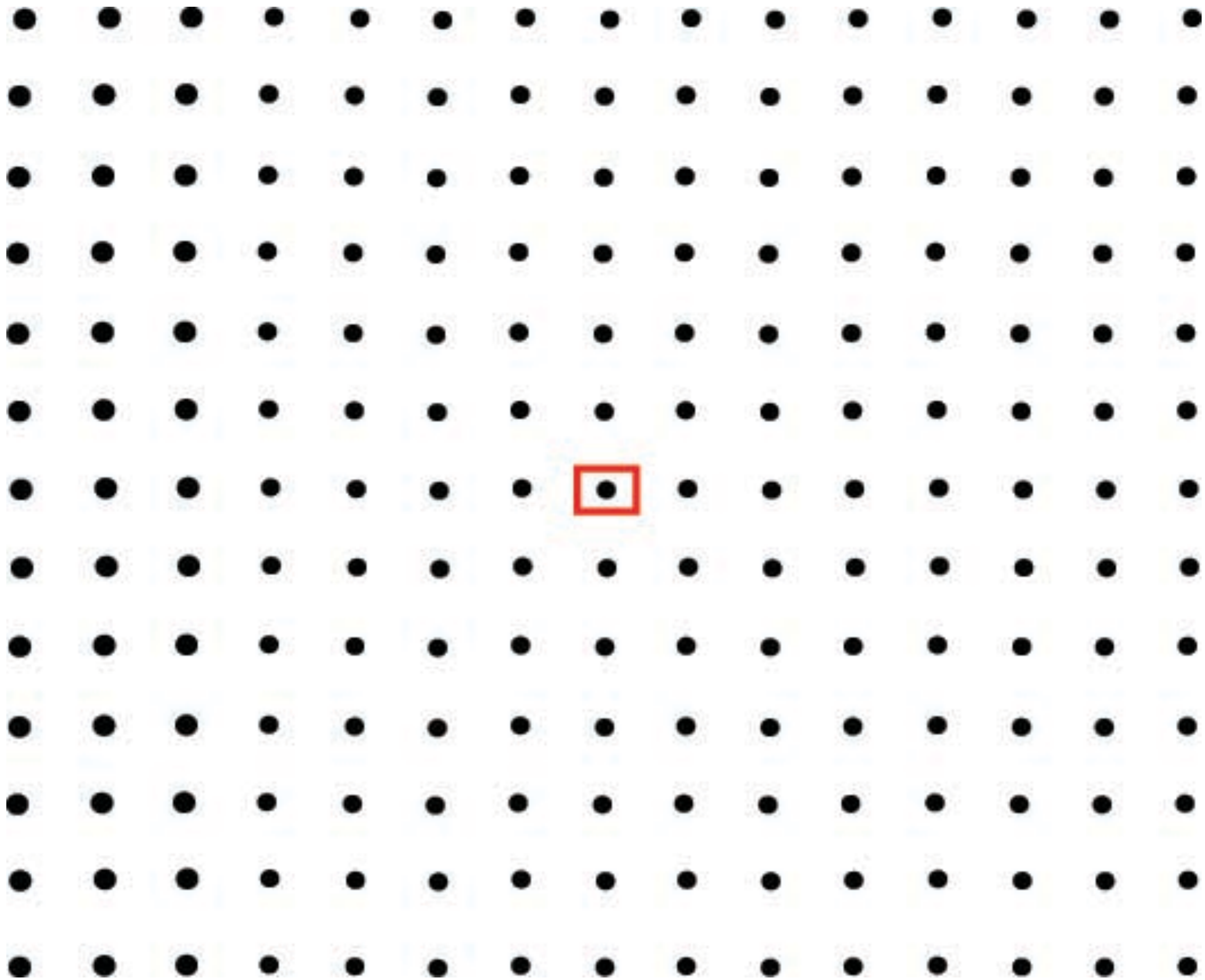


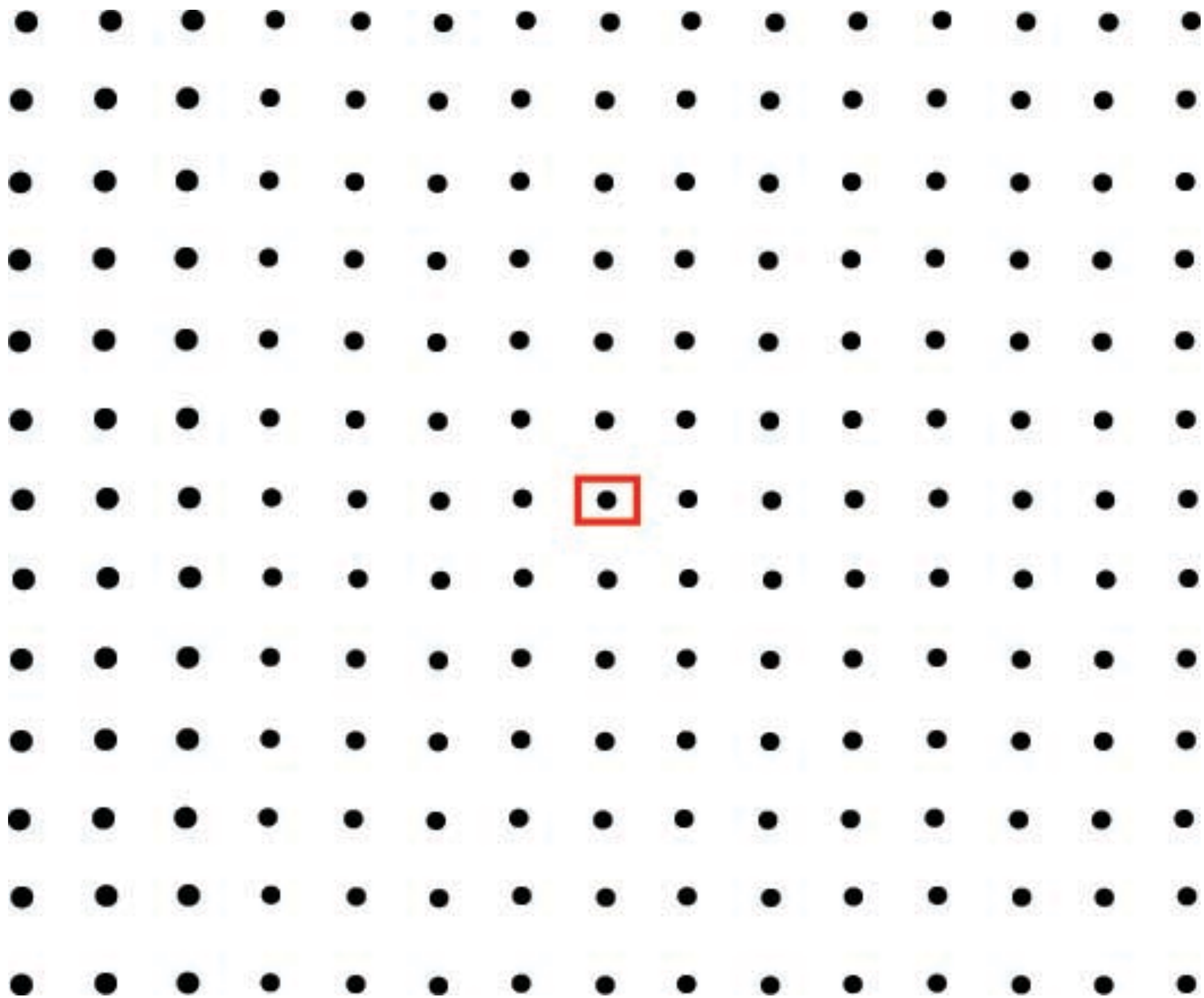


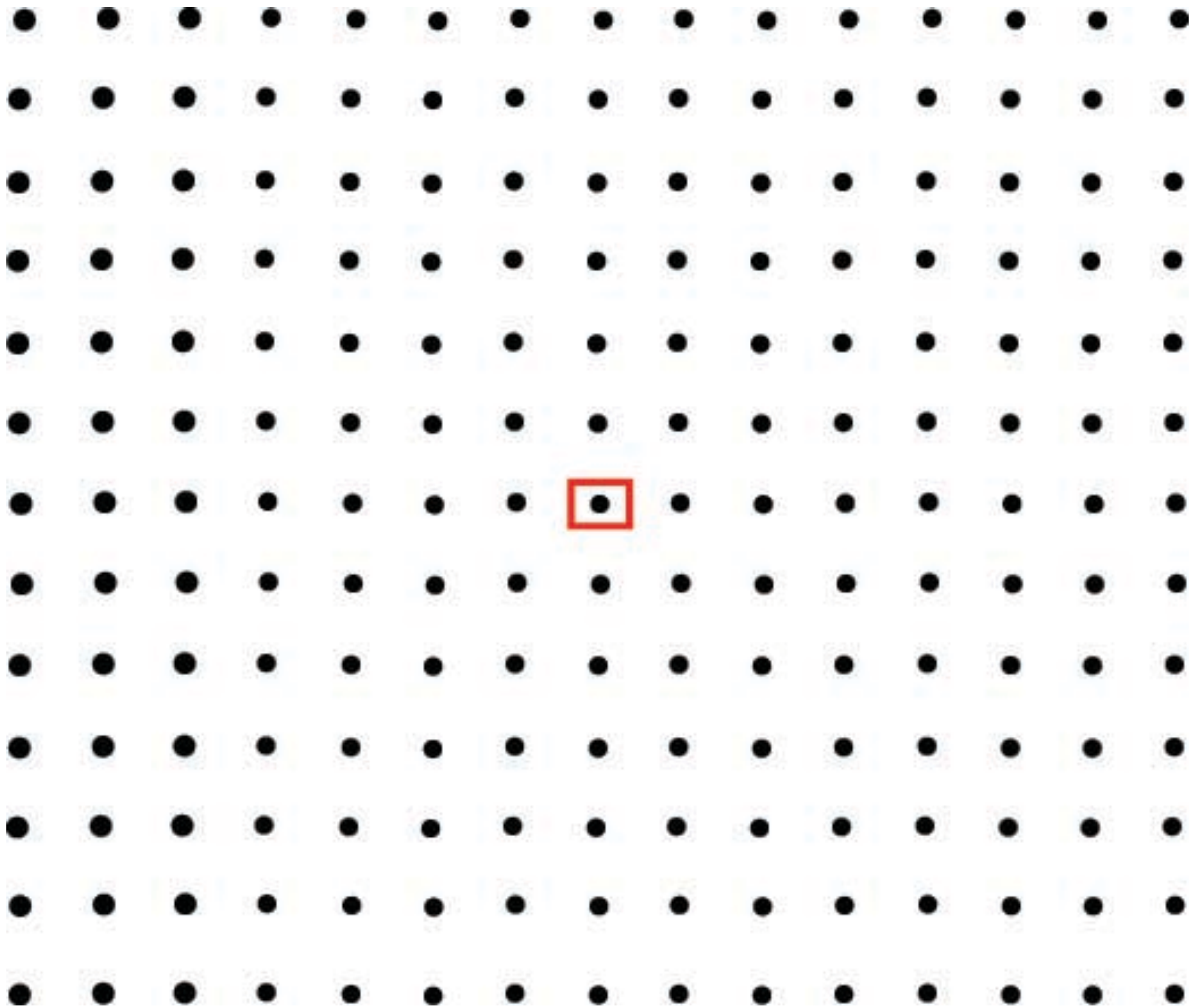




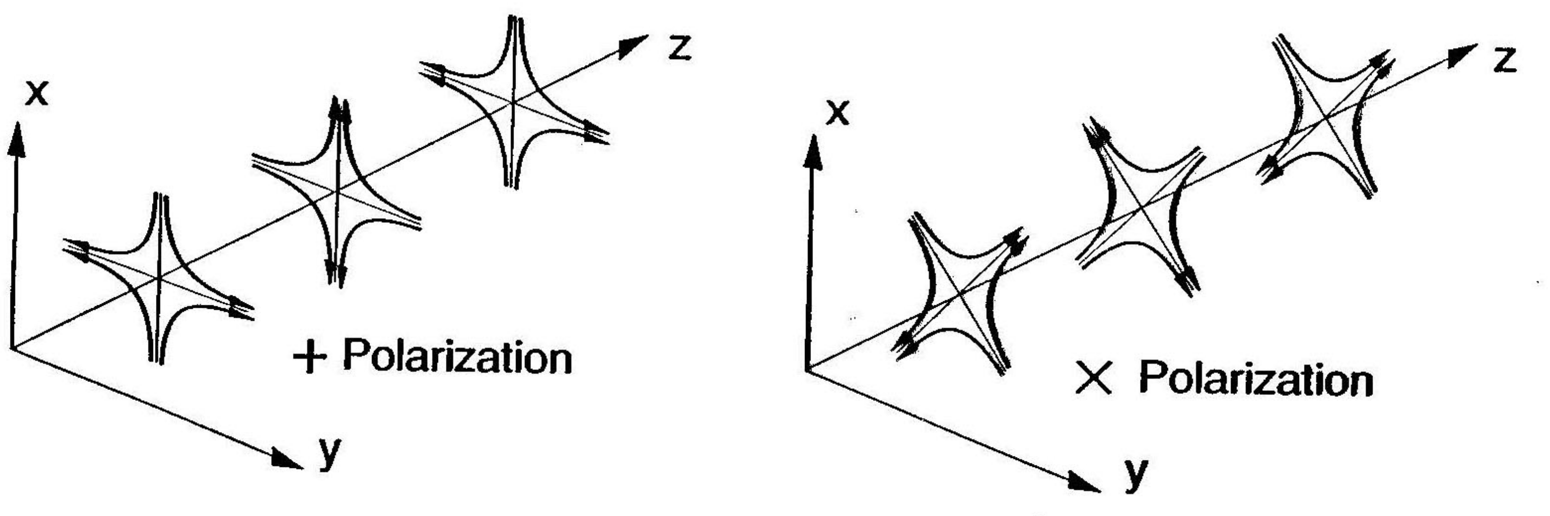








Polarization states



Relations for gravitational waves

Intensity: $$S_g = \frac{c^3}{16\pi G}\left\langle \dot{h}_+^2 + \dot{h}_x^2 \right\rangle$$ pseudo tensor

$$\frac{c^3}{16\pi G} = 7.8\text{x}10^{36}\,\text{erg sec/ cm}^2$$ space is very stiff

Power radiated: $$P_g = \frac{32Gm^2x_0^4\omega^6}{5c^5}$$ quadrupole formula

Relation to estimate GW amplitude: $$h \approx \frac{\varphi_{Newton}}{c^2}\frac{v^2}{c^2} = \frac{Gm}{Rc^2}\frac{v^2}{c^2}$$

Discouraging back of the envelope estimate 1916 example

binary star system

m_1=m_2= 1 solar mass
T_{orbit}= 1 day
R = 10 Kly

h ~ 10^{-23} @ ½ day period

$$Q = \frac{2\pi E_{stored}}{\Delta E_{1period}} \sim 10^{15} \quad \text{decay time} \sim 10^{13}\,\text{years}$$

scaling $$h \propto \frac{m^{\frac{5}{3}}}{RT_{orbit}^{\frac{2}{3}}}$$

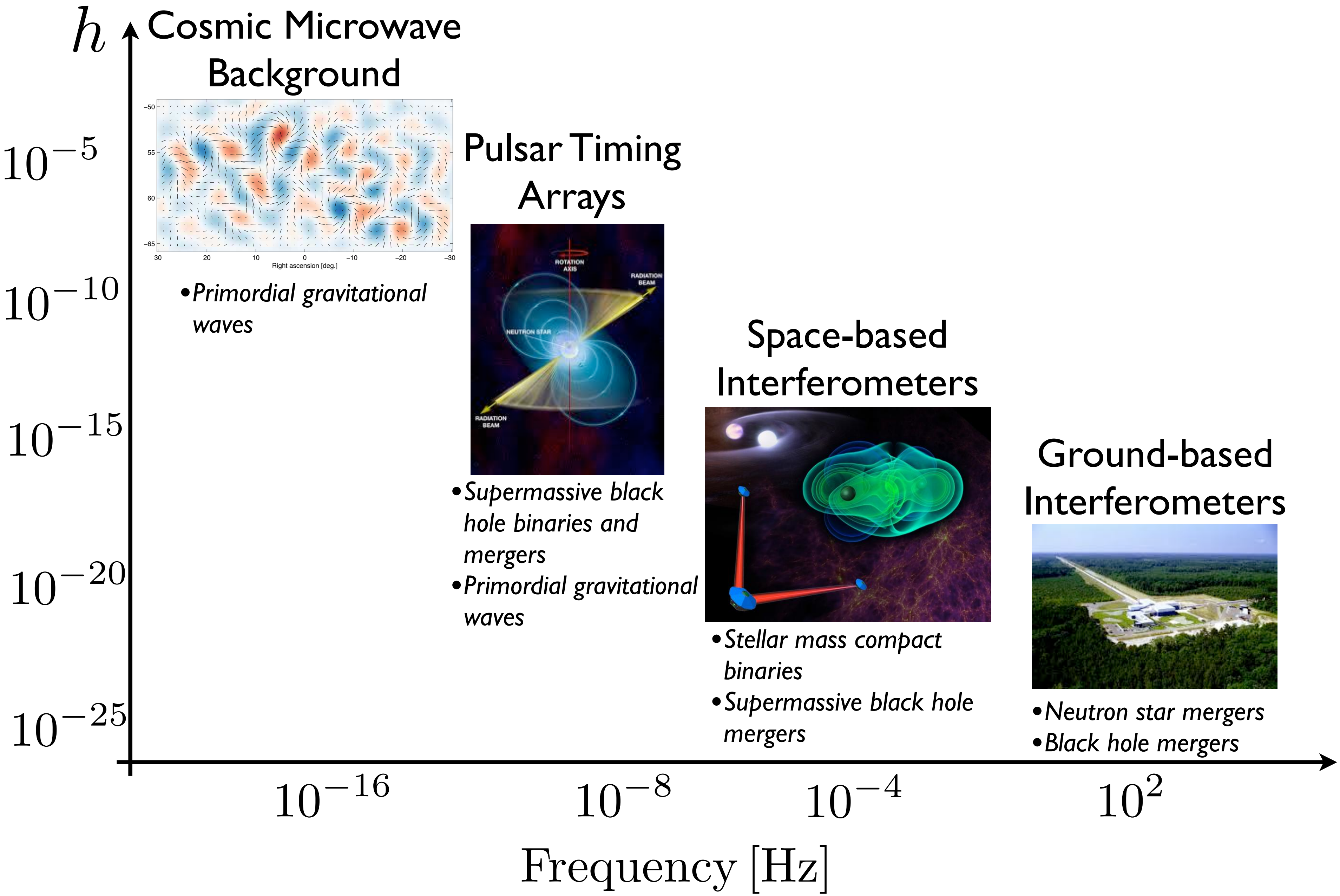
$$Q \propto \left(\frac{T_{orbit}}{m}\right)^{\frac{5}{3}}$$

What has changed over the past 100 years

- Nature has been kind: in addition to binary stars in our galaxy there are enormously powerful radiators:
- Discovery of compact objects in the universe
  - black holes $M_{sun} < M < 10^{11} M_{sun:}$ $10^4 < f\,(Hz) < 10^{-8}$
  - neutron stars: pulsars as good clocks and binary neutron stars as GW sources $10^{-3} < f(Hz) < 10^3$

    Hulse and Taylor settling the 40 year controversy
- Discovery of the cosmic background radiation
- Ability to model the sources
- These GW sources hold the promise of fundamental discoveries in physics and astronomy

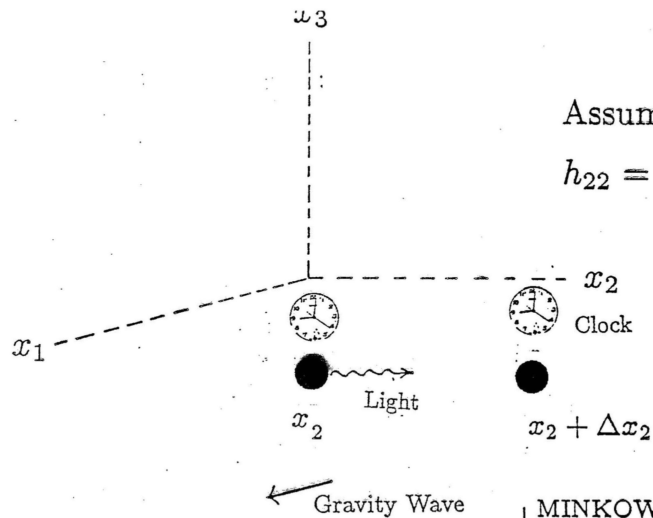
Gravitational wave spectrum



What else has changed over the past 100 years

- Ability to measure small strains
  - Frequency and amplitude stabilized high power lasers operating at the quantum limit
  - Tailored quantum states to modify momentum and phase noise – "squeezed " light
  - Low loss optics – a few ppm
- Ability to reduce stochastic forces
  - Feedback and feed forward from inertial references – reduction in seismic noise
  - Low mechanical loss suspensions and optics with few mechanical resonances in the gravitational wave band - reduction of thermal noise by noise "free" feedback damping.
- Ability to diagnose instrument performance by correlation techniques – digital control and filtering
- Ability to reduce 10^{14} bytes of data per year in the data analysis for gravitational wave signatures

Timing light in the gravitational wave



Assume + Polarization

$h_{22} = h \sin(kx_1 - \omega t)$

MINKOWSKI

$$\Delta s^2 = 0 = c^2 \Delta t^2 - \left(1 + h \sin(kx_1 - \omega t)\right) \Delta x_2^2$$

LIGHT RAY

Let $\quad \Delta t \ll \frac{1}{\omega} \qquad h \ll 1$

$$c\,\Delta t \cong \left(1 + \frac{h}{2} \sin(kx_1 - \omega t)\right) \Delta x_2$$

INFERRED DISTANCE BETWEEN POINTS

$$\frac{\delta(c\,\Delta t)}{\Delta x_2} = \frac{h}{2} \sin(kx_1 - \omega t)$$

Time Dependent **Strain**

$$\frac{\Delta l}{l} = \frac{h}{2}$$

The Measurable Quantity